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2010 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '10

Paris, France
July 25–31, 2010

Logic Colloquium '10, the 2010 European Summer Meeting of the Association for Symbolic Logic, was hosted by the University of Paris Diderot (Paris 7) from 25 July to 31 July 2010. The lectures were held on the University Campus at Halle aux Farines. The conference was organized by the following three laboratories of Paris Diderot University and CNRS:

- Équipe de Logique Mathématique (ELM)
- Preuves, Programmes et Systèmes (PPS)
- Laboratoire d'Informatique Algorithmique: Fondements et Applications (LIAFA)

The meeting was also supported by the Mathematics Department of Orsay University (Paris 11).

The sponsors of the meeting were: the Association for Symbolic Logic; CNRS: INSMI, INS2I and INSHS; University Paris Diderot – Paris 7; INRIA; DIGITEO, Région Ile de France; Ville de Paris; Orsay University – Paris 11; and the Kurt Gödel Society.

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, René Cori. The other members were Samy Abbes (PPS), Odile Ainardi (PPS), Vincent Balat (PPS), Khadija Bayoud (ELM), Luc Bélair (Montréal University), Thomas Colcombet (LIAFA), Arnaud Durand (ELM) Thomas Ehrhard (co-chair, PPS), Martin Hils (ELM), Ramez Labib-Sami (ELM), Richard Lassaigne (ELM), Yves Legrandgérard (PPS), Guillaume Malod (ELM), Audrey Mansuet (PPS), Marie-Hélène Mourgues (ELM), Catherine Muhlrad-Greif (ELM), Donald Pelletier (ELM), Sylvain Perifel (LIAFA), Jean Eric Pin (co-chair, LIAFA), Françoise Point (ELM), Paul Rozière (PPS), Todor Tsankov (ELM), and Boban Velikovic (ELM). The Program Committee consisted of Albert Atserias (Barcelona), Zoé Chatzidakis (Paris Diderot), Thierry Coquand (Gothenburg), Pierre-Louis Curien (Paris Diderot), Michael Detlefsen (Notre Dame), Costas Dimitracopoulos (Athens), Juliet Floyd (Boston), Istvan Juhasz (Budapest), Menachem Magidor (Jerusalem), Michael Rathjen (Leeds, Chair), Thomas Scanlon (Berkeley), Alexandra Soskova (Sofia), and Yde Venema (Amsterdam).

The main conference topics were: *Bolzano's Beyträge*, *Computability Theory*, *Model Theory*, *Set Theory*, and *Simplicity (Complexity) of Proofs*. The program included two tutorial courses, nineteen invited plenary lectures, and twenty-three invited lectures in five special sessions corresponding to the five main topics. There were about hundred contributed papers and 293 registered participants from all over the world. More than sixty students and recent Ph.D's were assisted by local grants and the ASL.

The following tutorial courses were delivered:

Uri Abraham (Ben Gurion University, Beer-Sheva), *Introduction to the oracle and the oracle-proper forcing*.

Ted Slaman (University of California, Berkeley), *Randomness and Recursion Theory*.

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The following invited plenary lectures were presented:

Françoise Delon (University Paris Diderot), *C-minimal structures without density assumption*.

Nicola Gambino (Palermo University), *Homotopy-theoretic aspects of Martin-Löf type theory*.

Mai Gehrke (Nijmegen University), *Stone duality and recognizability*.

Jean-Yves Girard (University Marseille Méditerranée), *Normativity in logic*.

Moti Gitik (Tel-Aviv University), *On precipitous ideals*.

Valentin Goranko (Technical University of Denmark, Lyngby), *Interval temporal logics: straddling the decidability border*.

Ian Hodkinson (Imperial College, London), *Algebras of relations: some results and methods*.

Julia Knight (University of Notre Dame), *Comparing classification problems*.

Piotr Koszmider (Technical University of Lodz), *Set-theoretic topological methods in Banach spaces*.

Jan Krajíček (Charles University in Prague), *Towards hard tautologies*.

Angus MacIntyre (Queen Mary, University of London), *Some issues of definability for the complex exponential*.

Paulo Oliva (Queen Mary, University of London), *Sequential games and optimal strategies*.

Kobi Peterzil (University of Haifa), *Abelian varieties, their moduli spaces and o-minimality*.

Jan von Plato (University of Helsinki), *On the dimensionality of deductive arguments*.

Göran Sundholm (Leiden University), *Inference*.

Simon Thomas (Rutgers University), *A descriptive view of combinatorial group theory*.

Andreas Weiermann (Ghent University), *Recent trends in phase transitions for Gödel incompleteness*.

Thomas Wilke (Kiel University, Germany), *Logic of cryptography and cryptographic protocols*.

Alex Wilkie (University of Manchester), *Some model theory for expansions of the complex field by holomorphic functions*.

The proceedings of Logic Colloquium '10 will be published in a special issue of the ***Annals of Pure and Applied Logic***.

More information about the meeting can be found at the conference webpage, `http://logic2010.org/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
MICHAEL RATHJEN

Abstracts of invited plenary talks

► FRANÇOISE DELON, *C-minimal structures without density assumption*.
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A *C-relation* is the ternary relation induced by a meet-semi-lattice tree on the set of its branches, namely: $C(x; y, z)$ if the intersection of x and z is strictly contained in the intersection of y and z. An ultrametric distance, in particular a valuation on a field, defines a C-relation: $C(x; y, z)$ iff $d(x, y) < d(y, z)$. A C-structure is a set equipped with a C-relation and possibly additional structure. Following Haskell, Macpherson and Steinhorn, such a structure $\mathbb{M}$ is said to be C-minimal if, in any structure $\mathbb{N}$ elementarily equivalent to $\mathbb{M}$, definable sets in one-space (in one variable) are Boolean combinations of "cones" or "thick cones" (the generalisation of "open" and "closed" balls from ultrametric spaces).

In this contribution we review what is known about C-minimal structures. But we work in a more general framework than the one that had been considered previously: we allow isolated points in the topology defined by the C-relation. In this way, strong minimality and o-minimality become particular instances of C-minimality. This goes in the direction of the most recent developments of pure model theory, the generalization of methods from stability theory to the unstable context of the so-called dependent theories ("NIP"). This also allows us to understand in a much more precise way the analogies and differences beetween o-minimality and C-minimality as well as the connections to stability.

► NICOLA GAMBINO, *Homotopy-theoretic aspects of Martin-Löf type theory*.
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Over the past few years, there has been significant progress in making precise the tantalizing connections (originally suggested by Hoffman and Streicher in the '80s) between Martin-Löf type theory, Quillen's homotopical algebra and higher category theory. In particular, the analogy between identity types and path spaces can be explained by showing that the syntactic category of Martin-Löf type theory admits a weak factorisation system. Furthermore, it can be shown that, like topological spaces, types have a fundamental weak ω-groupoid, in which the weakness of the associativity and unit laws corresponds to the intensional (rather than extensional) formulation of identity types. The aim of the talk is to give an accessible survey of these results.

► MAI GEHRKE, *Stone duality and recognizability*.
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Recognizability is an important concept in computer science. It was originally introduced for finite words but has since been successfully extended to many other structures. Generally speaking, a set (called a language in this context) is recognizable if it is the preimage under an appropriate type of transformation of a subset of a *finite* structure. Where it applies, recognizability is a powerful tool for separating logical and/or computational complexity classes. It figures centrally in many decidability results, e.g., in automata theory. The main tools may be summed up as the Eilenberg–Reiterman theory. Stone duality on the other hand, is the fundamental mechanism connecting algebraic specification and spatial or geometric semantics—whether in mathematics, in logic, or in computer science. In logic it is useful in relating syntactic and semantic conceptions of logics. For propositional logics, access to relational semantics is typically useful in proving decidability results and tends to be computationally advantageous.

In joint work with Jean-Eric Pin and Serge Grigorieff, we have realized that there is an intrinsic connection between these two areas as minimal recognizers are dual spaces of certain enriched Boolean algebras or lattices generated by the objects to be recognized. This allows the formulation of Eilenberg–Reiterman theory to be generalized in two ways. Firstly, among the languages that can be treated, we may treat any collection closed under union and intersection (this is a very weak requirement; in terms of logical complexity classes, for example, this means any class closed under disjunction and conjunction). Secondly, we may in principle treat *any* sort of language—as long as we are willing to weaken the notion of recognizability from finiteness to compactness. In this general setting, recognition is no longer by finite algebras but by compact topological algebras or even compact topo-relational structures.

In this talk I will attempt to give an idea of the basic features and concepts both from the theory of recognizability and from duality theory for lattices with additional algebraic structure. Finally, I will try to explain the content of our results connecting the two illustrated by a few examples.

► JEAN-YVES GIRARD, *Normativity in logic*.
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Incompleteness—the absence of alternative natural numbers—can be ascribed to a ready-made normativity, inducing a rigid departure syntax/semantics. Geometry of Interaction, set in the non-commutative universe of von Neumann algebras, makes normative assumptions explicit, thus rending possible their internalisation, a possible way out from the semantic *aporia*. As an illustration, we define an alternative "model": logspace integers.

[1] J.-Y. GIRARD, ***Le point aveugle, tome 1: vers la perfection, tome 2: vers l'imperfection***, Visions des Sciences, Hermann, Paris, 2006–2007.

► VALENTIN GORANKO, *Interval temporal logics: straddling the decidability border*.
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Interval temporal logics are modal logics with modal operators that correspond to various relations between intervals in (usually, linearly) ordered relational structures, where truth of formulae is defined relative to time intervals, rather than time instants. Thus, formulae of interval logics are interpreted by sets of intervals, or equivalently, by binary relations, in ordered structures. Consequently, interval logics translate with respect to validity and satisfiability to fragments of dyadic, rather than monadic, second-order logic.

In particular, a very expressive and natural interval logic, introduced by Halpern and Shoham in 1991 (hereafter called HS), involves modal operators that correspond to all binary relations between intervals in a linear ordering (aka Allen's interval relations), viz., 'precedes', 'begins', 'ends', 'overlaps', 'contains, 'meets on the right', and their inverses. Not surprisingly, the logic HS was proved by Halpern and Shoham to be undecidable over all natural classes of linear orderings satisfying very weak assumptions. The undecidability remains the case amongst most, but not all, fragments of HS, as well as all other modal logics with essentially interval-based semantics. Thus, one of the central problems in that area is to delineate the border of undecidability and to identify the maximally expressive decidable cases of interval logics.

In this talk I will give a brief introduction to interval logics; then I will survey the most important and interesting decidability and undecidability results, and will discuss and illustrate the main techniques used so far to obtain such results. Finally, I will present some open problems and outline further perspectives in the area.

Most of the talk is based on a joint work with Angelo Montanari, Guido Sciavicco, Davide Bresolin, and several other collaborators on that topic over the past eight years.

► IAN HODKINSON, *Algebras of relations: some results and methods*.
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URL Address: `www.doc.ic.ac.uk/~imh`.

An algebra of relations on a set U consists of a non-empty set of relations on U of some fixed arity, endowed with (and closed under) the boolean operations and some choice of further operations, such as identity, converse, and composition for binary relations, or diagonals (equalities) and cylindrification (quantification) for n-ary relations.

Algebras of relations are a part of algebraic logic. Their modern development began with Tarski and his colleagues in the 1940s, although earlier work can be traced back to Boole and De Morgan. They form an algebraic counterpart to classical first-order logic, and techniques devised in work on them have been applied in areas such as modal and temporal logic: for example, the so-called cylindric relativised set algebras were the origin of the well known 'guarded fragment' of first-order logic [1].

