Workshop on Thin Films and Fluid Interfaces

January 30$^{\text{th}}$ – February 2$^{\text{th}}$, 2006

supported by NSF #0244498 FRG-Collaborative Research Grant:
New Challenges in the Dynamics of Thin Films and Fluid Interfaces
Held at the Institute for Pure and Applied Mathematics, UCLA

This workshop brings together an international group of researchers working on experimental, analytical, and computational problems related to thin films and fluid interfaces. Problems of current interest include solid-liquid-vapor interfaces, moving contact lines, and surface tension effects.

Conference organizers

- Robert Behringer, Duke University
- Andrea Bertozzi, UCLA
- Michael Shearer, NCSU
- Dejan Slepcev, UCLA
- Thomas Witelski, Duke University

Monday, January 30, 2006

8:45 am-9:00 am	Bertozzi	Welcome and opening remarks
9:00 am-9:30 am	Craster	*Flow down a vertical fibre*
9:30 am-10:00 am	Matar	*Dynamics, stability and pattern formation in surfactant-driven thin liquid films*
10:00 am-10:30 am	Coffee break	
10:30 am-11:30 am	Cazabat	*A short story of drops*
11:30 am-1:30 pm	Lunch	
1:30 pm-2:00 pm	Ben Amar	*Darcy versus Stokes: the case of suction*
2:00 pm-2:30 pm	Shen	*Evaporation and rheological effects on Sol-gel coating*
2:30 pm-3:00 pm	Witelski	*Fingering flows of Marangoni-driven thin films*
3:00 pm-3:30 pm	Coffee break	
3:30 pm-4:00 pm	Grün	*Thin-Film Flow Influenced by Thermal Fluctuations*
4:00 pm-4:30 pm	Giacomelli	*Microscopic and effective spreading rates for shear-thinning droplets*
4:30 pm-5:00 pm	Slepčev	*Coarsening in thin liquid films*
5:00 pm-7:00 pm	Conference reception	

Tuesday, January 31, 2006

9:00 am-9:30 am	Kondic	*Instability of a fluid strip*
9:30 am-10:00 am	Bertozzi	*Shocks in driven liquid films*
10:00 am-10:30 am	Coffee break	
10:30 am-11:30 am	Homsy	*Some heat pipe problems*
11:30 am-2:00 pm	Lunch & Poster session	
	Aryafar	*An interferometric study of spreading non-Newtonian fluids*
	Cook	*Behavior of Shock Solutions in Particle-Laden Thin Films*
	Cubaud	*Viscous Buckling in Microfluidics*
	Bertozzi	*Computing Fourth Order PDEs on Implicit Surfaces*
	Guena	*Spreading of nematic liquid crystal on hydrophobic and hydrophilic surfaces*
	Guena	*Evaporating mixtures drops in the situation of complete wetting*
	Knüpfer	*Regularity of the free boundary for a thin film equation*
	Lauga	*Measuring slip using diffusion*
	Lu	*A Diffuse Interface Model for Electrowetting Droplets In a Hele-Shaw Cell*
	Mukhopadhyay	*Precursors and Contact Line Dynamics in Thermally Driven Marangoni flows*
	Nierop	*Reactive spreading of oil on water*

	Ristenpart	*Coalescence of Spreading Droplets on a Wettable Substrate*
	Roper	*Surfactant driven wall-climbing in B. subtilis biofilms*
	Rump	*Coarsening in a droplet model*
	Schmidt	*Length-scale selection in viscous entrainment of stratified fluids*
	Shklyaev	*Stability of evaporating thin liquid films*
	Ulusoy	*An entropy dissipation entropy estimate for a thin film type equation*
	Ward	*Electrohydrostatic Wetting of Poorly-Conducting Liquids*
	Wey	*An Experimental Approach to Understanding Slurry Flows*
	Yang	*Steady 3D thermocapillary flows and dryout inside a V-shaped wedge*
	Yochelis	*Spontaneous motion of droplets on freezing or melting substrates: A novel mechanism*
	Zhang	*Marangoni Convection Experiment in Binary Mixtures*
2:00 pm-2:30 pm	Stebe	*Surfactant effects on drop detachment*
2:30 pm-3:00 pm	Daniels	*Starbursts and Wispy Drops: Surfactants Spreading on Gels*
3:00 pm-3:30 pm	Kavehpour	*An interferometric study of spreading non-Newtonian fluids*
3:30 pm-4:00 pm	Coffee break	
4:00 pm-4:30 pm	Shearer	*Thin film equations for fluid motion driven by surfactants*
4:30 pm-5:00 pm	Behringer	*Thin Film Flows with Maragoni Driving*

Wednesday, February 1, 2006

9:00 am-9:30 am	Thiele	*Structure formation in thin liquid films*
9:30 am-10:00 am	Braun	*Models for the Tear Film during a Blink*
10:00 am-10:30 am	Coffee break	
10:30 am-11:00 am	Wagner	*Slippage and viscoelastic effects for dewetting films*
11:00 am-11:30 am	Münch	*Dewetting laws, sharp interface models & stability analysis for films with slippage*
11:30 am-1:30 pm	Lunch	
1:30 pm-2:00 pm	Bernoff	*Domain Relaxation in Langmuir Films*
2:00 pm-2:30 pm	Glasner	*The quasistatic model for droplet spreading*
2:30 pm-3:00 pm	Nadim	*The Bretherton problem in a power-law fluid*
3:00 pm-4:00 pm	Coffee break	
4:00 pm-5:00 pm	Davis (Joint Applied Mathematics Colloquium)	
		Dynamics and Solidification of Metallic Foams
5:30 pm-7:30 pm	Conference Dinner Hacienda Room, Faculty Club, UCLA	

Thursday, February 2, 2006

8:30 am-9:00 am	Miksis	*Nonlinear Dynamics of a Two-dimensional Viscous Drop under Shear Flow*
9:00 am-9:30 am	Oron	*Longwave Marangoni instability of a binary-liquid film with the Soret effect*
9:30 am-10:00 am	Limat	*Contact line morphology for flows down an incline*
10:00 am-10:30 am	Coffee break	
10:30 am-11:30 am	Quere	*Stressed interfaces: from bubbly drops to jet impact*
11:30 am-1:30 pm	Lunch	
1:30 pm-2:00 pm	Lowengrub	*A level-set method for interfacial flows with surfactant*
2:00 pm-3:30 pm	Lab tours and discussion	

Conference Proceedings The organizers, in conjunction with the new journal Applied Mathematics Research eXpress is putting together a proceedings of the workshop. We are soliciting papers from workshop speakers and participants that address one of the following areas:

(a) New mathematical results connected with the scientific focus of the workshop. These results could include computational, asymptotic, analytical, or modelling approaches. Also new computational algorithms are welcome as well.

(b) Results from physical experiments for which a full scientific understanding is lacking and for which the authors believe some mathematical theory is needed. In this case we ask the authors to (1) document the physical results including data from experiments (this could be previously published data provided that the publication is carefully cited) and (2) provide a careful review of the state of the mathematical literature as it pertains to the problem of interest along with some explanation of what kind of results you would like to see in the future associated with the problem.

(c) Experimental confirmation of a recent mathematical theory for which the experimental data is new and has not been published. It is possible to combine ideas from (b) and (c) and we request that the authors clearly present how the mathematics connects to the physics (or might connect) in both cases.

