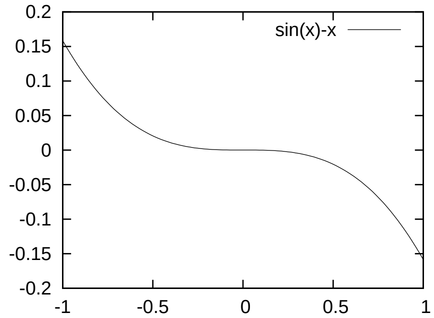
Solutions to Homework problems 3

3.5.4



We see from the graph that $f(x) = \sin x - x$ is positive at $x = -1$ but negative at $x = 1$ so the intermediate value theorem tells us that there must exist a $c \in (-1, 1)$ such that $f(c) = 0$, that is, $x = c$ is a solution to $\sin x = x$ in that interval. (Of course we know that $c = 0$ is a solution to this problem, but the intermediate value theorem only tells us that c lies in the interval $(-1, 1)$.

3.5.6 Let $f(x) = x - \cos x$. Then we note that $f(0) = 0 - \cos 0 = -1 < 0$ and $f(1) = 1 - \cos 1 > 0$ (since $\cos x \leq 1$ for all $x \in \mathbf{R}$ and $\cos 1 \neq 1$). Thus the intermediate value theorem guarantees that $f(x) = 0$ has a solution in the interval $(0, 1)$. If x is that solution then $\cos x = x$.

3.5.8 Let $f(x) = x - \cos x$. We make a table of values similar to the one on page 151 in the book.

a	$\frac{a+b}{2}$	b	$f(a)$	$f(\frac{a+b}{2})$	$f(b)$
0.0	0.5	1.0	-1.0	-0.3776	0.4597
0.5	0.75	1.0	-0.3776	0.0183	0.4597
0.5	0.625	0.75	-0.3776	-0.1860	0.0183
0.625	0.6875	0.75	-0.1860	-0.0853	0.0183
0.6875	0.71875	0.75	-0.0853	-0.0339	0.0183
0.71875	0.734375	0.75	-0.0339	-0.0079	0.0183
0.734375	0.7421875	0.75	-0.0079	0.0052	0.0183

From this we can see that $x = 0.74$ is a solution to $\cos x = x$ up to two decimal places.

4.1.14 Let $y = -2x^2$. Then the derivative of y with respect to x evaluated at

$x = 1$ can be calculated as follows:

$$\left.\frac{dy}{dx}\right|_{x=1} = \lim_{h\to 0}\frac{-2(1+h)^2 - (-2(1)^2)}{h} = \lim_{h\to 0}\frac{-2(1)^2 - 4\cdot 1h - 2h^2 + 2(1)^2}{h}$$
$$= \lim_{h\to 0}\frac{-4h - 2h^2}{h} = \lim_{h\to 0}(-4-2h) = -4$$

We note that when $x = 1$, $y = -2x^2$ gives $y = -2$ so $(1, -2)$ is on the graph of $y = -2x^2$.

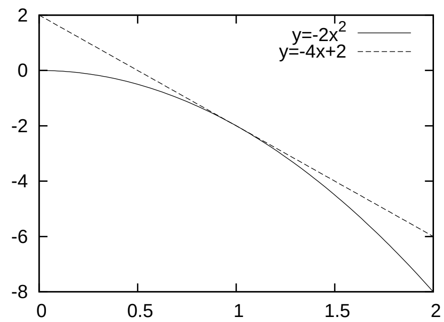
The tangent line to the graph at $(1, -2)$ is given by

$$y - (-2) = -4(x-1)$$

or

$$y = -4x + 2$$

Plotted in the same coordinate system these graphs look as follows



4.1.18 We calculate the derivative of $f(x) = \dfrac{1}{x+1}$ as follows:

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h} = \lim_{h\to 0}\frac{\frac{(x+1)-(x+h+1)}{(x+h+1)(x+1)}}{h}$$
$$= \lim_{h\to 0}\frac{-h}{h(x+h+1)(x+1)} = \lim_{h\to 0}\frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}.$$

In all of these calculations it is of course assumed that $x \neq -1$.

