

M05N Problems (Last Assignment, due 15/01/06) Revised 28/12/05

Problems 1-9 are worth 10 points each. Problems 10-13 are worth 15 points each.¹ Choose *any nine or ten* problems to solve; if you turn in solutions to more than ten, only the first ten will be corrected. Please work independently. I'll be glad to answer questions (by e-mail if necessary). You may want to consult the references listed on the class homepage.

1. In **CPC**, derive the $\neg E_i$ rule of **IPC**.
2. Define the Gödel-Gentzen negative translation $A \mapsto A^g$ for formulas A of $\mathcal{L}(\mathbf{Pd})$, and outline proofs of
 - (a) $\vdash_{\mathbf{CQC}} (A \leftrightarrow A^g)$.
 - (b) $\Gamma \vdash_{\mathbf{CQC}} A$ if and only if $\Gamma^g \vdash_{\mathbf{IQC}} A^g$, where $\Gamma^g = \{C^g \mid C \in \Gamma\}$.
3. Outline proofs of *Glivenko's Theorems* for all formulas A of the appropriate languages:
 - (a) $\vdash_{\mathbf{CPC}} A$ if and only if $\vdash_{\mathbf{IPC}} \neg\neg A$.
 - (b) $\vdash_{\mathbf{CQC}} A$ if and only if $\vdash_{\mathbf{IQC}+\text{DNS}_0} \neg\neg A$, where DNS_0 is the schema:

$$\forall x \neg\neg B(x) \rightarrow \neg\neg \forall x B(x).$$
4. (Exercise 4.9 in the revised notes.) Prove that for all formulas A and B of $\mathcal{L}(\mathbf{HA})$:
 - (a) $\vdash_{\mathbf{HA}} \neg A \leftrightarrow (A \rightarrow (1 = 0))$.
 - (b) $\vdash_{\mathbf{HA}} (A \vee B) \leftrightarrow \exists x [((x = 0) \rightarrow A) \ \& \ (\neg(x = 0) \rightarrow B)]$, where x is not free in A or in B .
5. Prove that the relation $\mathcal{R}(\alpha) \equiv (j_0(\alpha) = j_1(\alpha))$ is not continuous in the Baire topology on ${}^\omega\omega$.
6. Exercise 5.3 in the revised notes.
7. Exercise 5.5 in the revised notes.
8. Exercise 5.7 in the revised notes.
9. For each of the following continuous functions on Baire space, give a neighborhood function σ :
 - (a) $F_1(\alpha) = \alpha(\alpha(0) + \alpha(\alpha(0)))$.
 - (b) $F_2(\alpha) = (j_0(\alpha))(j_1(\alpha))$.
10. Exercise 4.10* in the revised notes.
11. Exercise 4.17* in the revised notes.
12. Exercise 5.4* in the revised notes.
13. Exercise 5.6* in the revised notes.

Good Luck!

¹A total of 180 points for *all* the exercises will give you a perfect 10 for the course, so here you have a chance to make up for lost points on the earlier exercises.