While we are not suggesting any particular page length for the articles, we ask the authors to use good judgement in having the page length be appropriate for the nature of the results presented in the paper. Both letter-length and full length articles will be considered. All speakers are invited to submit articles. In the case of student and postdoc participants, we ask that they consider submitting an article in collaboration with a senior mentor or with approval of the manuscript by a senior mentor prior to submission. All manuscripts will be peer reviewed according to the guidelines of AMRX. Participation in the workshop is not an automatic guarantee of acceptance of your manuscript for publication in AMRX. Deadline for submitting manuscripts is July 1, 2006.

Titles and Abstracts for Talks and Posters[1]

1 Monday, January 30, 2006

- **Richard V. Craster**, Department of Mathematics, Imperial College London

Flow down a vertical fibre

We consider a model axisymmetric flow, somewhat akin to viscous jet or thread breakup but with an interior rigid core; the fluid flows, under the influence of gravity, down the exterior of a rigid circular fibre. The modelling is similar to that usually utilized for thin-films, but adapted to axisymmetry. This flow is accompanied by rich dynamics, even at low Reynolds numbers, that manifests itself via the formation of droplets, or beads, driven by a Rayleigh mechanism modulated by the presence of gravity. These droplets propagate down the fibre and undergo coalescence with preceding droplets. We derive an evolution equation for the interface in the long-wavelength approximation, for low flow rates, which captures the flow characteristics of the system. Analytical and numerical solutions of the evolution equation yield information regarding the shape and propagation speeds of the droplets, which is in good agreement with available experimental data. At higher flow rates, the fibre problem becomes similar to the classical falling film problem, but with azimuthal curvature. It is possible to generate evolution equations for this case too; solutions of these equations will be presented if time permits.

- **Omar K. Matar**, Department of Chemical Engineering, Imperial College London

Dynamics, stability and pattern formation in surfactant-driven thin liquid films

The presence of surfactant in thin liquid films can often have a dramatic influence on the flow. We illustrate this by focusing on the spreading of surfactant on thin liquid layers for concentrations both above and below the critical micelle concentration. We use lubrication theory to develop a model for the dynamics of the liquid film thickness and the concentration of surfactant, which is present in the form of monomers that can exist both in the bulk and at the interface, as well as micellar aggregates. This model accounts for Marangoni stresses, capillarity, surface and bulk diffusion, solubility, sorption kinetics, micelle formation and breakup, micelle size and the nonlinearity of the surfactant equation of state. We show that the spreading process is accompanied by a fingering instability which leads to non-uniform coating. We also demonstrate that the characteristics of the unperturbed flow and of the fingering patterns depend critically on the mass of deposited surfactant and on a parameter which characterises the affinity of the surfactant to form micelles. Our numerical and analytical results are in good agreement with experimental trends.

- **Anne Marie Cazabat** (G. Guéna, C. Poulard, A. M. Cazabat), Collège de France

A short story of drops

It is well known that discussing the spontaneous spreading of drops on solid substrates raises the paradox of the moving contact line. We are interested in complete wetting situations when a second process interferes with the spontaneous spreading of the drops. For volatile liquids, in the case of a stationary, diffusion-controlled evaporation process, the evaporation flux diverges at the contact line. During the receding motion of pure liquids, well defined power laws are observed for the drop radius, with exponents close to 0.5. The behaviour of the contact angle is more complex, noticeably at late times. The occurrence of thermal gradients, or concentration gradients in liquid mixtures, may drastically change the dynamics. For complex fluids like liquid crystals, with nematic elasticity, unusual spreading laws are observed, together with convex interface profiles close to the contact line. Moreover, contact line instabilities develop on hydrophilic substrates, in some cases interfering with the anchoring defects present on the substrate. Experiments are presented and available models, developed in collaboration with M.Ben Amar, are summarised. The volatile liquids used are alkanes and their mixtures, water, and PDMS oligomers. The liquid crystal is the nematic phase of the short-chain cyanobiphenyl 5CB.

[1]Each listed by presenter, co-authors given in parentheses and/or in abstracts.

- **Martine Ben Amar**, Laboratoire de Physique Statistique de l'Ecole Normale Supérieure

Darcy versus Stokes: the case of suction

Suction of a circular viscous blob can produce a contour instability. For Darcy's flow, it is known that when air pushes oil, the boundary does not remain circular (Bataille, Paterson). Suction of a viscous blob induces such flow and one can calculate the dispersion relation to derive the number of oscillations of the boundary , at early time. This dispersion relation can be tested in a lifting Hele-Shaw cell where the number of oscillations at time ($t = 0$) is few hundred. Nevertheless, a discrepancy remains between analytical predictions and measurements. Stokes flow is less known and one can wonder if such instability occurs. Moreover, bidimensional Stokes flow are really difficult to realize in a laboratory. I will discuss a very recent experiment of cholesterol pumping by high density lipo-proteins in an inhomogeneous vesicle. The suction occurs in the cholesterol-enriched membrane active zones called rafts. The HDL are added near unilamellar vesicles (GUV) under microscope and we observe the complete disappearance of the raft-like domain in few minutes. An instability with undulations of the boundary may occur for strong enough pumping. The suction induces in the GUV membrane a lateral hydrodynamic flux of matter towards the raft-like domain at the origin of the instability. We explain these few oscillations by an interfacial instability occurring in the radial geometry for inner Stokes flow whatever the sign of the viscosity contrast. We predict the number of undulations of the contour, which is compared to our experimental results. This work shows the significance of viscous hydrodynamics for the description of lateral displacements in the cell membrane, an area of membrane physics much less explored than elastic shape-deformations.

Work in collaboration with D. Bonn (Ecole Normale supérieure) and the experimental group of Miglena Angelova (Université Pierre & Marie Curie)

- **Amy Q. Shen** (C. Lee and A. Shen), Department of Mechanical & Aerospace Engineering, Washington University

Evaporation and rheological effects on Sol-gel coating

Evaporation induced self-assembly (EISA) accompanied with dip-coating is shown to be an easy and effective method to fabricate mesoporous silica thin films. Dip-coating is favored in fabrication of films due to easily achievable uniform film thickness. With a non-volatile Newtonian solution, the upward moving flux balances the downward gravitational drainage, and the fluid motion can easily be predicted. However with an EASA solution, which is composed of inorganic sol-gel, volatile solvent, and micelle forming surfactant, the fluid behavior is much complex and rheological parameters need to be considered. The film thinning by evaporation causes an increase in sol and surfactant concentration. The surfactant concentration gradient influences the local shear viscosity; hence, the viscosity term needs to be addressed in the modeling of an EISA fluid in dip-coating. Both experimental and theoretical results will be presented in this talk.

- **Thomas Witelski** (J. Sur, T. Witelski, R. Behringer), Department of Mathematics, Duke University

Fingering flows of Marangoni-driven thin films

We give experimental and computational results showing the existence of finite-amplitude fingering solutions in a thin film flow of a viscous fluid driven by thermal Marangoni stresses. Using well controlled experiments, spatially periodic perturbations to the contact line of an initially uniform thin film flow are shown to lead to the development of steady-profile two-dimensional traveling wave fingers. Using an infrared laser and scanning mirror, we impose thermal perturbations with known wavelength to an initially uniform advancing fluid front. As the front advances in the experiment, we observe convergence to fingers with the initially prescribed wavelength. Experiments and numerical computations show that this family of solutions arises from a subcritical bifurcation.