However, the study of these algebras in their own right is of interest. The main problems are to do with characterising the class of abstract algebras that are isomorphic to genuine algebras of relations. For example, is this class closed under various algebraic operations? Is it definable by equations? If so, are finitely many equations sufficient, and how complex do they need to be? How can one test whether an algebra is in the class? For finite algebras, is there an algorithm to decide this? Can we approximate the class? Are there variants with different behaviour? These problems are difficult and a range of methods have been used to solve them, including universal algebra, duality, model-theoretic forcing, undecidable tiling problems, and combinatorics (such as probabilistic constructions of graphs).

My talk will survey some of the known results and go into more detail on a few of the proofs. Some of them have appeared in [2] as joint work with Robin Hirsch.

[1] H. Andréka, J. van Benthem, and I. Németi, *Modal logics and bounded fragments of predicate logic*, ***Journal of Philosophical Logic***, vol. 27 (1998), pp. 217–274.

[2] R. Hirsch and I. Hodkinson, ***Relation algebras by games***, Studies in Logic and the Foundations of Mathematics, North-Holland, 2002.

► JULIA F. KNIGHT, *Comparing classification problems*.
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A great deal of work in mathematics involves trying to classify the objects in some class, up to some important equivalence relation. We focus on classes of countable structures of familiar kinds, with isomorphism as the equivalence relation. We consider concrete ways to compare the classification problems. Different branches of logic have provided different approaches. One model theoretic approach involves cardinality—we compare the number of isomorphism types. Thus, $\mathbb{Q}$-vector spaces are easier to classify than undirected graphs.

Friedman and Stanley [5] developed an approach that involves Borel embeddings and "Borel cardinality". This approach tells us that classification for fields of characteristic 0 and finite transcendence degree, and for Abelian p-groups, is easier than for graphs, or for linear orderings, even though each of these classes has $2^{\aleph_0}$ isomorphism types of countable members. Borel cardinality does not distinguish among classes with $\aleph_0$ isomorphism types.

In [1], there is another approach, involving Turing computable embeddings and "effective cardinality". This approach tells us that number fields are easier to classify than $\mathbb{Q}$-vector spaces. The results in [6] suggest a uniform way to look for non-embeddability results, considering the form of the sentences that distinguish among non-isomorphic members of the different. In [4], we re-work in this way the result of Friedman and Stanley on non-embeddability of graphs in Abelian p-groups. We say that a class K has "Ulm type" if non-isomorphic members of K can be distinguished by infinitary sentences lying in the thinnest admissible set that contains the ordinals computable from the structures.

In [3], S-D. Friedman, Fokina, and Törnquist considered Σ^1_1 equivalence relations on ω. Among these are the isomorphism problems for *computable* members of various familiar classes, where the structures are identified with their indices. We may use Turing computable embeddings and effective cardinality to compare equivalence relations. Some distinctions are lost in this setting. In [2], it is shown that Abelian p-groups now lie on top—the effective cardinality is the same as for graphs.

[1] W. Calvert, D. Cummins, J. F. Knight, and S. Miller, *Comparing classes of finite structures*, ***Algebra and Logic***, vol. 43 (2004), pp. 365–373.

[2] E. B. Fokina, S-D. Friedman, V. Harizanov, J. F. Knight, C. McCoy, and A. Montalbán, *Isomorphism and bi-bmbeddability relations on computable structures*, pre-print.

[3] E. B. Fokina, S-D. Friedman, and A. Törnquist, *The effective theory of Borel equivalence relations*, ***Annals of Pure and Applied Logic***, vol. 161 (2010), pp. 837–850.

[4] E. Fokina, J. F. Knight, C. Maher, A. Melnikov, and S. M. Quinn, *Classes of Ulm type, and relations between the class of rank-homogeneous trees and other classes*, pre-print.

[5] H. Friedman and L. Stanley, *Borel reducibility theory for classes of countable structures*, ***The Journal of Symbolic Logic***, vol. 54 (1989), pp. 894–914.

[6] J. F. Knight, S. Miller (Quinn), and M. Vanden Boom, *Turing computable embeddings*, ***The Journal of Symbolic Logic***, vol. 73 (2007), pp. 901–918.

▶ PIOTR KOSZMIDER, *Set-theoretic topological methods in Banach spaces*.
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We survey recent results in applications of forcing and other infinitary combinatorial methods in Banach spaces of the form $C(K)$. The existential results are often relevant to the entire class of Banach spaces. We propose an approach when set-theoretic topological methods can be practised with particular naturality: linear operators on $C(K)$ are treated as some sort of generalized continuous functions on K, via the adjoint operator, then some classical set-theoretic techniques can be often mimicked in this context.

▶ JAN KRAÍČEK, *Towards hard tautologies*.
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A proof system for propositional logic is a polynomial time map P whose range is exactly the set of propositional tautologies (Cook and Reckhow [1]). Any string w for which $P(w) = \tau$ is called a P-proof of formula τ. A proof system P is p-bounded if all tautologies admit a P-proof of the length bounded by a fixed polynomial in the length of the tautology. Cook and Reckhow [1] noted that a p-bounded proof system exists if and inly if the class NP is closed under complementation.

It is expected that no p-bounded proof system exists and it is a fundamental problem to prove this conjecture. This amounts to demonstrating super-polynomial lengths-of-proofs lower bounds for all proof systems. Such lower bounds were established for a variety of proof systems but all of them are weaker than the usual text-book propositional calculus based on a finite number of axiom schemes and inference rules, cf. [2].

A key issue in any lower bound proof is to come up with plausible candidate hard tautologies. For weaker systems such hard tautologies encode various combinatorial principles (e.g., the pigeonhole principle) but for stronger systems no combinatorial constructions were proposed. I shall expose a recent theory of proof complexity generators describing a class of possibly hard tautologies, cf. [3, Chapters 29 and 30].

[1] S. A. Cook and R. A. Reckhow, *The relative efficiency of propositional proof systems* ***The Journal of Symbolic Logic***, vol. 44 (1979), no. 1, pp. 36–50.

[2] J. Krajíček, *Proof complexity*, ***European congress of mathematics***, (Stockholm), (Ari Laptev, editor), European Mathematical Society, 2005, pp. 221–231.

[3] ———, ***Forcing with random variables and proof complexity***, London Mathematical Society Lecture Notes, Cambridge University Press, to appear.

► PAULO OLIVA, *Sequential games and optimal strategies*.
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In this talk I will describe a general notion of sequential (n rounds) game introduced in [1]. One of the main differences from the standard notion of sequential game is that we focus on the *objective* or *goal* of each round, rather than on the number of players—think of a lonely person playing both black and white in chess. If the objective at each even (respectively, odd) round is the same, but objectives at even and odd rounds are conflicting, we can think of this is a win-or-lose two-player game. If the objective at round i does not necessarily conflict with the objective at round j, then this can be thought of as an n-player game, as in the theory of Nash equilibrium. A *play* in such games is a sequence of moves, one for each round, and the *outcome function* computes the outcome of the game for each given play. In the case of a two-player game the outcome function determines who won the game, whereas in the n-player game the outcome function describes the payoff for each of the players.

The plan for talk is to (1) describe the theory selection functions, on which the definition of the game is based, (2) give the abstract definition of the game, (3) go through a few concrete instances of the game, and (4) describe how optimal strategies can be computed via the product of selection functions.

If time permits, we will also cover an even more general form of sequential game where the number of rounds is unbounded. In other words, the game carries on until a stopping condition is met. The particular case above corresponds to a constant stopping condition that simply checks whether round n has been reached or not. But in general the stopping condition might depend on how the game unfolds, so that plays in the game might be arbitrarily long (but always finite). As it turns out, the computation of optimal plays in the unbounded case is a form of bar recursion, which is strong enough to interpret full classical analysis, over Gödel's system T (cf. [2]).

This talk is based on recent joint work with Martín Escardó.

[1] Martín Escardó and Paulo Oliva, *Selection functions, bar recursion, and backward induction*, ***Mathematical structures in computer science***, vol. 20 (2010), no. 2, pp. 127–168.

[2] ———, *Computational interpretations of analysis via products of selection functions*, ***Proceedings of Computability in Europe 2010***, (F. Ferreira and B. Lowe and E. Mayordomo and L. M. Gomes, editors), Springer Lecture Notes in Computer Science, 2010.

► YA'ACOV PETERZIL, *Abelian varieties, their moduli spaces and o-minimality*.
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Several classical mathematical constructions involve the action of an infinite discrete group Γ on a complex manifold M. In o-minimal structures one cannot define infinite discrete sets, therefore it might seem at first that these constructions are beyond the reach of o-minimality. Still, in some important cases there is a definable subset of M which intersects every Γ-orbit in a finite set (so-called a fundamental set) and in these cases one may hope to capture the construction of the quotient space ${}_\Gamma\backslash M$ in an o-minimal structure.

In [2] we investigated in this framework the analytic family of elliptic curves and their definability in the o-minimal structure $\mathbb{R}_{an,exp}$. In this talk I will discuss the construction of families of complex tori, of polarized abelian varieties, and their moduli space within various o-minimal structures (see [1]).

Recent work by Pila-Zannier [4] and by Pila [3] makes use of such constructions in the application of o-minimality to arithmetical problems in algebraic geometry. If time permits I will discuss these ideas as well.

This is joint work with Sergei Starchenko.

[1] Y. Peterzil and S. Starchenko, *The definability of the Riemann theta functions, abelian varieties and their moduli space*, in preperation.

[2] ———, *Uniform definability of the Weierstrass $\wp$-functions and generalized tori of dimension one*, ***Selecta Mathematica, New Series***, vol. 10 (2004), pp. 525–550

[3] J. Pila, *Rational points of definable sets and results of Andre–Oort-Manin–Mumford type*, ***International Mathematical Research Notices***, vol. 13, (2009), pp. 2476–2507.

[4] J. Pila and U. Zannier *Rational points in periodic analytic sets and the Manin-Mumford conjecture*, ***Rend. Lincei Mat. Appl.***, vol. 19 (2008), pp. 149–162.

► GÖRAN SUNDHOLM, *Inference*.
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The term inference is ambiguous between an epistemic act of inference, that is, an act of passage from judgement to judgement, and an inference *scheme* ("blue-print") for such acts. Furthermore, to the epistemic notion of inference there corresponds an ontological ("alethic") notion of consequence among propositions.

The lecture investigates the relation between the *rightness* of the act and the *validity* of the schema. Similarly, the related notions of *consequence* and *conditional* will be explored. The distinction between the (logical) *holding* ("in all variations") of a consequence and the *validity* of the matching inference-scheme is noted and insisted upon. Familiar works of Bolzano, Frege, and Gentzen are considered and given a place in the resulting framework.

The epistemic role of inference is emphasized in an explanation of validity in terms of the epistemic notion of *immediate evidence*, where analogues to Aristotle's notion of a *perfect syllogism* are noted. Finally, themes from John Austin's Theory of Speech Acts are brought to bear on the validity of inference, where phenomena pertaining to the communicative roles of sender and receiver, as well as possible connections to Dialogue Theory, are noted.

► SIMON THOMAS, *A descriptive view of combinatorial group theory*.
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It is well-known that descriptive set theory provides a framework for measuring the relative complexity of many naturally occurring classification problems. But it is less well-known that descriptive set theory also provides a framework for explaining the inevitable non-uniformity of many classical constructions in mathematics. In this talk, I will illustrate this point by considering two constructions from combinatorial group theory related to the word problem for finitely generated groups and the Higman–Neumann–Neumann Embedding Theorem.

► ANDREAS WEIERMANN, *Recent trends in phase transitions for Gödel incompleteness*.
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First we survey phase transitions related to classical independence results for first order Peano arithmetic. In particular we treat refinements of results by H. Friedman and by J. Paris and L. Harrington. These refinements will depend on an interaction between classical proof theory and analytic combinatorics.

In the second part we will cover recent developments on phase transitions which emerge from Kristiansen style subrecursive degree theory. The idea is to refine the study of degrees of PA provably recursive functions and to induce a degree structure on threshold functions.

