Math 3A Final Practice Problems

1. Sequences
Let the sequence $\{a_n\}$ be defined explicity by $a_n = 2^n + 1$ on the domain $n = 0, 1, 2, \ldots$.

a. Rewrite $\{a_n\}$ as a recursively defined sequence.

b. Rewrite $\{a_n\}$ as an explicitly defined sequence on the the domain $n = 1, 2, 3, \ldots$.

2. Continuity
What does it mean for a function $f(x)$ to be continuous at $x = c$? What does it mean for a function to be continuous everywhere?

3. Sandwich Theorem
When we used the sandwich theorem to compute

$$\lim_{x \to 0} \frac{\sin x}{x}$$

we used some tricky geometry to get the correct inequality. Why didn't we use the fact that $-1 \leq \sin(x) \leq 1$?

4. Mean Value Theorem
Use the Mean Value Theorem to show that $f(x) = x^2 - x + 2$ has a critical point somewhere in the interval $(0, 1)$.

5. Some limits
Compute the following limits

a.

$$\lim_{h \to 0} \frac{2^{5h} - 1}{h}$$

b.

$$\lim_{x \to \infty} \frac{3x - 2x^3}{2x^2 + 4}$$

c.

$$\lim_{x \to \infty} e^{-x}$$

6. Some derivatives
Compute the derivative of each of the following functions

a.

$$3^{\sin(x)}$$

b.

$$\tan(\cos(x))$$

c.

$$\log_2(x)$$

7. L'Hospital's rule Compute the following limits.

a.

$$\lim_{x\to\infty} \frac{x^2+2}{3x-x^3}$$

b.

$$\lim_{x\to\infty} [\ln x]^{1/x}$$

8. Antidifferentiation
Consider the differential equation $\frac{dy}{dx} = \frac{1}{x}$.

a. Find the general solution to this differential equation.

b. Find the solution that fits the initial condition $y(1) = 1$.

9. More Antiderivatives Antidifferentiate the following functions:

a.

$$\frac{1}{1+x}$$

b.

$$2e^{2x}$$

c.

$$\cos(\frac{x}{5})$$