- **Günther Grün**, Institute for Applied Mathematics, University of Bonn

Thin-Film Flow Influenced by Thermal Fluctuations

We will be concerned with the effects thermal fluctuations have on thin-film (de)wetting. Starting from incompressible Navier-Stokes equations with noise, we use long-wave approximation and Fokker-Planck-type arguments to derive a fourth-order degenerate parabolic stochastic partial differential equation – the stochastic thin-film equation. We propose a discretization scheme and give both formal and numerical evidence for our conjecture that thermal fluctuations may resolve discrepancies with respect to time-scales of dewetting which have been observed recently in comparing physical experiments and deterministic numerical simulations. This is joint work with K. Mecke and M. Rauscher.

- **Lorenzo Giacomelli**, Dipartimento Me.Mo.Mat. Universit di Roma (La Sapienza)

Microscopic and effective spreading rates for shear-thinning droplets

We discuss a few seemingly relevant outcomes of the analysis of the capillarity-driven spreading of shear-thinning droplets. In lubrication approximation, the simplest shear-thinning models lead to thin-film equations which degenerate also with respect to the pressure gradient. These operators apparently lack entropy estimates. However, in one space dimension it turns out that global and local energy estimates suffice both to construct zero contact-angle solutions and to describe their qualitative behavior (upper bounds on the spreading rate, lower bounds on the waiting time, decay rates). In particular, it is rigorously shown that shear-thinning liquids are not affected by the well-known contact-line paradox. In case of a weakly non-Newtonian rheology, in which the nonlinear behavior takes over only at a “high” threshold shear, formal asymptotics suggest a very weak (logarithmic) dependence on the threshold. Comparisons with known results for slip models and open questions will be presented, too.

- **Dejan Slepčev**, Department of Mathematics, UCLA

Coarsening in thin liquid films

Thin, nearly uniform, layers of some liquids can destabilize under the effects of intermolecular forces. After the initial phase, the liquid breaks into droplets connected by an ultra-thin fluid film. This structure coarsens over time. The characteristic distance between droplets and their average size grow, while their number is decreasing.

The thin-film equation that models this process is a gradient flow, that is the steepest descent in the energy landscape. I will discuss coarsening in thin-film equation with mobility equal to the height of the fluid in which case the equation is a gradient flow in the Wasserstein metric. I will describe how information on the geometry of the energy landscape yields a rigorous upper bound on the coarsening rate. This is joint work with Felix Otto and Tobias Rump.

2 Tuesday, January 31, 2006

- **Lou Kondic**, Department of Mathematical Sciences, NJIT

Instability of a fluid strip

Consider a long fluid strip positioned on a horizontal partially wetting substrate. This strip is subject to the capillary, van der Waals and gravitational forces. In analogy to the classical problem of a liquid jet, one may expect the strip to become unstable with respect to Rayleigh-type of instability. However, it turns out that presence of the substrate and finite length of a strip modify the energy balance of the problem and influence strongly the development of instability. Recent experiments have shown that the instability does not proceed simultaneously all along the strip, but it propagates from the ends towards the center. Thus, a process of strip pinch-off and drop formation finally leads to a linear array of sessile droplets.

The talk concentrates on computational results obtained under lubrication framework that show the main features of instability development. We find that details of the solid-liquid interaction strongly influence the droplet formation process and may in some cases prevent break-up of the strip. We will discuss the connection between the instability of a finite and infinite strip, in addition to illustrating the importance of spatial dimensionality of the problem, i.e., the differences between 2D and 3D results. These results will be also of relevance to the more general problem of dewetting which has recently generated a lot of interest.

- **Andrea Bertozzi**, Department of Mathematics, UCLA

Shocks in driven liquid films

Driven contact line problems in thin liquid films are an active area of research. The mathematical theory of shock waves has recently been shown to play an important role in our understanding of basic properties of the contact line motion. I will present the theory for two recently studied experimental systems: (1) Thermally driven films counterbalanced by gravity are described by a scalar conservation with a non-convex flux. Such systems are known to produce 'undercompressive shocks' in which characteristics emerge from the shock on one side. (2) A related problem is that of particle laden flow driven by gravity. The differential settling rate of the particles with respect to the fluid results in the formation of double shock fronts which are solutions of a system of two conservation laws for the motion of the species. Comparison between theory and experiment will be discussed, along with open mathematical problems directly related to the experiments.

- **G. M. Homsy**, Department of Mechanical Engineering, University of California, Santa Barbara

Some heat pipe problems

- POSTER SESSION

H. Aryafar (H. Aryafar, A. S. Lukyanets, H. P. Kavehpour), Mechanical and Aerospace Engineering, UCLA

An interferometric study of spreading non-Newtonian fluids

Using high speed digital camera, we investigated the rupture of a liquid film between a drop and an interface between two immiscible fluids during the coalescence process. This interfacial film, formed during the settling of drop on interface, prevents the drop from coalescing into the lower liquid. At a certain critical thickness the interfacial film becomes unstable and a liquid bridge forms connecting the drop with the lower phase. An experimental study has been performed to investigate the time dependent behavior of the radius of the bridge. The bridge is found to have two distinct power-laws with time depending upon coalescence conditions. The dimensionless parameter, the Ohnesorge number, is the important parameter in this process. For the Ohnesorge number less than unity, the process is inertia dominated, and the radius followed a power-law with time to the power of a half. For values of the Ohnesorge number much greater than unity, the liquid bridge was viscous dominated and followed a power-law with time to the power of unity. Dimensional analysis produced front factors which matched those predicted and measured from previous drop/drop coalescence experiments.

Andrea L. Bertozzi(J. B. Greer, A. L. Bertozzi, G. Sapiro), Department of Mathematics, UCLA

Computing Fourth Order PDEs on Implicit Surfaces

We extend a recently introduced method for numerically solving PDEs on implicit surfaces to fourth order PDEs including the Cahn-Hilliard equation and a lubrication model for curved surfaces. By representing a codimension-one manifold as the level set of a smooth function, we compute the PDE using only finite differences on a standard Cartesian mesh in the embedding space. The higher order equations introduce a number of challenges that are of less concern when applying this method to first and second order PDEs. Many of these problems, such as time-stepping restrictions and large stencil sizes, are shared by standard fourth order equations in Euclidean domains, but others are caused by the extreme degeneracy of the PDEs that result from this method and any complexities of the geometry.