If time is left we will discuss some amazing phase transition results related to the Ackermann function.

► ALEX WILKIE, *Some model theory for expansions of the complex field by holomorphic functions*.
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The motivating problem is, of course, Zilber's conjecture: is the expansion of the complex field by the exponential function *quasi-minimal*, i.e., is every definable subset of $\mathbb{C}$ is either countable or uncountable? I shall discuss a possible approach to this and related questions, as well as discussing some local theory for the complex exponential field where the theory of polynomially bounded o-minimal structures plays a role.

Abstracts of invited talks in the Special Session on Computability Theory

► KLAUS AMBOS-SPIES, *Joins and meets in the computably enumerable cl-degrees*.
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Recently two variants of strongly bounded Turing reductions have been introduced. An *identity bounded Turing* reduction (ibT-reduction for short) is a Turing reduction where no oracle query is greater than the input while a *computable Lipschitz* reduction (cl-reduction for short) is a Turing reduction where the oracle queries on input x are bounded by $x + c$ for some constant c.

In our talk we discuss some algebraic properties of the partial orderings of the strongly bounded Turing degrees of the computably enumerable sets. In particular, we show that, for $r = ibT, cl$, the five element nonmodular lattice N_5 can be embedded into the partial ordering of the c.e. r-degrees by a map which preserves the least element whereas there is no such embedding of the five element nondistributive modular lattice M_3. Moreover we show that every c.e. r-degree is meet reducible. It follows that, in the language $L(\leq, \vee, \wedge)$, the elementary Σ_1-theory of the partial ordering of the c.e. r-degrees differs from the corresponding theories of the c.e. weak truth-table degrees and c.e. Turing degrees.

The results we present are joint work with Philipp Bodewig, Thorsten Kräling, and Liang Yu.

► JOHANNA N. Y. FRANKLIN, *Difference randomness*.
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In this talk, I will define new notions of randomness based on the difference hierarchy. First, I will describe various ways in which a real can avoid all effectively given tests consisting of n-r.e. sets for some given n. In each case, the n-r.e. randomness hierarchy collapses for $n \geq 2$. One of the resulting notions, difference randomness, results in a class of random reals that is a strict subclass of the Martin-Löf random reals and a proper superclass of both the Demuth random and weakly 2-random reals. In fact, the difference random reals are precisely the Turing incomplete Martin-Löf random reals. I will then define a notion of ω-r.e. randomness and discuss other subclasses of the reals that are ω-r.e. random.

This work is joint with Keng Meng Ng.

► HRISTO GANCHEV, *$\mathcal{K}$-pairs and the theory of the Σ^0_2 enumeration degrees*.
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A Kalimullin pair (briefly $\mathcal{K}$-pair) of sets is a pair $\{A, B\}$ for which there is a c.e. set W, such that

$$A \times B \subseteq W \;\; \& \;\; \overline{A} \times \overline{B} \subseteq \overline{W},$$

In this talk we shall survey some results about the local theory of the enumeration degrees, involving $\mathcal{K}$-pairs. Among these are some recent results concerning the definability of the classes of the downwards properly Σ_2 and of the upwards properly Σ_2 enumeration degrees, as well as the interpretation of true arithmetic in the theory of the Σ^0_2 enumeration degrees.

► ISKANDER KALIMULLIN, *Limitwise monotonicity and the degree spectra of structures*.
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For a set A we study the spectrum of degrees in which A is limitwise monotonic. The talk is devoted to new results in this direction recently obtained by the speaker jointly with B. Khoussainov and A. Melnikov.

Abstracts of invited talks in the Special Session on Model Theory

► EMMANUEL BREUILLARD, *Approximate groups*.
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Approximate groups were introduced by Terence Tao a few years ago in order to describe finite subsets A of an ambient group whose product set AA is not much larger than A. A large literature exists on the problem in abelian groups, and only recently did people start to investigate thoroughly the non-commutative case. A major question, the "non-commutative Freiman problem", consists in describing the rough structure of such sets. Recently Ehud Hrushovski made important progress in this area bringing tools from model theory and algebraic groups. In this talk, I will describe some recent work joint with Ben Green and Terence Tao in which we refine part of Hrushovski's work, giving explicit bounds, and present some applications of this result to various questions in group theory.

[1] E. Breuillard, *Approximate groups: a very brief introduction*, notes from a mini-course, to be found at `http://www.math.u-psud.fr/~breuilla/cour.html`

[2] E. Breuillard, B. Green, and T. Tao, *Approximate subgroups of linear groups*, arXiv preprint, `arXiv:1005.1881`

[3] E. Hrushovki, *Stable group theory and approximate subgroups*, arXiv preprint, `arXiv:0909.2190`

► PHILIPP HIERONYMI, *Dichotomies for expansions of the real field*.
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Let $\overline{\mathbb{R}} = (\mathbb{R}, +, \cdot)$ be the field of real numbers. In this talk, the following characterization of expansions of $\overline{\mathbb{R}}$ by closed and discrete sets will be presented.

Theorem 1. [2] *Let $\mathcal{R}$ be an o-minimal expansion of $\overline{\mathbb{R}}$ and let $D \subseteq \mathbb{R}$ be closed and discrete. Then either*

- *$(\mathcal{R}, D)$ defines $\mathbb{Z}$ or*
- *every subset of $\mathbb{R}$ definable in $(\mathcal{R}, D)$ has interior or is nowhere dense.*

We will discuss possible generalizations of this theorem. As an application, we consider the following extension of the results from [1].

THEOREM 2. [3] *Let* Γ *be a finitely generated subgroup of* $\mathbb{C}^{\times}$. *Then either*

- $(\overline{\mathbb{R}}, \Gamma)$ *defines* $\mathbb{Z}$ *or*
- *there is* $a \in \mathbb{R}^{>0}$ *such that every open subset of* $\mathbb{R}$ *definable in* $(\overline{\mathbb{R}}, \Gamma)$ *is already definable in* $(\overline{\mathbb{R}}, a^{\mathbb{Z}})$.

[1] OLEG BELEGRADEK and BORIS ZILBER, *The model theory of the field of reals with a subgroup of the unit circle*, ***Journal of the London Mathematical Society***, vol. 78 (2008), no. 2, pp. 563–579.

[2] PHILIPP HIERONYMI, *Defining the set of integers in expansions of the real field by a closed discrete set*, ***Proceedings of the American Mathematical Society***, vol. 138 (2010), pp. 2163–2168.

[3] ———, *Expansions of the real field by finitely generated subgroups of the complex field*, in preparation.

► MICHAEL C. LASKOWSKI, *Data compression and definability of types in stable and dependent formulas*.
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If a first-order formula has Uniform Definability of Types over Finite Sets (UDTFS) then the associated uniformly definable family of subsets is *compressible*, which has direct applications to learning theory and data compression.

It is easily seen that any formula φ with UDTFS must be dependent, but the converse remains open. Several years ago, Shelah proved that every stable formula φ has UDTFS, but we improve his argument by showing that the number of parameters needed is bounded by the Shelah 2-rank $R_{\varphi}(x = x, 2)$. We discuss ideas of Vincent Guingona [1], which, among other things, imply that every formula in a dp-minimal theory has UDTFS. This generalizes older results of Hunter Johnson and the speaker [2].

[1] VINCENT GUINGONA, *On uniform definability of types over finite sets*, www.math.umd.edu/~vincentg/UDTFS.pdf, (submitted).

[2] H. R. JOHNSON and M. C. LASKOWSKI, *Compression schemes, stable definable families, and o-minimal structures*, ***Discrete and Computational Geometry***, vol. 43 (2010), pp. 914–926.

► JEAN-MARIE LION, *Hausdorff limits of Rolle leaves*.
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Let $\mathcal{A}$ an o-minimal structure and $\mathcal{L}(\mathcal{A})$. We call a set nested subpfaffian over $\mathcal{A}$ if it is the projection of a positive boolean combination of definable sets and nested Rolle leaves over $\mathcal{A}$. Assuming that $\mathcal{A}$ admits analytic cell decomposition, we prove that the Haussdorf limit of a sequence of nested Rolle leaves of a nested distribution is a nested subpfaffian set over $\mathcal{A}$.

This is joint work with Patrick Speissegger.

► HENRY TOWSNER, *Amalgamation and regularity*.
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The Szemerédi regularity lemma and its generalization, the hypergraph regularity lemma, are powerful tools in extremal graph theory. If, instead of working in large finite graphs, we pass to an ultraproduct model with a measure, various parameters drop out, and the regularity lemma essentially says that the type of almost every point in this ultraproduct model is *wide* (contains no definable sets of measure 0). Proofs which use the regularity lemma are then translated into arguments requiring amalgamation properties on these types. We will describe this relationship between the regularity lemma and amalgamation properties.

► YIMU YIN, *Integration in algebraically closed valued fields with sections*.
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I will describe how to construct Hrushovski–Kazhdan style motivic integration in certain expansions of ACVF(0, 0). Such an expansion is typically obtained by adding a full section from the RV-sort into the VF-sort and some (arbitrary) extra structure in the RV-sort. The construction of integration, that is, the inverse of the lifting map $\mathbb{L}$, is rather straightforward. What is a bit surprising is that the kernel of $\mathbb{L}$ is still generated by one element, exactly as in the case of integration in ACVF(0, 0). The overall construction is more or less parallel to the main construction in the original paper of Hrushovski and Kazhdan.

Abstracts of invited talks in the Special Session on Set Theory

► ANDRÉS E. CAICEDO, *G_0-dichotomies*.
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The graph G_0 of Kechris–Solecki–Todorčević [3] is a key object in descriptive set theory. The theory $\mathsf{ZF}+\mathsf{DC}_{\mathbb{R}}+$there is a fine σ-complete measure on $\mathcal{P}_{\omega_1}(\mathbb{R})$ holds in the version of Solovay's model obtained by Levy collapsing a measurable cardinal to ω_1, and in all known models of the axiom of determinacy AD.

Continuing [1], Richard Ketchersid and I prove in this theory a version of the G_0-dichotomy for ∞-Borel sets, and use it to show that the quotient of $\mathbb{R}$ by the equivalence relation E_0 is a successor of $\mathbb{R}$, a result previously known under $\mathsf{AD}_{\mathbb{R}}$, but (perhaps surprisingly) not in $L(\mathbb{R})$. We generalize this dichotomy to an appropriate "local" version, which allows us to recover the AD^+-version of a dichotomy theorem of Hjorth [2], and to an infinite dimensional setting, previously investigated for analytic graphs by Lecomte and Miller.

[1] Andrés E. Caicedo and Richard Ketchersid, *A trichotomy theorem in natural models of* AD^+, ***Proceedings of the conference Boise Extravaganza in Set Theory***, (Liljana Babinkostova, Andrés Caicedo, Stefan Geschke, Richard Ketchersid, Marion Scheepers, editors), American Mathematical Society, to appear.

[2] Greg Hjorth, *A dichotomy for the definable universe*, ***The Journal of Symbolic Logic***, vol. 60 (1995), no. 4, pp. 1199–1207.

[3] Alexander Kechris, Sławomir Solecki, and Stevo Todorčević, *Borel chromatic numbers*, ***Advances in Mathematics***, vol. 141 (1999), no. 1, pp. 1–44.

► STEFAN GESCHKE, *Continuous n-colorings on Polish spaces*.
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An *n-coloring* on a set X is a map $c\colon [X]^n \to 2$. Continuous 2-colorings on Polish spaces have been studied in [3] and in [2]. Given an n-coloring c, the *homogeneity number* of c is the least size of a family $\mathcal{H}$ of c-homogeneous subsets of X with $\bigcup\mathcal{H} = X$. A coloring c is *uncountably homogeneous* if its homogeneity number is uncountable.

