Homework 4

Due Monday, April 29.

(1) Read the section "Application to Cell Complexes" (pp. 50-52) in Hatcher. Pay special attention to Proposition 1.26.
(2) Hatcher, Section 1.2, Exercise 6.
(3) Prove that the cell complex X obtained by attaching an n-cell D^n to a 0-cell $D^0 = \{p\}$ via the attaching map $\phi : \partial D^n \to D^0 = \{p\}$, $\phi(x) = p$, is homeomorphic to $Y = S^n$ defined as $\{|x| = 1\} \subset \mathbf{R}^{n+1}$.
(4) The *real n-dimensional projective space* $\mathbf{RP}^n$ is the quotient of $S^n = \{|x| = 1\} \subset \mathbf{R}^{n+1}$ by the equivalence relation $x \sim -x$. Using the cell complex structure of $\mathbf{RP}^n$ described in Example 0.4, compute $\pi_1(\mathbf{RP}^n)$ for all $n \geq 2$.
(5) Prove that if X is both locally path-connected and semi-locally simply-connected, then for all $x \in X$ and an open set $V \ni x$, there is a path-connected open set $V \supset U \ni x$ such that the inclusion map $\pi_1(U) \to \pi_1(X)$ is trivial.
(6) Prove that if $\pi : \tilde{X} \to X$ is a covering space and X is connected, then the cardinality of $\pi^{-1}(x)$ does not depend on $x \in X$.
(7) Hatcher, Section 1.3, Exercises 3, 4, 9, 14, 17, 18.