Homework 8

Due Wednesday, May 29.

(1) Prove that the commutator subgroup $[G, G]$ of a group G is (i) a normal subgroup of G, and (ii) the smallest normal subgroup H of G so that G/H is abelian.

(2) Verify that the following diagram for the pair (X, A) is commutative:

$$\begin{array}{ccccccccc}
H^{\Delta}_{n+1}(X,A) & \longrightarrow & H^{\Delta}_{n}(A) & \longrightarrow & H^{\Delta}_{n}(X) & \longrightarrow & H^{\Delta}_{n}(X,A) & \longrightarrow & H^{\Delta}_{n-1}(A) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
H_{n+1}(X,A) & \longrightarrow & H_{n}(A) & \longrightarrow & H_{n}(X) & \longrightarrow & H_{n}(X,A) & \longrightarrow & H_{n-1}(A)
\end{array}$$

(3) Let X be a Δ-complex and X^k be its k-skeleton. Then prove that $H_n(X^k, X^{k-1}) = 0$ if $k \neq n$ and is the free abelian group generated by the k-dimensional simplices $\sigma_\alpha : \Delta^k \to X$ of the Δ-complex, if $k = n$.

(4) Prove the Five Lemma (without consulting any references).

(5) Hatcher, Section 2.1, Exercise 27.

(6) Hatcher, Section 2.2, Exercises 1, 2, 4, 8, 27.

(7) Hatcher, Section 2.3, Exercise 1.