In [3] it was shown that there is an uncountably homogeneous, continuous 2-coloring $c_{\min}$ on 2^ω that embeds into every uncountably homogeneous, continuous 2-coloring on any Polish space. It turns out that the situation is slightly more complicated in higher dimensions. For every $n > 2$ there is a finite collection $\mathcal{B}$ of uncountably homogeneous, continuous n-colorings on 2^ω such that every uncountably homogeneous, continuous n-coloring on any Polish space contains an isomorphic copy of one element of $\mathcal{B}$. For $n = 3$, a lower bound for the size of such a *basis* $\mathcal{B}$ is 6.

The existence of finite bases is proved using a generalization of a result of Blass [1] on continuous n-colorings on 2^ω that we derive from Milliken's theorem [4].

If time permits, some things might be said about universal continuous n-colorings.

[1] A. BLASS, *A partition theorem for perfect sets*, ***Proceedings of the American Mathematical Society***, vol. 82 (1981), no. 2, pp. 271–277.

[2] S. GESCHKE, M. GOLDSTERN, and M. KOJMAN, *Continuous Ramsey theory on Polish spaces and covering the plane by functions*, ***Journal of Mathematical Logic***, vol. 4 (2004), no. 2, pp. 109–145.

[3] S. GESCHKE, M. KOJMAN, W. KUBIŚ, and R. SCHIPPERUS, *Convex decompositions in the plane and continuous pair colorings of the irrationals*, ***Israel Journal of Mathematics***, vol. 131 (2002), pp. 285–317.

[4] K. MILLIKEN, *A Ramsey theorem for trees*, ***Journal of Combinatorial Theory. Series A***, vol. 26 (1979), pp. 215–237.

► DOMINIQUE LECOMTE, *Borel colorings and related problems*.
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The analytic graphs of countable Borel chromatic number have been characterized by Kechris, Solecki and Todorcevic, who found a minimum counter-example G_0 and proved the following dichotomy result: an analytic relation on a Polish space has either a countable Borel coloring, or there is a continuous homomorphism from G_0 into it. When the relation is Borel and admits a countable Borel coloring, we can ask about the relation between the Borel rank of the relation and that of the coloring. This motivates the study of countable colorings of a given Borel level. In particular, we study the Baire class one countable colorings, i.e., the countable partitions into F_σ sets. Such a partition gives a covering of the diagonal into countably many F_σ squares. This leads to the study of countable unions of F_σ rectangles. We give a Hurewicz-like dichotomy for such countable unions.

► ASSAF RINOT, *Higher Souslin trees constructions*.
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Three classical results of Jensen reads as follows:

1. GCH + $\diamondsuit_{E^{\kappa^+}_\kappa}$ implies the existence of a κ^+-Souslin tree, for a regular cardinal κ;
2. GCH + $\square_\kappa$ implies the existence of a κ^+-Souslin tree, for a singular cardinal κ;
3. $\square^*_\kappa$ is equivalent to the existence of a special κ^+-Aronszajn tree, for every κ.

In this talk, we shall introduce a combinatorial principle that implies the existence of Souslin trees over successor cardinals, and which is general enough to follow from either of the hypotheses of items 1 and 2 above. We then study its validity.

We shall also outline the proof of [1] that a special $\aleph_{\omega_1+1}$-Aronszajn tree may (consistently, from large cardinals) be introduced by a forcing that shoots a club to $\aleph_2$.

[1] ASSAF RINOT, *A cofinality-preserving small forcing may introduce a special Aronszajn tree*, ***Archive for Mathematical Logic***, vol. 48 (2009), no. 8, pp. 817–823.

[2] ———, *A unified approach to higher souslin trees constructions*, work in progress.

► KATHERINE THOMPSON, *Factors affecting universality behaviour in graphs*.
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A universal model for λ-graphs (graphs of size λ) is one which embeds all other λ-graphs. Under GCH the theory of graphs has a universal model in every uncountable cardinal, as do first-order definable restriction to smaller classes of graphs (such as triangle-free graphs). However, for nonelementary (not first-order definable) classes of graphs, it is often the case that under GCH there are no universal models. For instance $\lambda^{<\kappa} = \lambda$ for λ, κ regular implies that there is no universal λ-graph which omits κ-cliques. This line separating elementary versus nonelementary classes of graphs becomes blurred in models of set theory with the failure of the GCH. In this case several factors come into play to influence universality behaviour, such as various cardinal arithmetic assumptions, model-theoretic or combinatorial properties of the specific class of graphs (e.g., amalgamation properties) and model construction (forcing techniques). The distictions in some cases can be independent of whether the class is elementary or not.

Abstracts of invited talks in the Special Session: Symposium on the Beyträge of Bernard Bolzano

► PAUL RUSNOCK, *Bolzano on the primitive elements of deductive systems*.
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Bolzano's 1810 essay *Contributions to a better-grounded presentation of mathematics* contains one of the first modern treatments of axiomatics. Of particular interest is Bolzano's discussion of the primitive elements of axiomatic systems (indefinable concepts, simple inferences, and axioms [*Grundsätze*]). In contrast with classical authors such as Aristotle, Descartes and Pascal, Bolzano makes no attempt to characterize primitives in extra-logical terms (e.g., as the fundamental principles of being or as things so clearly and distinctly understood that nothing more clear could be found to illuminate them). Instead, he consistently looks for logical and semantic criteria of primitiveness. Afterwards, he observes that there is no reason to suppose that what he calls the "objective connections" in axiomatic theories will line up with epistemological or metaphysical relations in any obvious way. In my talk, I will discuss the evolution of Bolzano's views on primitives from the *Contributions* to the *Theory of Science* (1837). It will emerge that, for good reasons, Bolzano's views on simple inferences and axioms (later, basic truths [*Grundwahrheiten*]) became less specific, while his views on simple concepts or ideas became more so.

[1] BERNARD BOLZANO, ***Beyträge zu einer begründeteren Darstellung der Mathematik. Erste Lieferung***, Prague, 1810.
[2] ———, ***Wissenschaftslehre***, Seidel, Sulzbach, 1837.

► STEVE RUSS, *The mathematical motivation for Bolzano's logic*.
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It is now fairly well known that Bernard Bolzano made numerous fundamental technical contributions to mathematics (especially in analysis and set theory) and that he did so as a partial consequence of some strict conceptual constraints. For example, he adopted and used effectively (in 1816) notations and arguments similar to the ϵ-δ methods usually credited to Weierstrass in the 1860's, he came as close as possible at the time to a rigorous proof of the intermediate value theorem (1817). He developed a theory of real numbers and constructed a function everywhere continuous and nowhere differentiable in the 1830's, long before the Weierstrassian example. His major work "Theory of Science" [***Wissenschaftslehre***] appeared in 1837 and contained numerous remarkable contributions to logic. (See the Stanford Encyclopedia entry "Bolzano's Logic" by Jan Sebestik for details.) One of the central ideas of that work was his notion of an objective "ground-consequence" [***Abfolge***] relation. This notion is introduced informally, and for the first time, in the publication we are celebrating in this session, his 1810 ***Beyträge zu einer begründeteren Darstellung der Mathematik***.

In this paper we shall examine the idea that the "objective connection" proposed by Bolzano in 1810 was strongly motivated as a remedy for the profound disorder in mathematics —especially in geometry—that Bolzano had diagnosed in his earlier publication (1804) on geometry and which he continued to emphasise in his 1810 work. We shall draw evidence for our case from several volumes of the on-going publication of the Bernard Bolzano ***Gesamtausgabe***. Further details of Bolzano's mathematical work may be found in Paul Rusnock, ***Bolzano's Philosophy and the Emergence of Modern Mathematics***, Rodopi, 2000; English translations of the mathematical works mentioned here are in Steve Russ, ***The Mathematical Works of Bernard Bolzano***, Oxford University Press, 2004.

► JAN SEBESTIK, *The significance of Bolzano's Beyträge zu einer begründeteren Darstellung der Mathematik*.
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Two of the first Bolzano's publications have permanent interest: the Rein analytischer Beweis (1817) which inaugurates the arithmetization of analysis, and the Beyträge zu einer begründeteren Darstellung der Mathematik (1810). While the first one was noticed by Weierstrass, the second one, going against the spirit of the dominant Kantian and postkantian philosophy, was completely neglected.

Written in the Leibnizian spirit, the Beyträge develop a new philosophy of mathematics and should influence the philosophy of the XIXth century in a similar way as Wittgenstein's Tractatus did in the XXth century. For various reasons, this did not happen. His main innovations consist in a new definition of mathematics, the claim of the objectivity of mathematics, the first modern study of an axiomatic system, an innovative theory of definition, a proof theory whose core is the the idea of grounding (Begründung), the criticism of Kant's philosophy of mathematics and, in the second unfinished issue, published only in 1975, the first outline of his doctrine of collections.

Under the title Universal mathematics, Bolzano tries to determine the laws governing all things. It is in fact a problem of ontology and Bolzano starts by examining the ontological principles established in the Leibnizian–Wolffian school and in Kant. According to Bolzano, the validity of all these principles is limited and therefore he sets up his own principles: (1) the principle of thinking-together (Zusammendenken) according to which any thing can be thought together with any other thing, (2) the principle of relation stating that any thing is in a certain relation with any other thing. The first principle reminds the famous Cantor's definition of a set. Bolzano's concept of collection thus becomes the fundamental concept of mathematics. His doctrine will be worked out in the Grössenlehre in the 1830ties and in the Paradoxes of the infinite (1851) which will be one of the most important sources of Cantor's and Dedekind's set theories.

Abstracts of invited talks in the Special Session: Symposium on Simplicity (Complexity) of Proofs— Mathematical and Philosophical

► MATTHIAS BAAZ, *Generalization*.
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In this lecture we compare generalizations based on logical transformations to generalizations in mathematics. Concerning logical transformations we focus on

- generalizations based on invariants of proofs wrt to conclusions/premises
- generalizations based on invariants of theories.

In addition we consider the problem of confluence wrt generalizations based on proof transformations. This is connected to the relation of the cut-free proof after cut-elimination to the original proof.

► ALESSANDRA CARBONE, *Proof structures and complexities*.
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The genus of a proof is a measure of proof complexity and through it we discuss a few geometrical properties of logical flow graphs of proofs, with and without cuts, with the purpose in mind to represent how complicated a cut-free proof can be. The main result says that arbitrarily complicated non oriented graphs, that is graphs of arbitrarily large genus, can be encoded in a cut-free proof. This fact was proved by Richard Statman in his thesis written in the early seventies and never published. We reformulate Statman's result in a purely graph theoretical language and will present the intuition of the proof. We show that there are several ways to embed non oriented graphs of arbitrary complexity into cut-free proofs and provide some other direct embeddings of arbitrarily complex non oriented graphs into proofs possibly with cuts. We also show that given any formal circuit, we can codify it into a proof in such a way that the graph of the circuit corresponds to the logical flow graph of the encoding proof [2].

Time permitting, we shall describe how certain finitely presented groups are encoded in the structure of proofs with cuts. The cyclic structure that they form and the impact in proof complexity that such cycles induce will be explained [1].

[1] A. Carbone, *Asymptotic cyclic expansion and bridge groups of formal proofs*, ***Journal of Algebra***, vol. 242 (2001), pp. 109–145.

[2] ———, *Logical structures and genus of proofs*, ***Annals of Pure and Applied Logic***, vol. 161 (2009), no. 2, pp. 139–149.

