Problem Set 6
Due Friday, May 16.

Mathematical Logic

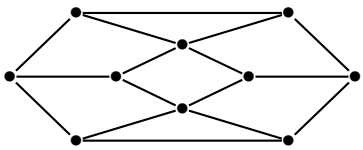
Math 114L, Spring Quarter 2008

1. (20 pt.) Exercise 17 (a) in Section 2.2 of the textbook.
2. (20 pt.) Write down sentences (in the language introduced in Problem Set 4, Exercise 7) that express the axioms for vector spaces over $\mathbb{Q}$.
3. (20 pt.) Consider a first-order language containing the 2-place predicate symbol R. Write down an S-formula which expresses that R is the graph of a 1-place function. (The *graph* of a 1-place function $f\colon A \to B$, where A and B are sets, is the set $\{(a,b) \in A \times B : b = f(a)\}$.)
4. (40 pt.) Consider the first-order language with a single 2-place predicate symbol E. A structure $\mathfrak{G} = (G, E^{\mathfrak{G}})$ is called an undirected **graph** if

$$\mathfrak{G} \models \forall x \neg Exx, \quad \mathfrak{G} \models \forall x \forall y (Exy \leftrightarrow Eyx).$$

The elements of G are called the **vertices** of $\mathfrak{G}$. We visualize a graph $\mathfrak{G} = (G, E^{\mathfrak{G}})$ by thinking of its vertices as points in the plane, with vertices a and b satisfying $(a,b) \in E^{\mathfrak{G}}$ connected by a line (called an **edge** of $\mathfrak{G}$).

(a) Describe

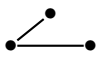


as a structure $\mathfrak{G}$.

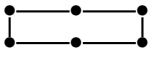
(b) Prove or disprove: for every assignment s for $\mathfrak{G}$ as in (a) we have $\mathfrak{G} \models \varphi[s]$, where φ is the formula

$$(Exy_1 \wedge Exy_2 \wedge Exy_3 \wedge Exy_4 \rightarrow \\ y_1 = y_2 \vee y_1 = y_3 \vee y_1 = y_4 \vee y_2 = y_3 \vee y_3 = y_4)$$

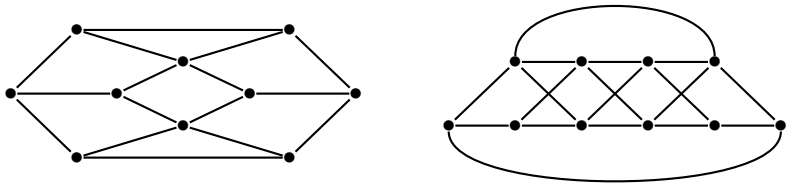
(c) Show that the following (undirected) graphs are not isomorphic (see p. 97 in the textbook):

i. $\mathfrak{A} =$  and $\mathfrak{B} =$ 

ii. $\mathfrak{A} =$  and $\mathfrak{B} =$ 

iii. $\mathfrak{A} =$  and $\mathfrak{B} =$ 

(d) Are the following graphs isomorphic?



5. (20 pt. extra credit.) Exercise 17 (b) in Section 2.2 of the textbook.