Benjamin Cook (B. Cook, A. Bertozzi), Department of Mathematics, UCLA

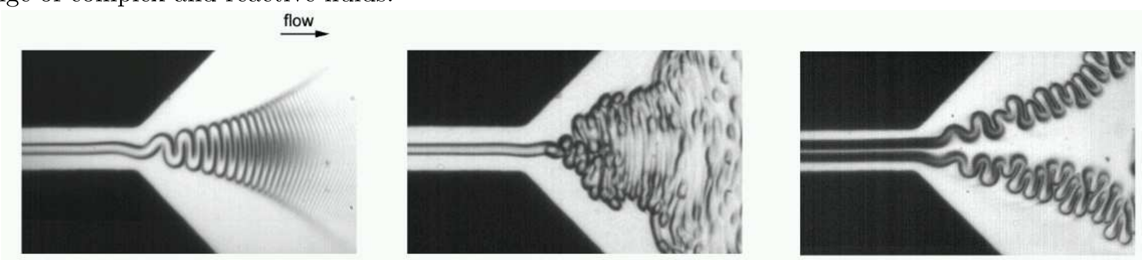
Behavior of Shock Solutions in Particle-Laden Thin Films

Zhou, Dupuy, Bertozzi, and Hosoi presented in 2005 a lubrication model for particle-laden thin films in which the particles experience Richardson-Zaki settling in the direction of the flow and the mixture is characterized by a local viscosity depending on particle concentration. Assuming a precursor layer of unknown thickness b, they found the fourth-order lubrication system to be well approximated by the associated first-order hyperbolic system, which for step initial data has double-shock solutions similar to a particle-rich ridge observed in experiments. We calculate numerical solutions and analytic shock solutions of the hyperbolic system, and find that for small enough b, such solutions do not exist. We propose a modified settling velocity that, rather than vanishing at volume fraction 1, vanishes at the same concentration where the viscosity is singular, representing a packing fraction of spheres. With this modified settling velocity we find double-shock solutions for arbitrarily small b, with the thickness in the “ridge” region between the shocks exhibiting power-law behavior as b goes to zero.

Thomas Cubaud (T. Cubaud and T. G. Mason), Chemistry and Biochemistry Dept, 2Physics and Astronomy Dept., UCLA

Viscous Buckling in Microfluidics

Microfluidic devices are useful in many disciplines, including chemistry, physics, biology, and medicine. Great effort has been devoted in recent years to exploring the formation of complex soft matter in these systems. Our study shows that miscible multiphase flows of liquids having widely different viscosities can be precisely manipulated and rapidly mixed in microfluidics. This study opens up routes to control and exploit viscous instabilities in systems strongly dominated by viscous forces. We investigate hydrodynamic instabilities of viscous threads surrounded by less viscous liquids. The threads are formed by hydrodynamic focusing of more viscous liquid flows by less viscous liquid side flows into microchannels. Depending upon the flow rates, nanoscale viscous threads can be formed and deformed using this technique. This approach allows studying viscous buckling instabilities when viscous forces dominate over buoyancy, inertia, and surface tension. These experiments explore the analogy between viscous flows and deformations of elastic solids. Viscous buckling occurs over several orders of magnitude in domains ranging from geophysics to material processing. When viscous threads enter a diverging microchannel, extensional shear stresses cause the threads to bend and fold rather than dilating in order to minimize dissipation and conserve mass. We show that the nanoscale diffusive layer at the boundary of the threads can dramatically modify the microscale folding flow morphologies. We relate the folding frequency to the characteristic shear rate in the channel. We are able to produce and control new types of flows such as viscous heterogeneous flows and viscous dendrites flows. As opposed to purely diffusive flow, viscous folding offers the practical ability to rapidly mix liquids having different viscosities. This can be of potential use in many fields, since numerous industrial and biological liquids are both miscible and can have widely different viscosities. The hydrodynamic coupling between multiple threads is investigated. The size and the distance between the threads are controlled independently. We investigate the phase-locking between the threads and their rupturing into arrays of viscous swirls, reminiscent of the Kelvin-Helmholtz instability. This approach reveals a rich variety of dynamics. This system provides an idealized environment for achieving dynamic control over the shape of miscible liquid interfaces and for passively enhancing the mixing between liquids having different viscosities at the micro and at the nano scale. Buckling instabilities in microfluidics are promising for producing intricate fluids morphologies and for studying of dynamic interactions between a wide range of complex and reactive fluids.



Single folding in viscous medium — Heterogeneous viscous flow — Viscous dendrites flow

Geoffroy Guéna, Ecole Normale Superieur, Paris

Spreading of nematic liquid crystal on hydrophobic and hydrophilic surfaces

The spontaneous spreading of nematic liquid crystals on hydrophobic silica substrates has been investigated at various scales by combining ellipsometry, profilometry and interferometry. While anchoring defects play a major role on hydrophilic substrates at the macroscopic scale, making the behaviour of films highly complex, they do not show up on these hydrophobic substrates. Then, the main specificity of the nematic films is the elastic energy associated to the long range orientational order. Experiment shows that the macroscopic spreading laws differ from the ones of simple wetting liquids. Moreover, at the microscopic scale, a sharp transition between a mesoscopic, nematic film, and a molecularly thin precursor, is observed. The length of the precursor scales as the inverse of the macroscopic velocity, as expected for adiabatic wetting films.

Geoffroy Guéna, Ecole Normale Superieur, Paris

Evaporating mixtures drops in the situation of complete wetting: the case of alkane mixtures

Alkane mixtures drops are model systems in which the influence of surface tension gradients during the spreading and the evaporation process can be easily studied. The surface tension gradients are mainly induced by concentration gradients, mass diffusion being a stabilising process. Depending on the relative concentration of the mixture, a rich pattern of behaviours is obtained.

Hans Knüpfer, University of Bonn

Regularity of the free boundary for a thin film equation

We present results on the regularity of the free boundary for the thin film equation. We view the thin film equation as a classical free boundary problem: It consists of finding a classical solution up to the boundary and a condition for the velocity of the free boundary which has to be satisfied. A standard technique for existence of regular solutions bases on linearizing around the initial data and then proving maximal regularity for the linearized operator. We have focused on the simplest case: the Lubrication approximation of the Hele–Shaw flow with zero contact angle. When the initial datum is a perturbation of the stationary solution, existence of a unique classical solution can be proved. The solution has for all times the same spatial regularity as the initial datum. the initial data. Regularity of the free boundary follows.

Eric Lauga, Hatsopoulos Microfluids Laboratory Department of Mechanical Engineering, MIT

Measuring slip using diffusion

In this work, we propose a novel experimental method to probe slip boundary conditions in hydrodynamics. Instead of relying on externally-induced flow or probe motion, we suggest that colloidal diffusivity near solid surfaces contains signatures of the degree of fluid slip exhibited on those surfaces. We present a number of calculations quantifying this idea and discuss different experimental setups. [This work was done in collaboration with Todd Squires, UC Santa Barbara].

Hsiang-Wei Lu (H.-W. Lu, K. Glasner, A. L. Bertozzi, and C.J. Kim), Department of Mechanical and Aerospace Engineering, UCLA

A Diffuse Interface Model for Electrowetting Droplets In a Hele-Shaw Cell

Electrowetting has recently been explored as a mechanism for moving small amounts of fluid in confined spaces. We propose a diffuse interface model for droplet motion, due to electrowetting, in Hele-Shaw geometry. In the limit of small interface thickness, asymptotic analysis shows the model is equivalent to Hele-Shaw flow with a voltage-modified Young-Laplace boundary condition on the free surface. We show that details of the contact angle significantly affect the timescale of motion in the model. We measure receding and advancing contact angles in the experiments and derive their influences through a reduced order model. These measurements suggest a range of timescales in the Hele-Shaw model which include those observed in the experiment. The shape dynamics and topology changes in the model agree well with the experiment, down to the length scale of the diffuse interface thickness.