► PANU RAATIKAINEN, *On the complexity vs. the length of a proof*.
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After a brief survey of the various measures of the "complexity" of a proof in the logic literature, it is suggested that all the standard notions measure in reality the length, or size, rather than the complexity of a proof; or, at least, there is a kind of complexity that such more familiar notions fail to capture. A fresh notion of complexity, which bears some analogy to Kolmogorov complexity, but relative to the formulas of formalized theories rather than to the programs of idealized computers, is then introduced. For simplicity, the concept and its basic logical properties are first discussed in the context of numbers and arithmetical theories; most importantly, complexity is not a decidable property. The approach is then generalized and applied to formalized metamathematical theories (such as Feferman's FM_0, or FS_0) and derivations they talk about, and it is proposed that this provides a rather natural notion of the complexity of a proof. Some connections to Dummett's concepts of canonical and non-canonical proofs are also pointed out.

Abstracts of contributed talks

► NIKOLAY BAZHENOV, *On Δ^0_α categoricity of the Boolean algebra $I(\omega^\beta \times \eta)$*.
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Ash [1] determined for which ordinals α a given superatomic Boolean algebra is Δ^0_α categorical. One can ask the same question considering other types of computable Boolean algebras instead of superatomic. The main goal of this paper is to answer the question considering the interval algebra $I(\omega^\beta \times \eta)$.

We use basic notions and definitions from [2] and [3]. Let δ be either a limit ordinal or 0, $\delta < \omega_1^{CK}$ and $n \in \omega$.

First we show that $I(\omega^{\delta+n} \times \eta)$ has a formally $\Sigma^0_{\delta+2n+1}$ Scott family. Then we consider standard back-and-forth relations in Boolean algebras and prove the following

LEMMA 1. *Let β, γ be arbitrary ordinals, and let $l, k < \omega$, $l \neq 0$. Then*
(a) *Let $\delta + k \geq 1$.*
(1) $I(\omega^\beta \times \eta) \leq_{\delta+2k} I(\omega^\gamma \times l)$ *if and only if* $\beta \geq \delta + k$ *and* $\gamma \geq \delta + k$.
(2) $I(\omega^\gamma \times l) \leq_{\delta+2k} I(\omega^\beta \times \eta)$ *if and only if* $\beta + 1 \geq \delta + k$ *and* $\gamma \geq \delta + k$.
(b) (1) $I(\omega^\beta \times \eta) \leq_{\delta+2k+1} I(\omega^\gamma \times l)$ *if and only if* $\beta \geq \delta + k$ *and* $\gamma \geq \delta + k$.
(2) $I(\omega^\gamma \times l) \leq_{\delta+2k+1} I(\omega^\beta \times \eta)$ *if and only if* $\beta \geq \delta + k$ *and* $\gamma \geq \delta + k + 1$.

Then we can apply theorems from [1] to prove the main result of the paper:

THEOREM 2. *Boolean algebra $I(\omega^{\delta+n} \times \eta)$ is $\Delta^0_{\delta+2n+1}$ categorical and not $\Delta^0_{\delta+2n}$ categorical.*

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[1] C. J. ASH, *Categoricity in hyperarithmetical degrees*, ***Annals of Pure and Applied Logic***, vol. 34 (1987), no. 1, pp. 1–14.

[2] C. J. ASH and J. KNIGHT, ***Computable structures and the hyperarithmetical hierarchy***, Studies in Logic and the Foundations of Mathematics, Elsevier Science, Amsterdam, 2000.

[3] S. S. GONCHAROV, ***Countable Boolean algebras and decidability***, Siberian School of Algebra and Logic, Consultants Bureau, New York, 1997.

▶ LUC BÉLAIR AND FRANÇOISE POINT, *Definability in valued Ore modules*.

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We consider valued fields with a distinguished operator having a close interaction with the valuation, as valued modules over a skew polynomial ring (Ore ring). This amounts to study linear equations with respect to the distinguished operator. Under certain assumptions on the residue field, but in all characteristics, we study quantifier elimination in natural languages, and the absence of the independence property.

▶ HOVHANNES BOLIBEKYAN AND IGOR ZASLAVSKY, *On the relation between two systems of fuzzy intuitionistic logic*.

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The relation between two systems of fuzzy intuitionistic logic is investigated. The first system of fuzzy intuitionistic logic is one considered in [2] and further we refer to it as *IF*. The second system is introduced in [3] and further denoted by *EFCL*. We use the notions of recursively enumerable fuzzy set (REFS) and other concepts of constructive mathematical analysis as they defined in [1], [3].

The notion of a classical fuzzy set ideal (CFS-ideal) plays a key role for our main result.

An n-dimensional CFS-ideal is defined as a non-empty set I of n-dimensional recursively enumerable fuzzy sets that satisfies the following conditions:
(1) if $\varphi \in I$ and $\psi \in I$ then $max(\varphi, \psi) \in I$;
(2) if $\psi \in I$ and $\varphi \subseteq \psi$ then $\varphi \in I$.
For every CFS-ideal I we introduce the notion of its majorant as $\varphi_0(x_1, \dots, x_n) = \sup\limits_{\varphi \in I} \varphi(x_1, \dots, x_n)$.
We define union ($\bigcup$), intersection ($\bigcap$), Cartesian product ($\times$), projection ($\downarrow_i^n$), generalization ($\uparrow_i^n$), trasposition (T_{ij}^n), and substitution (Sub_{ij}^n) operations over CFS-ideals in a way these operations are defined over REFSs in [3].

THEOREM. *IF is obtained from EFCL if in the latter one instead of CFS-ideals their majorants are considered as logical values.*

[1] B. A. KUSHNER, ***Lectures on constructive matehamtical analysis***, Translations of Mathematical Monographs, vol. 60, American Mathematical Society, 1984.
[2] G. TAKEUTI and S. TITANI, *Intuitionistic fuzzy logic and intuitionistic fuzzy set theory*, ***The Journal of symbolic logic***, vol. 49 (1984), no. 3, pp. 851–866.
[3] I. D. ZASLAVSKI, *Extended fuzzy constructive logic*, ***Zapiski nauchnykh seminarov POMI*** (in Russian), to appear.

► ELENA BUNINA, *Elementary, typical and second order equivalence of derivative models*.
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The first result on elementary equivalence of derivative structures was theorem of A. I. Maltsev (see [3]) in 1961 who proved that two linear groups $\mathcal{G}_n(K)$ and $\mathcal{G}_m(L)$ ($m, n \geqslant 2$, K, L are infinite fields) are elementary equivalent if and only if $n = m$ and the fields K and L are elementary equivalent.

Later these results were extended by K. I. Beidar and A. V. Mikhalev in 1992 (see [1]). They used the construction of ultraproduct and Keisler–Shelah Isomorphism Theorem and proved Maltsev-type theorems for different structures: linear groups over skewfields, over associative rings, lattices of submodules and so on.

E. I. Bunina, A. V. Mikhalev, P. P. Semenov, A. S. Dobrokhotova–Maikova in 1998–2009 proved similar results for other derivative structures: unitary linear groups over rings, incidence algebras over rings, semigroups of invertible nonnegative matrices over ordered rings. All these results have similar sense: some derivative structures are elementary equivalent if and only if the initial structure are elementary equivalent and some other parameters of construction coincide (for example, dimension or something else).

There are results of another sort, when elementary equivalence of derivative structures is equivalent to some other conditions for initial structures. For example, there is the following theorem:

THEOREM (E. I. Bunina and A. V. Mikhalev, [2]). *Endomorphism rings of Abelian p-groups A_1 and A_2 are elementary equivalent if and only if the groups A_1 and A_2 are second-order equivalent.*

Similar results were proved for categories of modules over rings, endomorphism rings, automorphism groups and submodule lattices of free modules of infinite rank, and other structures.

Also there are some Maltsev-type results about different types of equivalences of linear groups (typical equivalence, universal equivalence).

[1] K. I. BEIDAR and A. V. MIKHALEV, *On Malcev's theorem on elementary equivalence of linear groups*, ***Contemporary Mathematics***, vol. 131 (1992), pp. 29–35.

[2] E. I. Bunina and A. V. Mikhalev, *Elementary equivalence of endomorphism rings of Abelian p-groups*, ***Journal of Mathematical Sciences***, vol. 137 (2006), no. 6, pp. 5212–5274.

[3] A. I. Maltsev, *On elementary properties of linear groups*, ***Problems of mechanics and mathematics, Novosibirsk***, (1961), pp. 110–132.

► VEDRAN ČAČIĆ, *Towards the normal form theorem for Interpretability logic*.
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Interpretability logic (IL) is a modal logic extending Provability logic, defined with the purpose of describing interpretability between theories. While normal forms of closed formulas have been discovered for Provability logic, and for some extensions of IL (for ILF, by P. Hájek and V. Švejdar), we are not aware of any work concerning normal forms for the whole IL. Although the general normal form still seems far away, we prove many simplifications for various families of formulas of IL, enabling us to search for normal forms more systematically. The main part of the work is done semantically, by analyzing possible structure of Veltman models, for which IL is complete.

► ANAHIT CHUBARYAN, ARMINE CHUBARYAN, AND ASHOT ABAJYAN, *On numerical properties of determinative disjunctive normal forms for tautologies*.
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The representations of tautologies in disjunctive normal forms (DNF) are used in many proof systems of propositional logic, therefore these representations can be interesting for proof complexities.

It is well-known that a Boolean function can be represented by different DNF: perfect, abridged, dead-end, shortest and minimal. It is known also that every conjunct from abridged (therefore from every dead-end, every minimal and at least one shortest) DNF corresponds to some prime implicant. If we consider a tautology in the capacity of the Boolean function, then it has unique prime implicant, which corresponds to empty conjunct, but it is well-known, that there are "hard" and "simple" tautologies, therefore the representation of tautology in some above varieties of DNF is not entirely correct.

Earlier the following notions were introduced by the first coauthor of this abstract.

Let φ be a propositional formula, $X = \{x_1, x_2, \dots, x_n\}$ be the set of distinct variables of φ and $X' = \{x_{i_1}, x_{i_2}, \dots, x_{i_m}\}$ $(1 \leqslant m \leqslant n)$ be some subset of X.

Definition 1. Given $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_m) \in E^m$, the conjunct $K^\sigma = \left\{x_{i_1}^{\sigma_1}, x_{i_2}^{\sigma_2}, \dots, x_{i_m}^{\sigma_m}\right\}$ is called ***determinative*** for φ if the assignment of values σ_j $(1 \leqslant j \leqslant m)$ to each x_{ij} we obtain the value of φ (0 or 1) independly of the values of the remaining variables.

Definition 2. Determinative for φ conjunct $K^\sigma = \left\{x_{i_1}^{\sigma_1}, x_{i_2}^{\sigma_2}, \dots, x_{i_m}^{\sigma_m}\right\}$ is called ***minimal determinative*** if no subset of K^σ is determinative for φ.

Definition 3. DNF $\mathcal{D} = \{K_1, K_2, \dots, K_\ell\}$ is called determinative DNF (dDNF) for φ if $\varphi = \mathcal{D}$ and every conjunct K_j $(1 \leqslant i \leqslant \ell)$ is determinative for φ.

Basing on these notions we introduce for tautologies the notions of determinative versions for above mentioned DNF and search the numerical characteristics of dDNF and characteristics connecting various types of dDNF of a given tautology.

We search also the relation between the complexities of different dDNF and proof complexities in different proof systems for a given tautology.

► ANAHIT CHUBARYAN, ARMINE CHUBARYAN, AND HAKOB NALBANDYAN, *On the sizes in Frege proofs and substitution Frege proofs*.
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It is known that the minimal number of the steps in a proof of a tautology in a Frege system can be exponentially larger than in a substitution Frege system, but it is an open problem whether Frege systems can polynomially simulate substitution Frege systems by sizes. Many people conjecture that the answer is **no**. We prove that the answer is **yes**.

We use the generally accepted concept of Frege system, the well-known notions of proof complexities (size and lines) and the notion of polynomial equivalence (by size and by lines) of the proof systems.

