Problem Set 8
Due Friday, June 6.

Mathematical Logic

Math 114L, Spring Quarter 2008

1. (10 pt.) Suppose f is a 2-place function symbol. Is v_2 substitutable for v_0 in $\forall v_2 f v_2 v_2 = v_0$? If so, what is $(\forall v_2 f v_2 v_2 = v_0)^{v_2}_{v_0}$?

2. (10 pt.) Let x be a variable, and let t be a term. Show that for every formula φ:

   (a) x is substitutable for x in φ, and $\varphi^x_x = \varphi$;

   (b) if $x \notin \text{free}(\varphi)$, then t is substitutable for x in φ, and $\varphi^t_x = \varphi$.

   (Proceed by induction on the construction of φ.)

3. (20 pt.) Problem 3 in Section 2.4 of the textbook.

4. (20 pt.) Problem 4 in Section 2.4 of the textbook.

5. (20 pt.) Problem 13 in Section 2.4 of the textbook.

6. (20 pt.) Problem 14 in Section 2.4 of the textbook.