Shomeek Mukhopadhyay, Department of Physics, Duke University

Precursors and Contact Line Dynamics in Thermally Driven Marangoni flows

Ernst van Nierop, Harvard University

Reactive spreading of oil on water

We observe oil drops containing oleic acid to spread, then recoil, on a substrate of water containing NaOH. Surfactant is produced at the interface during spreading, and spreading is observed to be faster than it would be with no reaction. The spreading rate and maximum radius attained grows with the concentration of the reagents which are of order 1 mM. After ~ 10 seconds, the drops reach a their maximum radius (typically 3-4 times the initial radius), and then recoil with apparent power-law dynamics. The rate of recoil is found to depend on the concentration of NaOH in the substrate.

Bill Ristenpart, Division of Engineering and Applied Science, Harvard University

Coalescence of Spreading Droplets on a Wettable Substrate

We investigate experimentally and theoretically the coalescence dynamics of two spreading droplets on a highly wettable substrate. Upon contact, surface tension drives a rapid motion perpendicular to the line of centers that joins the drops and lowers the total surface area. We find that the width of the growing meniscus bridge between the two droplets exhibits power-law behavior, growing at early times as $t^{1/2}$. Moreover, the growth rate is highly sensitive to both the radii and heights of the droplets at contact, scaling as $h_o^{3/2}/R_o$. This size dependence differs significantly from the behavior of freely suspended droplets, in which the coalescence growth rate depends only weakly on the droplet size. We demonstrate that the scaling behavior is consistent with a model in which the growth of the meniscus bridge is governed by the viscously hindered flux from the droplets.

Marcus Roper (M. Roper, P. Dechadilok, S. Branda, R. Kolter, M. P. Brenner), DEAS, Harvard University

Surfactant driven wall-climbing in B. subtilis biofilms

Bacteria grown in still liquid media swim to the free surface and associate there into thick pellicles bound together by exopolysaccharides. Different bacterial strains are associated with different pellicle morphologies. We take a first step toward relating the pellicle phenotype to gene expression at the level of individual bacteria by analysing expression of surfactin (a biological surfactant) in B. subtilis biofilms. Marangoni stresses can drive the edges of the swarm against gravity up an angled substrate: biofilms created by sfp mutants do not spread on such substrates. The surfactin polar group is expressed by a convoluted chemical pathway so that effective quantities are produced only when a critical number density of bacteria is exceeded, and we demonstrate how this quorum sensing mechanism aids wall-climbing.

Tobias Rump, Institut für Angewandte Mathematik, University of Bonn

Coarsening in a droplet model

Certain liquids on a solid substrate form a configuration of droplets connected by a thin precursor layer. This configuration coarsens: The average droplet size increases while the number of droplets decreases and the typical distance between droplets grows. The evolution of the liquid film – described by the time-dependent film height $h > 0$ – is driven by the reduction of energy. We consider the model

$$\partial_t h + \nabla \cdot (M(h) \nabla(\Delta h - \mathcal{U}'(h))) = 0$$

on an n-dimensional substrate, where $\mathcal{U}$ models the intermolecular forces. The mobility function is given by $M(h) = h^q$, $q > 0$.

K. Glasner and T. Witelski (in PRE 67(1):016302, 2003) gave – based on asymptotic analysis – a heuristic argument for the case $n = 1$ and $q = 3$ that the system has a statistically self-similar behavior characterized by a single scaling exponent. We prove rigorously an upper bound on the coarsening rate for the case $q = 1$ (accepted for SIAM J. Math. Anal.).

As was also shown by Glasner and Witelski (in Physica D 209, 2005) for 1-d, there are two competing coarsening mechanisms: *Collapse* and *collision* of droplets.

- *Collapse* relies on mass exchange through the precursor layer: Larger droplets grow at the expense of the smaller ones. Eventually, the smaller droplets disappear, only the largest one survives. This is known as Ostwald ripening.
- Droplets migrate on the precursor layer, which can lead to coarsening by *collision* of two droplets. The mobility of a droplet depends on the choice of the mobility function in the thin film approximation; for our choice the mobility increases with height.

We study the role of migration in the coarsening process for the whole range of mobility exponents $q > 0$ in case of a two-dimensional substrate. In particular, we identify the scaling of the droplet velocity in terms of the droplet radius and the mobility exponent q.

This is joint work with F. Otto and D. Slepčev.

Laura E. Schmidt (L. E. Schmidt and W. W. Zhang), Physics Department, University of Chicago

Length-scale selection in viscous entrainment of stratified fluids

When two stratified fluid layers undergo vigorous thermal convection, a thin tendril of one fluid can become entrained in the other fluid. The persistence of these tendrils has been proposed as an explanation for long-lived, fixed structures in the Earth's mantle, such as hotspots. We obtain a self-consistent description of the entrainment process by requiring that the nearly flat interface far from the tendril joins smoothly with the nonlinear steady-state shape of the tendril. For strongly stratified layers the tendril radius is proportional to the interface deflection height for the case when no fluid is entrained.

Oleg E. Shklyaev (O. E. Shklyaev and E. Fried), Washington University in St. Louis

Stability of evaporating thin liquid films

In this study, we revisit the problem of evaporating thin film in the presence of surfactant. Instead of the conventional Hertz-Knudsen-Langmuir equation we impose a configurational momentum balance. This balance, which supplements the conventional conditions enforcing the balances of mass, momentum, and energy on the film surface, arises from a consideration of configurational forces within a thermodynamical framework [1]. In addition to classical term involving the difference between the temperatures of the film surface and the adjacent vapor, the configurational momentum balance includes two additional terms. One of these resembles a term considered previously by Ajaev and Homsy [2] and involves the pressure at the film surface relative to the pressure of the vapor. The other term involves the difference between the surfactant concentration at the film surface and the saturation value of the surfactant concentration. For simplicity, we refer to these terms as the effective pressure and surfactant activity.

We find that time-dependant base state of evaporating liquid film is affected by the effective pressure and the surfactant activity terms. Both effects influence the disappearance time. In particular, we find that the effective pressure strongly affects the film rupture processes and is important factor in consideration liquid films with thicknesses of one or two monolayers. These factors lead to a revised understanding of the stability of an evaporating film. Parameter domains where the contribution of the newly introduced terms is important are determined.

[1] Eliot Fried, Morton E. Gurtin and Amy Q. Shen. Theory for solvent, momentum and energy transfer between a surfactant solution and a vapor atmosphere (submitted into Physical Review E).
[2] Vladimir S. Ajaev and G. M. Homsy. Steady Vapor Bubbles in rectangular Microchannels. J. Colloid Interface Sci. 240, 259-271 (2001).

Suleyman Ulusoy, Department of Mathematics, Georgia Institute of Technology

An entropy dissipation entropy estimate for a thin film type equation

We prove a lower bound on the rate of relaxation to equilibrium in the H^1 norm for a thin film equation. We find a two stage relaxation, with power law decay in an initial interval, followed by exponential decay, at an essentially optimal rate, for large times. The waiting time until the exponential decay sets in is explicitly estimated.

Thomas Ward, Department of Mathematics, UCLA

Electrohydrostatic Wetting of Poorly-Conducting Liquids

We theoretically predict and experimentally study wetting behavior of poorly-conducting fluids between two parallel conducting plates of different heights. In the absence of an applied potential, the interface shape and height are determined by gravity and parameterized using the Bond number. In the presence of an electric field at sufficiently high values on the order of (1 kV/mm), there is modification of the interfacial stresses and liquid level. Here we measure the liquid level height as a function of the hydrostatic and electric Bond numbers.