Let $\mathcal{F}$ be some Frege system. The substitution Frege system $S\mathcal{F}$ consists of Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution $\sigma = \begin{pmatrix} \varphi_{i_1} & \varphi_{i_2} & \dots & \varphi_{i_m} \\ p_{i_1} & p_{i_2} & \dots & p_{i_m} \end{pmatrix}$, where p_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional variables, φ_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional formulas, and $A\sigma$ denotes the result of applying of the substitution σ to formula A. Such substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions. If the depths of formulas φ_{i_j} $(1 \leqslant j \leqslant m)$ are restricted by some fixed $(d \geqslant 0)$, then we have d-restricted substitution rule and we denote the corresponding system by $S^d\mathcal{F}$. If $m = 1$, then we have *single* substitution. We denote Frege system with d-restricted single substitution rule by $S^d_1\mathcal{F}$.

We prove that

(1) given arbitrary d, the systems $S^d\mathcal{F}$, $S^d_1\mathcal{F}$ and $S\mathcal{F}$ are polynomially equivalent by size,
(2) $S^0_1\mathcal{F}$ and $\mathcal{F}$ are polynomially equivalent both by lines and by size, and therefore
(3) $S\mathcal{F}$ and $\mathcal{F}$ are polynomially equivalent by size.

► ANDRÉS CORDÓN–FRANCO, ALEJANDRO FERNÁNDEZ–MARGARIT AND F. FÉLIX LARA–MARTÍN, *On conservation results for conditional axiom schemes in first order arithmetic*.
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A set of conditional axioms, E, is a set of formulas (in a first order language) of the form $\alpha(\vec{v}) \to \beta(\vec{v})$. For such a set E an inference rule, E–Rule, is defined by the instances:

$$\frac{\forall \vec{v}\, \alpha(\vec{v})}{\forall \vec{v}\, \beta(\vec{v})}, \quad \text{for each } \alpha(\vec{v}) \to \beta(\vec{v}) \in E.$$

In addition, two new sets of conditional axioms can be defined:

$$E^- = \{\varphi \in E : \varphi \text{ is a sentence}\} \text{ and } UE = \{\forall \vec{v}\, \alpha(\vec{v}) \to \forall \vec{v}\, \beta(\vec{v}) : \alpha(\vec{v}) \to \beta(\vec{v}) \in E\}.$$

In this work (inspired by L. Beklemishev's work in [2], [4] and [5]), for a base theory T, we study conservation theorems among the theories $T + E$, $T + UE$, $T + E^-$ and $T + E$–Rule (the closure of T under first order logic and applications of the E–Rule). We use a hierarchical version of the notion of an existentially closed model to develop an abstract version of J. Avigad's model–theoretic approach to conservation results (see [1], there Avigad introduced a model–theoretic analogue of Herbrand Analysis). The resulting model–theoretic framework allows us to isolate fairly general conditions that E must satisfy in order to derive conservation results among the theories previously associated to E. Applications include simple and uniform model–theoretic proofs of conservation results for Σ_{n+1}–collection, Δ_{n+1}–induction and Uniform Reflection (obtained by Beklemishev in [2], [4] and [5]) and new conservation results for parameter free Δ_{n+1}–induction and parameter free Π_{n+1}–induction. In particular, a model–theoretic proof of Beklemishev's conservation theorem between parameter free Π_2–induction and Σ_1–induction (see [3]) can be clearly described in this framework.

[1] J. Avigad, *Saturated models of universal theories*, ***Annals of Pure and Applied Logic***, vol. 118 (2002), no. 3, pp. 219–234.

[2] L. D. Beklemishev, *A proof–theoretic analysis of collection*, ***Archive for Mathematical Logic***, vol. 37 (1998), no. 5–6, pp. 275–296.

[3] ———, *Parameter free induction and provably total computable functions*, ***Theoretical Computer Science***, vol. 224 (1999), no. 1–2, pp. 13–33.

[4] ———, *On the induction schema for decidable predicates*, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 1, pp. 17–34.

[5] ———, *Proof–theoretic analysis by iterated reflection*, ***Archive for Mathematical Logic***, vol. 42 (2003), no. 6, pp. 515–552.

► WAGNER DE CAMPOS SANZ AND THOMAS PIECHA, *A criticism of the BHK interpretation.*
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We criticize the BHK interpretation by considering basic and non-basic extensions of natural deduction for intuitionistic logic NJ. For any basic extension we present a construction (which depends on the disjunction property) that converts each proof of the premiss $\neg A \to (B \vee C)$ of Harrop's rule into a proof of its conclusion $(\neg A \to B) \vee (\neg A \to C)$. According to the BHK interpretation for implication the corresponding Kreisel–Putnam formula $(\neg A \to (B \vee C)) \to ((\neg A \to B) \vee (\neg A \to C))$ is then assertable. However, the Kreisel–Putnam formula is not derivable in any basic extension of NJ. Therefore the BHK interpretation and NJ mismatch for basic extensions.

Next we consider non-basic extensions. In intuitionistic (Heyting) arithmetic HA the open formula $Odd(x) \vee Even(x)$ has a proof in strictly normal form whose last step is an application of the induction rule. The disjunction property does not hold for this formula since neither $Odd(x)$ nor $Even(x)$ are provable in HA. Thus the construction used in the case of basic extensions fails for non-basic extensions like HA. That is, if the BHK interpretation is intended to be adequate for all intuitionistically acceptable extensions, including non-basic ones, then the aforementioned mismatch disappears. However, the BHK interpretation cannot then be taken as the correct description of what constitutes an assertion of a complex formula anymore, since $Odd(x) \vee Even(x)$ is provable in HA and thus HA assertable, but its assertion is not obtained in accordance with the BHK interpretation for disjunction. Therefore the BHK interpretation is at least incomplete with respect to HA. As a consequence, the thesis that the BHK interpretation gives the operational meaning of the logical constants would have to be given up if the induction rule is considered an acceptable extension.

► MICHIEL DE SMET, *"Thin implies Ramsey" template as a source of unprovability results.*
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We start with introducing two basic concepts we deal with. Consider a family $\mathcal{F}$ of finite subsets of natural numbers. Then $\mathcal{F}$ is called *thin* if s is not an initial segment of t, for every two distinct elements s and t of $\mathcal{F}$. Define $\mathcal{F}$ to be *Ramsey* if for every finite partition

$$\mathcal{F} = \mathcal{F}_0 \cup \cdots \cup \mathcal{F}_k$$

there is an infinite set $M \subseteq \mathbb{N}$ such that at most one of the restrictions

$$\mathcal{F}_0 \cap \mathcal{P}(M), \ldots, \mathcal{F}_k \cap \mathcal{P}(M)$$

is non-empty.

Given the previous definitions, we look at the following theorem of Nash-Williams (see e.g., [3]):

"Thin families of finite subsets of natural numbers are Ramsey".

First, we generalise thinness and Ramseyness to *labelled structures*, which are sets with relations on them and number labels attached to their elements. Among many examples of these structures, there are words in a finite or infinite alphabet, graphs and labelled trees. Using *combinatorial forcing*, we extend the Nash-Williams theorem to labelled structures. In this way, "thin implies Ramsey" becomes a template, which yields a large class of theorems of the same shape, all of them possessing proof-theoretic strength. Second, we give finite versions of theorems of this class in such a manner that proof-theoretic strength is preserved.

Another topic we touch upon deals with Schreier families (see [1] for a definition). We introduce a finite version of the Ramsey partition theorem extended to countable ordinals (see [2]) which implies the Paris–Harrington statement (and thus is unprovable in Peano arithmetic).

This is joint research with Andrey Bovykin.

[1] Spiros A. Argyros and Stevo Todorcevic, ***Ramsey methods in analysis***, Advanced Courses in Mathematics, CRM Barcelona, Birkhäuser Verlag, Basel, 2005.

[2] Vassiliki Farmaki, *Ramsey and Nash-Williams combinatorics via Schreier families*, 2004, arXiv:math/0404014v1.

[3] Crispin St. J. A. Nash-Williams, *On well-quasi-ordering transfinite sequences*, ***Proceedings of the Cambridge Philosophical Society***, vol. 61 (1965), pp. 33–39.

► ANGEL DITCHEV, *On a notion of a structure and its degree*.
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In [1, 2] there have been considered partial structures and it's been given necessary and sufficient conditions for a partial structure to have an e-degree(T-degree). Those conditions are in the terms of so called type($\exists$-type) of an element of the universe of a structure. When we consider only unary partial structures, all types are determined of codes of the family of all definable sets. It directs that we can consider a structure as a enumerated family of subsets of a fixed set. This fixed set plays role of the universe of the structure. On the other hand we can consider this enumerated family as an "effective topological space". Similar way it could be defined type and $\exists$-type in new structures and to prove the similar theorem, giving necessary and sufficient conditions for new structures to have e-degrees. In the new structures it can be defined relations of the kind "e-reducibility" and least upper bound. For a fixed e-degree $\mathbf{a}$ there is a universal structure such that for every structure with degree $\leq \mathbf{a}$ there exist an embedding in the universal one.

[1] A. V. Ditchev, *Least enumerations of unary partial structures*, ***The Bulletin of Symbolic Logic***, vol. 16 (2010), no. 1, pp. 114–115.

[2] ———, *Least enumerations of partial structures*, ***Journal of Logic and Computation***, Special issue from ***Logic Colloquium 2009***, 2010, accepted for publication.

► VLADIMIR FILATOV, *Trajectory characterization for particular classes of hybrid automata*.
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Hybrid automata are dynamic systems with both continuous and discrete dynamics. A run of a hybrid automaton is a sequence of steps. Within each step the system state evolves continuously according to a dynamical law until a transition occurs. At this point discrete state changes and continuous state experiences a jump. Such automata are widely used to model systems where environment with continuously evolving parameters is controlled by a computer.

In this paper a particular type of hybrid automata is considered which is defined by the following objects $(X, \Phi_1, \dots, \Phi_m, A, M)$, $m \in \mathbb{Z}$, where X is a state space, $\Phi_1, \dots, \Phi_m$ are families of mappings $\mathbb{R}_+ \to X$, A is an alphabet (a finite set of a special form) and map M is a mapping $X \to 2^A$. A trajectory of the system consists of pieces. Each piece is defined by a mapping from some family Φ_i, $i \in \{1, \dots, m\}$, while map M defines states where a mapping family is changed.

Such a hybrid automaton is considered not only as a dynamic system, but as a computational system also. It is a real-time analog of Turing machine where state space is $\mathbb{R}^n$, $n \in \mathbb{N}$, or generally any set (instead of a tape) to elements of which letters of an alphabet are assigned that define a choice of a trajectory followed by the machine head.

In this paper complete characterization of trajectories is provided for the cases when X is a finite set and when X is $\mathbb{R}$. For the case when X is $\mathbb{R}^2$ it is stated that all mappings from $\mathbb{Z}^2$ to $\mathbb{Z}^2$ can be computed in a certain sense using hybrid automata. Some open questions are considered.

[1] R. Alur, C. Courcoubetis, N. Halbawachs, T. A. Henzinger, P. H. Ho, X. Nicollin, A. Olivero, J. Sifakis, and S. Yovine, *The algorithmic analysis of hybrid systems*, ***Theoretical Computer Science***, vol. 138 (1995), pp.3–34.

[2] R. Alur and D. L. Dill, *Automata for modeling real-time systems*, ***Lecture Notes in Computer Science***, vol. 443 (1990), pp. 332–335.

► DANDOLO FLUMINI, *Weak well-orders and recursion*.
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After introducing the notion of weak well-orders and the associated principles of weak induction and weak minimality, we present a formal theory that extends NBG by a schema ETR^+. This new principle takes the form: If A is a weak well-order then there exists a class H that satisfies some property $\mathsf{P}_\varphi(H, A)$. We then give a short argument that in cases where A is a well-order this principle is a very "natural"translation of the principle of transfinite recursion along A. We continue by proving that, when we restrict ETR^+ to well-orders, it is indeed equivalent to the principle of elementary transfinite recursion. Further on, it is indicated how the schema ETR^+ can be obtained from NBG plus the assertion that fixed points of elementary definable and monotone operators exist (as classes). In the end, we present some work in progress that aims at proving the reverse direction of this implication as well.

