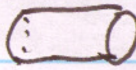
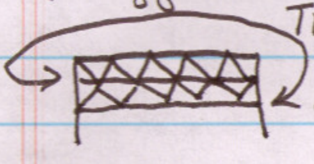
Homework III : Math. 120B Spring 2008

1. Show that for a triangulation of a cylinder (including two circular boundary edges ) $F - E + V = 0$.
(Suggestion: Cap off ends with hemispheres to get a sphere and see how this changes $F - E + V$).

2. Show that, for a cylinder as in problem 1, $\int K \, d(\text{Area}) = 0$ if boundary curves are geodesics

(Suggestion: Use small triangulation of this form


and add up "excess"es from them all).

3. Show, as in problem 2, that if boundary curves are maybe not geodesics, then $\int K \, d(\text{area})$ may be nonzero, and find an expression for $\int K \, d(\text{area})$ involving the geodesic curvatures of the boundary curves.

4. Check your formula of problem 3 for the cylinder $\{(x, y, z) : x^2 + y^2 + z^2 = 1, \ |z| < \alpha\}$ where $0 < \alpha < 1$.

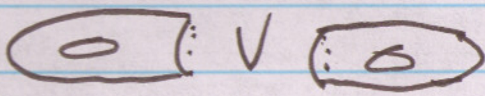
5. Verify your formula of problem 3 for the more general case of a surface of revolution
$S(u, v) = (x(u), \ y(u) \cos v, \ y(u) \sin v)$ $\quad a \leq u_0 \leq b$.

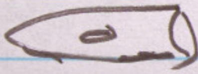
4. Noting that, by Gauss–Bonnet Theorem, $F - E + V$ is independent of triangulation of a given surface, find $F - E + V$ (Euler characteristic) for

(a) torus 

(b) "two-hole" surface 

(Hint: Triangulate so it looks like



and figure out what $F-E+V$ is for  by capping off the by a hemisphere. to get a torus).

(c) Inductively, find $F - E + V$ for g-hole surface.

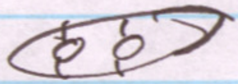
5. Verify your formula of problem 3 for the case of a surface of revolution

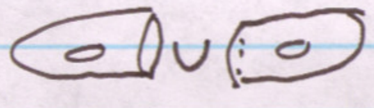
$$S(u,v) = (x(u),\ y(u)\cos v,\ y(u)\sin v),$$

the portion between $u = u_0$ and $u = u_1$, $u_0 < u_1$. (You need to compute k_g for the boundary curves).

6. Recall that, by the Gauss-Bonnet Theorem $F - E + V$ (the "Euler characteristic") is independent of triangulation of a given surface. Find the Euler characteristic of

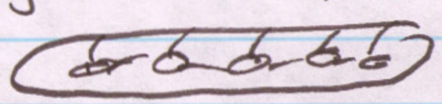
(a) the torus

(b) the surface of genus 2 

(Suggestion: Think of it as 

and figure out $F-E+V$ for each part by capping off with a hemisphere and then figure out what happens when the two pieces are glued together)

(c) Inductively, find $F-E+V$ for the g-hole surface ("genus g surface")

 g hole $g \geq 2$

7. What is the Euler characteristic of a disc with k smaller discs removed ($k \geq 0$) from its interior (so there are $(k+1)$ boundary curves, the outer circle and k smaller inner circles)? What kind of curvature/geodesic curvature formula do you get in this case? Illustrate the formula for a disc in the plane with k discs removed.

8. Illustrate problem 7 for the upper hemisphere of the unit sphere with k discs removed.

9. What is the geodesic curvature of the circle of Poincare radius R around (0,0) in the Poincare unit disc. [Compute this from ~~Sss~~ formula etc.].

10. Check (using the F − E + V = 1 for a disc with 0 discs removed) that the problem 7 formula works out in this case (you need to find the area of the disc of radius R: use G(r, θ) coordinates!)