Chi Wey (C. Wey, T. Ward, A. Bertozzi, A. E. Hosoi), Department of Mechanical and Aerospace Engineering, UCLA

An Experimental Approach to Understanding Slurry Flows

We present experimental results for slurry flow on an inclined plane. Our experiment uses particles that are denser than the surrounding fluid, which results in particle segregation due to gravity. By analyzing the average front position versus time, we can study the following: effects of varying inclination angles, effects of particle concentrations, and the effects of volume portions. Further investigation gives us insight in effective viscosity trends based on particle concentrations, and a departure from the classical Huppert similarity solution for clear fluids flowing down an inclined plane.

Li Yang (L. Yang and G. M. Homsy), Department of Mechanical and Environmental Engineering, University of California, Santa Barbara

Steady 3D thermocapillary flows and dryout inside a V-shaped wedge

We consider a liquid meniscus inside a wedge of included angle 2β that wets the solid walls with a contact angle θ. Under an imposed axial temperature gradient, the Marangoni stress moves fluid toward colder regions while capillary pressure gradients drive a reverse flow, leading to a steady state. The fluxes driven by these two mechanisms are found by numerical integration of the parallel flow equations. Perturbation theory is applied to derive an expression for the capillary pressure, which is typically dominated by the transverse curvature of the circular arc inside the cross section perpendicular to the flow axis, and corrected by a higher order axial curvature resulting from the axial variation of the interface. Lubrication theory is then used to derive a thin film equation for the shape of the interface. Solutions are determined by two primary parameters: D, a geometric parameter giving the relative importance of the two curvatures and M, a modified Marangoni number. Numerical solutions indicate that for sufficiently large M, the Marangoni stress creates a virtual dry region. The value of M at dryout is found to depend linearly on D. A simplified analytical mode l is developed which agrees very well with the exact solution for large values of D. It is found that dryout occurs more easily for larger wedge and/or contact angles except for a special case of $\beta + \theta = \pi/2$. In that case the axial curvature dominates and the dependence of the dryout condition on β and θ is non-monotonic, but only weakly so.

Arik Yochelis, Department of Physics, University of California, Berkeley

Spontaneous motion of droplets on freezing or melting substrates: A novel mechanism

A fluid droplet may support a spontaneous self-propelled translation by modifying the wetting properties of the substrate, resulting in asymmetry in the contact angles. We propose a novel mechanism for droplet propagation upon a terraced landscape of ordered layers formed as a result of surface freezing. The droplet velocity is derived analytically in the isothermal case using a lubrication approach and found to involve contributions from both the terraced layer thickness and molecular interactions via the disjoining potential. Heat fluxes involved in the melting or freezing of the terrace edge are shown to support spontaneous and continuous translation of the droplet. This process is limited by the heat absorbed/released in surface layer phase transition and can result in a joint droplet-terrace motion.

Jie Zhang (J. Zhang, R. Behringer, A. Oron), Dept. of Physics and Center for Nonlinear and Complex Systems, Duke University

Marangoni Convection Experiment in Binary Mixtures

The Marangoni instabilities in binary mixtures are believed to be different from pure liquids (Bhattacharjee (1994), Joo (1995), Oron and Nepomnyashchy (2004), Podolny et al, (2005)). In contrast with a large amount of experimental work on Marangoni convection in pure liquids, such experiments in binary mixtures are not available in the literature. Using commonly available binary mixtures such as ethanol/water, sodium chloride/water and potassium chloride/water, we have systematically investigated the pattern formation for a set of substrate temperature and solute concentrations. A liquid film is drawn on the surface of a silicon wafer which maintains a constant substrate temperature, while the upper interface of the film opens directly to the room air with the room temperature fixed. The film is first covered for a sufficiently long time to allow the binary mixture to reach an equilibrium. After the cover is removed, the onset of the motion sets in and the patterns evolve with time, driven by surface tension fluctuations due to evaporation and the Soret effect, while the air liquid interface does not deform. A standard shadow graph method is used to keep track of the pattern formation as a function of time. The patterns are mainly composed of polygons and rolls which can be either stationary or traveling. We also observe a novel target-pattern traveling wave. The magnitude of convection generally increases with the initial solute concentration of the binary mixture and with the substrate temperature. Our findings differ from those of the theoretical predictions in which evaporation is neglected.

- **Kathleen J. Stebe** (Fang Jin, Nivedita R. Gupta, Kathleen J. Stebe), Department of Chemical and Biomolecular Engineering, Johns Hopkins University

Surfactant effects on drop detachment

When a buoyant viscous drop is injected into a viscous fluid, it evolves to form a distended shape that detaches via the rapid formation and pinching of a neck. The effects of surfactants in altering this process are studied numerically. In the absence of surfactants, surface contraction is fastest near the neck. Thus, when surfactants are present, they accumulate there and alter the ensuing dynamics by reducing the surface tension that drives the contraction. The surface tension is described by a nonlinear surface equation of state that accounts for the maximum packing of surfactant in a monolayer. When surfactant adsorption-desorption is very slow, interfaces dilute significantly during drop expansion, and drops form necks that are only slightly perturbed in their dynamics from the surfactant-free case. When adsorption-desorption dynamics are comparable to the rate of expansion, the necks form slowly, with a variety of shapes, depending on the amount of surfactant present in solution. Significantly, they fail to neck at all at elevated coverages. When surfactant adsorption-desorption kinetics are rapid, the surface remains in equilibrium with the surrounding solution, and drops break like

surfactant-free drops with a uniform surface tension. These arguments are used to construct a phase diagram of drop neck shapes as a function of surfactant coverage. A map of neck/no-neck thresholds is also constructed as a function of surfactant coverage and sorption dynamics, suggesting that drop detachment can be used as a means of characterizing surfactant dynamics. Preliminary work on the impact of these effects on satellite drop distribution is presented.

- **Karen E. Daniels**, Department of Physics, North Carolina State University

Starbursts and Wispy Drops: Surfactants Spreading on Gels

Droplets spread differently on liquids than on prewet solids. We conduct experiments on an intermediate case, spreading on a deep visco-elastic substrate (gel agar), in which by changing the agar concentration we can tune the substrate from liquid-like to solid-like behavior. We find that the instabilities of a spreading drop of surfactant are influenced by both the substrate elasticity and the surfactant properties. For weak gels (shear modulus $G < 30$ Pa), we find a novel branching instability in which the drop develops a starburst of spreading arms, and identify that the onset is controlled by the ratio of surface tension gradient to the gel's shear modulus. In contrast, well above the gelation transition ($G > 30$ Pa) the surfactant drops have a circular shape as on a pre-wet solid substrate, although after long times some demonstrate a wispy morphology. We investigate the morphology and dynamics in the starburst phase, and quantitatively describe the effects of substrate stiffness and droplet surface tension, viscosity, and volume. For all starbursts, the arms grow on average as $L \sim t^{3/4}$ (as in axisymmetric oil spills), with the prefactor controlled only by the width of the arms, and not the shear modulus, surface tension, viscosity, or droplet volume.