[1] Jeremy Avigad, *On the Relationships between ATR_0 and $\widehat{ID}_{<\omega}$*, ***The Journal of Symbolic Logic***, vol. 61 (1996), no. 3, pp. 768–779.

[2] Stephen G. Simpson, ***Subsystems of second order arithmetic***, Perspectives in Logic, Cambridge University Press, 2009.

► MARCO FORTI, *Quasi-selective and weakly Ramsey ultrafilters*.
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Throughout this paper $\mathcal{U}$ is a nonprincipal ultrafilter on $\mathbb{N}$, all functions are in $\mathbb{N}^{\mathbb{N}}$,and

$$\mathfrak{S} = \{\sigma \colon \mathbb{N} \to \mathbb{N} \mid \sigma \text{ 1-1}\}, \quad \mathfrak{F} = \{f \colon \mathbb{N} \to \mathbb{N} \mid f \text{ nondecreasing}\}.$$

It is well known that $\mathfrak{U}$ is selective iff every finite colouring of $[\mathbb{N}]^2$ has a homogeneous set $U \in \mathfrak{U}$ iff every function is nondecreasing on some $U \in \mathfrak{U}$. Natural weakenings of these properties led to the notions of *weakly Ramsey* and of *quasi-selective* ultrafilter, introduced and studied in [1] and [2], respectively. Here we consider the relations holding between weakly Ramsey and (a natural generalization of) quasi-selective ultrafilters.

DEFINITION 1. A *nonselective* ultrafilter $\mathfrak{U}$ is *weakly Ramsey* (shortly WR) if for every finite colouring of $[\mathbb{N}]^2$ there is $U \in \mathfrak{U}$ s.t. $[U]^2$ has *only two colours*.
$\mathfrak{U}$ is *g-quasiselective* (shortly g-QS) if for all $h \leq g$ there exists $f \in \mathfrak{F}$ s.t. $f \equiv_{\mathfrak{U}} h$.
$\mathfrak{U}$ is *quasi-selective* (shortly QS) if it is id-QS.
$\mathfrak{U}$ is *weakly quasi-selective* (shortly WQS) if it is g-QS for *some, but not all* g.

THEOREM 2. *Let $g \in \mathbb{N}^{\mathbb{N}}$ and $\sigma \in \mathfrak{S}$ be unbounded modulo $\mathfrak{U}$. Then*

1. $\mathfrak{U}$ *is g-QS* $\Longrightarrow g\mathfrak{U}$ *is QS*;
2. $\mathfrak{U}$ *is σ-QS* $\Longleftrightarrow \sigma\mathfrak{U}$ *is σ^{-1}-QS*;
3. *if $\sigma \equiv_{\mathfrak{U}} f \in \mathfrak{F}$, then $\mathfrak{U}$ is $(g \circ \sigma)$-QS* $\Longleftrightarrow \sigma\mathfrak{U}$ *is g-QS*;
4. *if $\mathfrak{U}$ is g-QS, then $\mathfrak{U}$ is selective* $\Longleftrightarrow \mathfrak{U}$ *is rapid.*

For unbounded $p \in \mathfrak{F}$ put

$$p^{-}(x) = \min\{y \mid p(y) = p(x)\}, \quad b_p^U(x) = |U \cap [0, p^{-}(x)]|, \quad \mathfrak{B}_p^{\mathfrak{U}} = \{b_p^U \mid U \in \mathfrak{U}\},$$

and

$$p^{+}(x) = \max\{y \mid p(y) = p(x)\}, \quad c_p^U(x) = |U \cap [p^{-}(x), p^{+}(x)]|, \quad \mathfrak{C}_p^{\mathfrak{U}} = \{c_p^U \mid U \in \mathfrak{U}\}.$$

THEOREM 3. *Let $\mathfrak{U}$ be WR, and let $S^{\mathfrak{U}}$, $F^{\mathfrak{U}}$, $B_p^{\mathfrak{U}}$, and $C_p^{\mathfrak{U}}$ be the (lower parts of the) cuts generated in the ultrapower $\mathbb{N}^{\mathbb{N}}_{\mathfrak{U}}$ by $\mathfrak{S}$, $\mathfrak{F}$, $\mathfrak{B}_p^{\mathfrak{U}}$, and $\mathfrak{C}_p^{\mathfrak{U}}$ respectively. Then*

$$\mathbb{N} \leq B_p^{\mathfrak{U}} \leq S^{\mathfrak{U}} \leq F^{\mathfrak{U}} \leq C_p^{\mathfrak{U}}$$

for all nonconstant p that are not 1-1 *modulo* $\mathfrak{U}$.
Moreover exactly one of the following three cases occurs:

1. $\mathbb{N} < B_p^{\mathfrak{U}} < C_p^{\mathfrak{U}} \Longleftrightarrow \exists \sigma \in \mathfrak{S}$ *s.t.* $\mathfrak{U}$ *is σ-QS* $\Longleftrightarrow \exists \mathfrak{U}' \cong \mathfrak{U}$ *s.t.* $\mathfrak{U}'$ *is QS*;
2. $\mathbb{N} < B_p^{\mathfrak{U}} = C_p^{\mathfrak{U}} \Longleftrightarrow \exists g$ *s.t.* $\mathfrak{U}$ *is g-QS* & $\neg\exists \mathfrak{U}' \cong \mathfrak{U}$ *s.t.* $\mathfrak{U}'$ *is QS* (*so* $g \notin \mathfrak{S}$);
3. $\mathbb{N} = B_p^{\mathfrak{U}} = C_p^{\mathfrak{U}} \Longleftrightarrow \neg\exists g$ *s.t.* $\mathfrak{U}$ *is g-QS* $\Longleftrightarrow \mathfrak{U}$ *is rapid.*

In particular $\mathfrak{U}$ is QS if and only if $B_p^{\mathfrak{U}} < [id]_{\mathfrak{U}} < C_p^{\mathfrak{U}}$.

Remark that both WR and WQS ultrafilters are P-points, so the above results are nontrivial only when such ultrafilters exist. However mild hypotheses, like CH or MA, suffice in making all these classes rich and distinct (see [1, 2]).

[1] A. BLASS, *Ultrafilter mappings and their Dedekind cuts*, ***Transactions of the American Mathematical Society***, vol. 188 (1974), pp. 327–340.

[2] A. BLASS, M. DI NASSO, and M. FORTI, *Quasi-selective-ultrafilters and asymptotic numerosities*, submitted.

[3] M. DI NASSO, *Fine asymptotic densities for sets of natural numbers*, ***Proceedings of the American Mathematical Society***, in print.

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We are told that there are standards of rigour, and are told that the standard has increased over the centuries. This is fairly clear. But rigour has also changed its nature. This is so in two respects, one concerns the interplay between arithmetic and geometry. The other concerns developments in logic. We concentrate on the former in this paper. Aristotle sought to rid proofs of all arithmetical content in favour of geometry since these were the better (more rigourous) proofs. Descartes showed us that we could re-describe geometry using arithmetic. Hilbert sought to rid all geometric proofs of geometrical intuition, and replace them with formal, uninterpreted calculation. So we had a complete reversal, where first geometry set the higher standard of rigour and then algebra set the higher standard. All this was done in the name of increasing rigour.

In the paper, we will (1) list some of the different motivations for increasing rigour of a mathematical argument, then (2) we will search for a characterisation of rigour, by comparing it to the notion of gapless proof. (3) We then draw some conclusions from what we discovered in (1) and (2). In the conclusions we confirm that rigour is a relative term, and it varies with respect to quantity and quality (rigour in geometry is not obviously the same as rigour in arithmetic). To try to make an ordinal comparison between different (prima facie) qualities of rigour we have to reduce one type of proof to the other or express both proofs in a foundational theory and, philosophically, this is not a trivial matter. But *rigour* is not, for all that, an unconstrained term. There are completely definite things we can say, and some ordinal comparisons can be made, provided we understand the context for the comparison.

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A proof interpretation I of a theory T_1 in a theory T_2 is a function mapping a formula A of T_1 to a formula $A_I(x)$ of T_2 (with distinguished variables x) verifying $\mathsf{T}_1 \vdash A \Rightarrow \mathsf{T}_2 \vdash A_I(t)$ for some terms t extracted from a proof of A. Proof interpretations are used in:

- consistency results (e.g., if $\bot_I(x) = \bot$, then $\mathsf{T}_1 \vdash \bot \Rightarrow \mathsf{T}_2 \vdash \bot$, i.e., T_1 is consistent relatively to T_2);
- conservation results (e.g., if $\mathsf{T}_2 \vdash A_I(x) \to A$ for Π^0_2 formulas A, then $\mathsf{T}_1 \vdash A \Rightarrow \mathsf{T}_2 \vdash A$, i.e., T_1 is conservative over T_2 for Π^0_2 formulas);
- closure under rules (e.g., if $\mathsf{T}_1 = \mathsf{T}_2$, $(\exists y A(y))_I(x) = A_I(x)$ and $\mathsf{T}_1 \vdash A_I(x) \to A(x)$, then $\mathsf{T}_1 \vdash \exists y A(y) \Rightarrow \mathsf{T}_1 \vdash A(t)$ for some term t, i.e., T_1 has the existence property);
- extracting computational content from proofs (e.g., extracting t in the previous point).

The last two applications need $\mathsf{T}_1 \vdash A_I(x) \to A$ that can be achieved by:

- upgrading T_1 to the characterization theory CT that proves $\exists x A_I(x) \leftrightarrow A$;
- or hardwiring truth in I obtaining It verifying $\mathsf{T}_1 \vdash A_{It}(x) \to A$.

The first option doesn't work if:

- CT is classically inconsistent (e.g., bounded proof interpretations);
- or we want applications to theories weaker than CT.

So we turn to the second option and present a method to hardwire truth (in proof interpretations of Heyting arithmetic satisfying some mild conditions) by defining:

- a function c that replaces each subformula A of a formula by $A \wedge A_c$ where A_c is a "copy" of A;
- an "inverse" function c^{-1} that replaces A_c by A;
- $It = c^{-1} \circ I \circ c$.

As examples we hardwire truth in:

- modified realizability;
- Diller-Nahm functional interpretation;
- bounded modified realizability;
- bounded functional interpretation;
- slash.

This is based on joint work with Paulo Oliva [1].

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[1] JAIME GASPAR and PAULO OLIVA, *Proof interpretations with truth*, ***Mathematical Logic Quarterly***, forthcoming.

► ALEXANDER GAVRYUSHKIN, *New spectra of computable models*.
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The problem of description of spectra of computable models is nontrivial in the case where a theory is ω_1- and not ω-categorical. In the case of Ehrenfeucht theories, the computable models spectrum and the decidable models spectrum are both hard to describe. This paper adds several new spectra of computable models for the case of Ehrenfeucht theories to the list of known spectra.

A model is said to be *quasi-prime over a type p* if it is prime over a finite tuple of constants which realize the type p. A model $\mathfrak{M}$ is said to be *limit over a type p* if $\mathfrak{M} = \bigcup_{n\in\omega} \mathfrak{M}_n$, for some elementary chain $(\mathfrak{M}_n)_{n\in\omega}$ of quasi-prime models over p, and $\mathfrak{M} \not\cong \mathfrak{M}_p$. $\mathfrak{M}$ is said to be a *quasi-prime* (*limit*) *model* if it is quasi-prime (limit) over some type.

Every model of an Ehrenfeucht theory (a theory with finite, greater then 1, number of countable models) is either quasi-prime or limit.