4.1.20 If $y = 3/x$ then $\dfrac{dy}{dx} = -\dfrac{3}{x^2}$ so $\left.\dfrac{dy}{dx}\right|_{x=3} = -\dfrac{3}{3^2} = -\dfrac{1}{3}$. Therefore, the equation for the tangent line at $(3, 1)$ is

$$y - 1 = -\frac{1}{3}(x-3)$$

or

$$y = -\frac{x}{3} + 2.$$

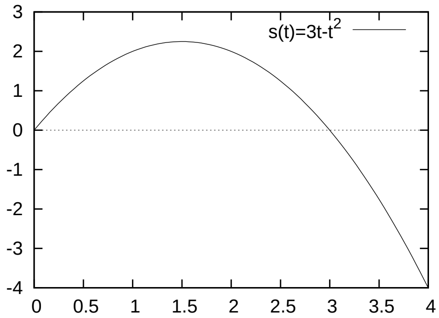
4.1.22 If $y = x^2 - 3x + 1$ then $y' = 2x - 3$ so $y'(2) = 2 \cdot 2 - 3 = 1$. Therefore, the equation for the tangent line at $(2, -1)$ is

$$y - (-1) = 1(x - 2)$$

or

$$y = x - 3.$$

4.1.34 Let $s(t) = 3t - t^2$. We plot s as a function of t.



From the graph we see that at time 0 the particle is at position $s = 0$ and it visits the position $s = 0$ again at time 3.

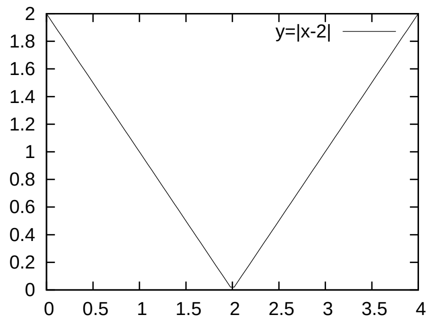
It goes as far to the right (up on the graph) as $s = 9/4 = 2.25$ and as time goes on the particle goes as far as we want to the left ($\lim_{t\to\infty} s(t) = -\infty$).

The velocity of the particle is positive for time less than 1.5 and it is negative for time greater than 1.5.

The velocity of the particle is given by the derivative of s, $s'(t) = 3 - 2t$.

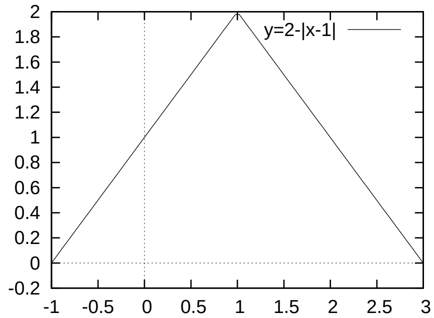
The velocity of the particle is equal to $1m/s$ at time 1.

4.1.46 $y = |x - 2|$



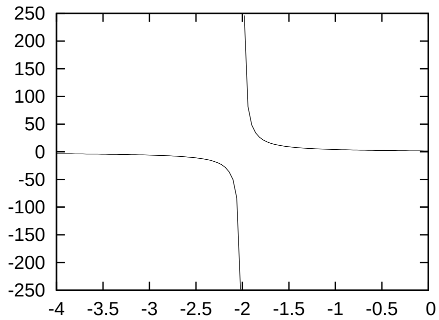
The function is differentiable for all $x \in \mathbf{R}$ except $x = 2$.

4.1.46 $y = 2 - |x - 1|$



The function is differentiable for all $x \in \mathbf{R}$ except $x = 1$.

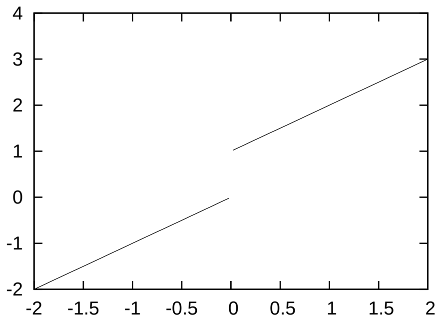
4.1.52 $y = \frac{3-x}{2+x}$



The function is differentiable for all $x \in \mathbf{R}$ except $x = -2$.

4.1.56

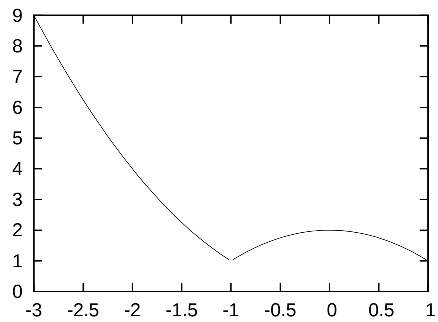
$$f(x) = \begin{cases} x & \text{for } x \le 0 \\ x + 1 & \text{for } x > 0 \end{cases}$$



The function is differentiable for all $x \in \mathbf{R}$ except $x = 0$.

4.1.58

$$f(x) = \begin{cases} x^2 & \text{for } x \leq -1 \\ 2 - x^2 & \text{for } x > -1 \end{cases}$$



The function is differentiable for all $x \in \mathbf{R}$ except $x = -1$.