- **H. P. Kavehpour**, Mechanical & Aerospace Engineering, UCLA

An interferometric study of spreading non-Newtonian fluids

The spreading of highly entangled polymer melts on a solid surface is investigated using a phase-shifted laser feedback interferometer. Our experiments confirm the existence of a non-Newtonian "foot" region in the vicinity of the moving contact line for highly entangled polymer melts. Our experimental results of the lateral and vertical scales of this "foot" are in fair agreement with available theoretical predictions. The transient spreading of an ideal elastic "Boger" fluid is also investigated. It is known shown that the spreading rate of this model elastic fluid is smaller than is observed for corresponding Newtonian fluid drop of similar size and viscosity due to the viscoelastic effects. A foot-like structure is detected at the leading edge of the droplet for these unentangled, elastic fluids as well.

- **Michael Shearer**, Department of Mathematics, North Carolina State University

Thin film equations for fluid motion driven by surfactants

In the lubrication approximation, the motion of a thin liquid film is described by a single fourth-order partial differential equation that models the evolution of the height of the film. When the fluid is driven by a Marangoni force generated by a distribution of insoluble surfactant, the thin film equation is coupled to an equation for the concentration of surfactant. In this talk, I show the basic structure of this system, and begin an analysis of wave-like solutions in the specific context of a thin film flowing down an inclined plane. Numerical simulations reveal an array of traveling waves, which persists when capillarity and surface diffusion are neglected. The analysis of the limiting system has some surprises, and in this talk, I show how far we have come in understanding the numerical results analytically, and the analytical results numerically.

- **Robert Behringer**, Duke University Department of Physics

Thin Film Flows with Maragoni Driving

This work will highlight recent experimental studies at Duke on Maragoni-driven flow. In initial studies, we studied the formation of undercompressive and reverse undercompressive shocks for thin films driven up an inclined substrate against gravity. Depending on the pinch-off thickness, we could observe either a trailing rarefaction wave or a compressive shock that separates from the reverse undercompressive shock. More recently, we have probed the prewetting layer that is just observable in front of the advancing Marangoni-driven front. As time permits, I will also give a preview of recent work on thin-layer convection involving Maragoni stresses and evaporation in which the fluid is a salt solution (a binary mixture). Depending on the concentration of the solution and the temperature gradient, patterns can be very localized convection states, target pattern states or roll-like states. See the poster by Jie Zhang for more details. This work has alse been carried out in collaboration with Andrea Bertozzi, Shomeek Mukhopadhyay, Alex Oron, Jeanman Sur, and Tom Witelski.

3 Wednesday, February 1, 2006

- **Uwe Thiele**, Max-Planck-Institut fr Physik komplexer Systeme, Dresden

Structure formation in thin liquid films: Beyond the case of a single evolution equation

After reviewing some basic experimental and theoretical results for a dewetting one-layer thin films we present long-wave models for two-layer thin films [1,2] and reactive droplets [3,4].

Whereas, the one-layer model represents an evolution equation for a conserved order parameter field, the models for the two-layer films and the reacting droplets correspond to coupled evolution equations of two conserved and one conserved and one non-conserved order parameter, respectively.

For the the two-layer system we distinguish different pathways of rupture (incorporating van der Waals interactions only) and coarsening (adding stabilizing short-range interactions) pathways.

For the reactive droplets the solution structure is analysed explaining different experimentally observed regimes of running droplets. Finally we discuss existence regions of running and sitting droplets and the related drift-pitchfork bifurcation.

[1] A. Pototsky, M. Bestehorn, D. Merkt and U. Thiele, Phys. Rev. E 70, 025201(R) (2004)
[2] A. Pototsky, M. Bestehorn, D. Merkt and U. Thiele, J. Chem. Phys. 122, 224711 (2005)
[3] U. Thiele, K. John and M. Bär, Phys. Rev. Lett. 93, 027802 (2004)
[4] K. John, M. Bär and U. Thiele, Eur. Phys. J. E 18, 183-199 (2005)

- **Richard Braun** (R. J. Braun and P. E. King-Smith), Department of Mathematical Sciences, University of Delaware

Models for the Tear Film during a Blink

Lubrication theory for model problems that approximate the tear film on the eye are developed and solved numerically. The models incorporate viscosity, capillarity, and surfactant transport. The blink is modeled by a moving endpoint of the film, and at a minimum, sinusoidal motion of the end of the film will be studied. The one-dimensional equations are put onto a fixed domain via a mapping, and the resulting PDEs are solved via the method of lines (finite differences in space, DASPK for the ODEs at the grid points). The results are very encouraging for some conditions that are observed in vivo. For example, partial blinks leave a nonuniform tear film thickness after the lid motion, and the lubrication model produces quite similar film thickness profiles.

- **Barbara Wagner**, Weierstrass Institute for Applied Analysis and Stochastics (WIAS), Berlin

Slippage and viscoelastic effects for dewetting films

Lubrication models for dewetting thin liquid polymer films are derived taking into account effects of possibly large slippage and viscoelastic properties of the film. These models are used to describe the the morphological transition from oscillatory to monotone decay of a dewetting rim.

- **Andreas Münch**, Department of Mathematics, Humboldt University of Berlin

Dewetting laws, sharp interface models, and stability analysis for dewetting films with slippage

Singular perturbation techniques are used to derive the dewetting rates for the dewetting of a liquid film after rupture for small to large slippage. The dewetting laws are compared to results from numerical simulations. These ideas are used to consider the finger instability that occurs at the dewetting rim. We derive sharp interface models and show how the linear stability analysis for the ridge compares to the stability analysis for the "full" lubrication models in the no-slip and the intermediate slip case.

- **Andrew J. Bernoff** (J. C. Alexander, A. J. Bernoff, E. K. Mann, J. Adin Mann, Jr, J. M. Pugh, L. Zou), Department of Mathematics, Harvey Mudd College

  Domain Relaxation in Langmuir Films

  We report on an experimental and theoretical study of a molecularly thin polymer Langmuir layers on the surface of a Stokesian subfluid. Langmuir layers can have multiple phases (fluid, gas, liquid crystal, isotropic or anisotropic solid); at phase boundaries a line tension force is observed. By comparing theory and experiment we can estimate this line tension. We first consider two co-existing fluid phases; specifically a localized phase embedded in an infinite secondary phase. When the localized phase is stretched (by a transient stagnation flow), it takes the form of a bola consisting of two roughly circular reservoirs connected by a thin tether. This shape relaxes to the minimum energy configuration of a circular domain. The tether is never observed to rupture, even when it is more than a hundred times as long as it is thin. We model these experiments by taking previous descriptions of the full hydrodynamics (primarily those of Stone & McConnell and Lubensky & Goldstein), identifying the dominant effects via dimensional analysis, and reducing the system to a more tractable form. The result is a free boundary problem where motion is driven by the line tension of the domain and damped by the viscosity of the subfluid. The problem has a boundary integral formulation which allows us to numerically simulate the tether relaxation; comparison with the experiments allows us to estimate the line tension in the system. We also report on the collapse of a localized hole of a gas phase embedded in a liquid phase. The hole collapses in a self-similar fashion, and we present a solution for the fluid flow by solving a dual integral equations. Comparisons with experiments again yield an estimate for the line tension.

