Problem Set 7
Due Friday, November 17.

Algebra

Math 110A, Fall Quarter 2017

1. Do problems 4.2.12, 4.2.13 in the textbook.
2. Do problems 4.3.4, 4.3.14, 4.3.22 in the textbook.
3. Do problems 4.4.8, (d) (f), 4.4.10, 4.4.16 in the textbook.
4. Recall that an **automorphism** of a ring R is a ring isomorphism $R \to R$. Let now R be an integral domain.
   (a) Describe all the automorphisms of $\mathbb{Z}$.
   (b) (5 pts. extra credit!) Show that given $a, b \in R$, where a is a unit, there is a unique automorphism φ of $R[X]$ such that $\varphi(r) = r$ for all $r \in R$ and $\varphi(X) = aX + b$. What is the inverse φ^{-1} of φ?
   (c) (5 pts. extra credit!) Show that conversely, for every automorphism φ of $R[X]$ such that $\varphi(r) = r$ for all $r \in R$ there exist $a, b \in R$, where a is a unit, such that $\varphi(X) = aX + b$.
   (d) Now describe all the automorphisms of $\mathbb{Z}[X]$. (Hint: use (a), (b), and (c).)