Homework 7

Due Wednesday, May 22.

(1) Prove that the barycentric subdivision operator $S : LC_m(\Delta^n) \to LC_m(\Delta^n)$ defined inductively by $S\lambda = b_\lambda(S\partial\lambda)$ and $S[w_0] = [w_0]$ coincides with the following definition:

$$S[w_0, \dots, w_m] = \sum_s (-1)^s s \left(\left[\left(\frac{1}{m+1}, \dots, \frac{1}{m+1} \right), \left(0, \frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m} \right), \left(0, 0, \frac{1}{m-1}, \dots, \frac{1}{m-1} \right), \dots, (0, \dots, 0, 1) \right] \right).$$

Here $(t_0, \dots, t_m)$ refers to $\sum_i t_i w_i$, the sum ranges over all permutations s of the coordinates, and $(-1)^s$ is 1 if s is even (a product of an even number of transpositions) and -1 if s is odd (a product of an odd number of transpositions).

(2) Suppose $f, g : (X, A) \to (Y, B)$ are homotopic as maps of pairs. Then prove that $f_* = g_* : H_n(X, A) \to H_n(Y, B)$.

(3) Given a triple (X, A, B), i.e., $X \supset A \supset B$, prove that there is a long exact sequence for the triple:

$$\dots \to H_n(A, B) \to H_n(X, B) \to H_n(X, A) \to \dots.$$

(4) Hatcher, Section 2.1, Exercises 16, 17, 18, 20, 21, 22.

(5) Hatcher, Section 2.2, Exercises 32, 34.