- **Karl Glasner**, Department of Mathematics, University of Arizona

  The quasistatic model for droplet spreading: dynamics, numerics and homogenization

  This talk focuses on the simplest global model for contact line motion. At sufficiently small capillary numbers, the fluid pressure can be treated as constant, and the fluid has the shape of a capillary (constant curvature) surface. The use of this approximation is reviewed, and is justified as a limiting problem for lubrication equation dynamics. A boundary integral method for numerical simulation is employed to understand the overall dynamics.

  The second part of this talk considers the effect of a periodically heterogeneous surface, modelled by a contact angle-velocity law with small spatial period. The homogenized dynamics of this problem is computed using a combination of multiple scales asymptotics and boundary integral numerics. Rigorous results concerning the existence and properties of the homogenized velocity are discussed.

- **Ali Nadim**, Keck Graduate Institute & Claremont Graduate University

  The Bretherton problem in a power-law fluid

  Understanding the movement of drops and bubbles in microchannels is increasingly important in the design and operation of microfluidic devices that involve two-phase flows. Thus, Bretherton's analysis of the motion of long bubbles in tubes [F P Bretherton, J Fluid Mech, 10:166-188 (1961)] and the associated profile of the wetting film around it are relevant. In this work, Bretherton's problem is revisited in the case where the wetting film between the bubble interface and the capillary wall is non-Newtonian and described by the power-law constitutive relation. Using the standard lubrication analysis, the equation for the thickness of the wetting film as a function of axial distance is derived and integrated to find the film thickness. The film thickness and pressure drop across the entire bubble are found to scale with the capillary number as $\mathrm{Ca}^{2/3}$, with a proportionality factor that depends on the power-law index. [Joint work with Ali Borhan.]

• **Stephen H. Davis**, Department of Engineering Sciences and Applied Mathematics, Northwestern University

Dynamics and Solidification of Metallic Foams

High-porosity solids, solid foams, are extremely useful in construction projects in which high strength and light weight are both essential. Many of these properties fit the needs of vehicles and armaments where weight considerations are directly related to fuel consumption or carrying weight. For some applications one would want an extremely regular array of pores while in others prescribed variations in space may be required.

Most of the efforts of modern researchers focuses on aqueous systems and hence surfactant dynamics. The analogous problem of freezing of metallic foams has received less attention. The characterization of solid foams is well studied but the solidification process itself is not well understood. We examine the case in which bubbles are numerous enough that they crowd each other and so have polygonal boundaries, but one in which the liquid channels drain and control the dynamics of the foam as a whole. Here, the liquid fraction is roughly in the 5%-15% range.

The microscale processes involve the liquid flows in channels. There are gravitational draining, capillary effects, as well as van der Waals attractions that could accelerate the film rupture that could cause the foam to collapse. There is also the freezing of such channels, which involves a solid-liquid front progressing into a channel with free boundaries and flowing liquid. In being able to study such effects one needs to examine the response of the liquid in channels to the imposition of temperature gradients. In the isothermal case there is a gravitational/capillary draining. In a temperature gradient, thermocapillary effects are superposed on this and one must determine the flow dynamics here. Given such dynamics, phase transformation is induced in a channel in which there is a complex fluid flow is present. The character and speed of freezing, and the microstructure created are strongly affected by the fluid flows. The mesoscale processes link together the dynamics of whole bubbles with that of the channels of a fluid network. In this talk we shall discuss new thinning models and rupture instabilities that generate coarsening, in both cases utilizing the "thin" nature of a lamella to develop approximations.

4 Thursday, February 2, 2006

- **Michael Miksis** (J. Zhang, M. J. Miksis and S. G. Bankoff), Department of Engineering Sciences and Applied Mathematics, Northwestern University

Nonlinear Dynamics of a Two-dimensional Viscous Drop under Shear Flow

The dynamics of a viscous drop moving along a substrate under the influence of shear flow in a parallel-walled channel is investigated. A front tracking numerical method is used to simulate a drop with moving contact lines. A Navier slip boundary condition is applied to relax the contact line singularity. Steady state solutions are observed for small Reynolds and Capillary number. Unsteady solutions are obtained with increasing Reynolds or Capillary number. For large values of the parameters, the interface appears to rupture, but for intermediate parameter values, time periodic drop interface oscillations are possible as the drop is moving along the bottom channel wall. These different states are identified in the Reynolds number - Capillary number plane for a specific range of physical parameters. The effects of density and viscosity ratio are also illustrated.

- **Alexander Oron**, Department of Mechanical Engineering, Technion

Pattern formation in the longwave Marangoni instability of a binary-liquid film with the Soret effect

The longwave Marangoni instability in a binary-liquid film with the Soret effect is investigated. Both monotonic and oscillatory instabilities attributed solely to the presence of the Soret effect are found in the case of a finite Biot number. Weakly nonlinear analysis in the case of the oscillatory instability shows the emergence of several kinds of stable supercritical standing and traveling waves in various parameter domains.

- **Laurent Limat** (in collaboration with A. Daerr, N. Le Grand, E. Rio, J. Snoeijer), Laboratoire PMMH, ESPCI, Paris, Laboratoire MSC, University Paris 7

Contact line morphology for flows down an incline

When a liquid is injected from a localized source and flows down an incline, different kind of free surface flows are observed for increasing flow rates: drop trains, straight rivulets, stationary meandering rivulets, unstationary meandering, braiding pattern... I will discuss two of these cases here: drop trains and stationary meandering of rivulets. In the first case, I will emphasize and describe singularities formed at the rear of the drops: formation of a "corner" where two inclined contact lines are meeting, onset of pearling... Despite several progresses, the full description of these singularities remain puzzling. In the second case (stationary meandering), I will show that simple arguments based upon a balance involving surface tension, wetting hysteresis and centrifugal effects are sufficient to recover the scalings followed by both the critical flow rate and the curvature radii of the meanders.

- **David Quéré**, Ecole Superieure de Physique et de Chimie Industrielles de la Ville de Paris

Stressed interfaces: from bubbly drops to jet impact

We consider a few situations where fluid interfaces are stressed in some way. We first show the analogy between the shape of falling rain drops and liquid globules deposited on very hot plates. Then, we discuss the air entrainment arising from the impact of a jet on a pool of same liquid. In both cases, we highlight the major role played by air, contrasting with most interfacial systems where air can be considered as a passive fluid.

Main contributors to this study: Etienne Reyssat, Anne-Laure Biance, Frdric Chevy and Elise Lorenceau.

- **John Lowengrub**, Department of Mathematics, University of California, Irvine

A level-set method for interfacial flows with surfactant

In this talk, I will present a level-set method for the simulation of fluid interfaces with surfactant. The method can be straightforwardly extended to the case of soluble surfactants. The method is based on an Eulerian formulation and couples a semi-implicit discretization of the surfactant equation with the immersed interface method for the flow-solver. Novel techniques are used to accurately conserve component mass and surfactant mass during the evolution. The method is applied in 2D to study the effects of surfactants on single drops, drop-drop interactions and interactions among multiple drops in Stokes flow under an applied shear. The non-monotonic behavior of the minimum distance between approaching drops versus capillary number is observed. Accordingly, a critical capillary number Ca* measuring the tendency to coalescence is determined as a function of the surfactant coverage. A transition in behavior is observed to occur at a critical coverage. This is joint work with J. Xu, Z. Li and H.-K. Zhao.