A pair $\langle X, F\rangle$, where X is a pre-ordered finite set and $F\colon X \to \omega$, is said to be *e-parameters* of the theory T if (1) X is isomorphic to the set of all *quasi-prime* models of the theory T with the relation of elementary embeddability $\hookrightarrow$; (2) for all $x \in X$, the number of limit models over a quasi-prime model corresponding to the element x coincides with $F(x)$.

Spectrum of computable (*decidable*) *models* of an Ehrenfeucht theory T with e-parameters $\langle X, F\rangle$ is a pair $\langle Y, G\rangle$ where Y is a subset of X corresponding to the computable (decidable) quasi-prime models of the theory T, $G(x) \leqslant F(x)$ for all x, and the values of G correspond to the number of computable (decidable) limit models.

THEOREM 1. *Let $n \geqslant 2$, $m \leqslant n-1$, and the pairs $\langle X, F\rangle$ and $\langle Y, G\rangle$ are defined as follows*:

$$X = \{x_0 < x_1 \leqslant \cdots \leqslant x_n \mid x_n \leqslant \cdots \leqslant x_1\}, F(x_0) = 0, F(x_1) = \cdots = F(x_n) = 1;$$

$$Y = \{x_1 \leqslant \cdots \leqslant x_m \mid x_m \leqslant \cdots \leqslant x_1\}, G(x_0) = 0, G(x_1) = \cdots = G(x_n) = 1.$$

Then there is the theory T satisfying the following conditions:

1. *$\langle X, F\rangle$ are e-parameters of the theory T*;
2. *T has exactly $n+2$ non-isomorphic countable models*;
3. *$\langle Y, G\rangle$ is a spectrum of computable models of the theory T*.

Thus, for any m and n, there exist n $\hookrightarrow$-equivalent quasi-prime models among which exactly m models are computable. In the case of decidable models we have the following. If a quasi-prime model $\mathfrak{A}$ is $\hookrightarrow$-equivalent to a decidable one then $\mathfrak{A}$ is decidable.

THEOREM 2. *Let $\langle X, F\rangle$ be such that $X = \{x_0 < x_1 < x_2\}$, $F(x_0) = F(x_1) = 0$, $F(x_2) = 3$, and $G\colon X \to \omega$ is defined by $G(x_0) = G(x_1) = 0$, $G(x_2) = 2$. Then there is a theory T satisfying the following conditions*:

1. *$\langle X, F\rangle$ are e-parameters of the theory T*;
2. *T has exactly* 6 *non-isomorphic countable models*;
3. *$\langle \{x_0\}, G\rangle$ is a spectrum of computable models of the theory T*.

Thus, there are limit over the same type models one of which is computable and the other does not. In the case of decidable models, the question of existence of such models is known as the Morley Problem.

► ALEXANDRA GAVRYUSHKINA, *On automatic and decidable linear orderings*.
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We are studying the following question of computable model theory for the class of automatic structures. If we have two computable (or decidable, or automatic) presentations of a structure, does there exist a computable isomorphism between them? If there exists one, we say that the structure is *computably categorical*. We focus our attention on the computable categoricity in the class of automatic presentations and the computable categoricity in the class of decidable presentations. We show using linear ordering that these two notions in a general case are not interrelated.

One of the most important properties of automatic structures is that they are decidable in the language extended by the additional quantifier "there exist infinitely many". We show that the class of automatic linear ordering does not coincide with the class of linear orderings which are decidable with the additional quantifier.

More precisely. By an *automatic structure* we mean the structure of finite predicate language which domain and relations are recognized by the finite number of finite string automata. Automata recognizing the relations work synchronously on n-tuples. A structure is said to be *computably categorical in the class of automatic* (*decidable*) *presentations* if there is a computable isomorphism between every two automatic (decidable) presentations of the structure.

Let $\mathcal{A}$ be the class of automatic linear orderings. Let $\mathcal{CA}$ be the class of linear orderings which are computably categorical in the class of automatic presentations. And let $\mathcal{CD}$ be the class of linear orderings which are computably categorical in the class of decidable presentations. It is obviously that $\mathcal{CD} \cap \mathcal{A} \subset \mathcal{CD}$. Since automatic structures are decidable then $\mathcal{CD} \cap \mathcal{A} \subset \mathcal{CA}$. It is easy to see that $\mathcal{CD} \cap \mathcal{A}$ is not empty. We show that there exist a linear order $\mathcal{L}_1 \in \mathcal{CA}$ such that $\mathcal{L}_1 \notin \mathcal{CD}$ and a linear order $\mathcal{L}_2 \in \mathcal{CD}$ such that $\mathcal{L}_2 \notin \mathcal{A}$. So the inclusions $\mathcal{CD} \cap \mathcal{A} \subset \mathcal{CD}$ and $\mathcal{CD} \cap \mathcal{A} \subset \mathcal{CA}$ are proper. Also, we construct an example of decidable with the additional quantifier $\exists^\infty$ linear order which is not automatic.

► ÇIĞDEM GENCER AND DICK DE JONGH, *Extensions of* **K4** *by closed formulas*.
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A uniform syntactic characterization was given for unifiability of formulas in **KD4** and its extensions using Ghilardi's [3,4] finitary results for unification in [5]. In [2] we extended and generalized the work on unifiability of [5]. We gave a semantic characterization for unifiabilty in the extensions of **K4**. We applied this in particular to extensions of **KD4**, **GL**, and **K4.3** to obtain a sytactic characterization in [1,2]. The characterization in [1,2] is based on the use of 0-universal models for the logics studied. 0-universal models can be seen as part of the 0-canonical model (constructed using closed formulas only). These models are useful only in case the logics do have the finite model property for closed formulas (0-fmp). To determine the set of unifiable formulas of extensions of **K4** it is sufficient to determine the set of unifiers of logics extending **K4** by closed formulas only [2,5]. As we mentioned in [2], it was not known whether each extension of **K4** by closed formulas or even any formulas has this property. In this paper we give an answer to this question. We prove that not all extensions of K4 by closed formulas have the 0-fmp property.

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[2] ———, *Unifiability in the extensions of K4*, ***Logic Journal of the IGPL***, vol. 17 (2009), no. 2, pp. 159–172.

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► JAKUB GISMATULLIN, *Absolutely connected groups*.
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We will introduce the class of *absolutely connected groups* i.e., G is absolutely connected if for an arbitrary first order structure on G, working in a saturated extension, G does not have any proper definable, type definable or invariant under automorphisms subgroup of bounded index i.e., $G = G^0 = G^{00} = G^{\infty}$. Examples of absolutely connected groups are [4]: Chevalley groups over infinite fields, connected perfect linear algebraic groups, infinite permutation and infinite dimensional general linear groups. The proofs involves: for algebraic groups—the Gauss decomposition of Chevalley group proved by Ellers and Gordeev [2, 3], for infinite dimensional general linear groups—Tolstykh description of a conjugacy class of a moietous involution [6] and for permutation groups—Bertram and Moran theorem on permutations with bounded support [1, 5].

We will give applications of our result.

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► SERGEY GONCHAROV, *Autostability relative to decidable representations for Ehrenfeucht theories*.
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The theory of decidable models is one of important approach in the theory of computable models [1]. The problem of autostability for strong constructivizations (decidable representations) was solved by A. Nurtazin [2]. We will consider the autostability relative to strong constructivizations for Ehrenfeucht theories.

Theorem 1. *There exists a Ehrenfeucht theory with autostable relative to decidable representations of a prime model and with non-autostable decidable almost prime model of this theory.*

Theorem 2. *There exists a Ehrenfeucht theory with non-autostable relative to decidable representations of an almost prime model and with autostable decidable almost prime model of this theory.*

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[2] A. T. NURTAZIN, *Strong and weak constructivizations and computable families*, ***Algebra Logic***, vol. 13 (1974), no. 3, pp. 177–184.

► MAGDALENA GRZECH, *The group of automorphisms of $P(\omega)/\text{fin}$.*
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It is well known that cardinality and structure of the group of automorphisms of $P(\omega)/\text{fin}$ depends on a model in which the group is considered. The cardinality is not less than $\mathbf{c}$ (the continuum) and not greater than $2^{\mathbf{c}}$ ([1], [2]) and the structure is determined by some additional axioms ([2], [3], [4]). We show that the structure of the group of the automorphisms of $P(\omega)/\text{fin}$ can be (almost) arbitrary. We construct a model in which the group is generated by the subgroup of trivial automorphisms and a subgroup isomorphic to arbitrary group (of appropriate cardinality).

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► GWYNETH HARRISON-SHERMOEN, *Independence relations in NTP1 theories.*
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We say that a theory has the *tree property of the first kind* ($TP1$) if there are a formula $\phi(\overline{x}, \overline{y})$ and tuples $\{\overline{a}_\alpha : \alpha \in {}^{<\omega}\omega\}$ such that for any incomparable $\alpha, \beta \in {}^{<\omega}\omega$, $\phi(\overline{x}, \overline{a}_\alpha) \wedge \phi(\overline{x}, \overline{a}_\beta)$ is inconsistent, but given any $\beta \in {}^{\omega}\omega$, $\{\phi(\overline{x}, \overline{a}_{\beta\restriction i}) : i \in \mathbb{N}\}$ is consistent. We call a theory $NTP1$ if it does not have the tree property of the first kind. (So the class of NTP1 theories contains that of simple theories.) There are a few examples of NTP1 unsimple theories that admit an "independence relation" satisfying most of the properties that forking independence satisfies in simple theories. Two such examples are the theory of infinite dimensional vector spaces with a bilinear form (explored in the Ph.D. thesis of Nicolas Granger) and the theory of ω-free pseudo-algebraically closed fields (studied by Chatzidakis). I will discuss these examples, and attempts to generalise the particular independence relations involved to one that works for all NTP1 theories, or (more realistically) for some smaller class of theories properly containing that of simple theories.

[1] ZOÉ CHATZIDAKIS, *Simplicity and Independence for Pseudo-algebraically closed fields*, ***Models and Computability (London Mathematical Society Lecture Note Series, no. 259)*** (S. Barry Cooper and John K. Truss, editors), Cambridge University Press, Cambridge, UK, 1997, pp. 41–61.

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[3] NICOLAS GRANGER, *Stability, simplicity and the model theory of bilinear forms*, Ph.D. Thesis, University of Manchester, 1999.

► ÅSA HIRVONEN, *Metric classes with generalized isomorphisms.*
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We present a new, abstract framework for the model theoretic study of metric structures.

The approach is based on Shelah's abstract elementary classes and generalizes earlier approaches to metric model theory (positive bounded formulas, Hausdorff CATs, continuous logic). The main motivation for this syntax-free approach is that it enables the built-in treatment of generalized isomorphisms (perturbations). These in turn may bring the class down in stability hierarchy so that a class which is nonsuperstable with respect to the infimum metric may become ω-stable. This enables the development of splitting theory giving better behaviour of independence and constructible models than one gets from stability or superstability. We show how to develop splitting theory in this context and how it relates to constructible models.

The talk is based on joint work with Tapani Hyttinen [1, 2].

[1] Å. Hirvonen and T. Hyttinen, *Categoricity in homogeneous complete metric spaces*, ***Archive for Mathematical Logic***, vol. 48 (2009), pp. 269–322.

[2] ———, *Metric abstract elementary classes with perturbations*, submitted.

► SIMON IOSTI, *A generalization of the Borel–Tits theorem for groupoids*.
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This talk will be about a generalization of a version of the Borel–Tits theorem for simple algebraic groups over algebraically closed fields which can be found in [4]. The original theorem was published in [1]. I tried to reformulate the proof of [4] using the notion of binding group, which appears in the context of internality. This brought me to study an article of Moshe Kamensky [3] presenting a proof of the Tannakian formalism using Model Theory, and an article of Ehud Hrushovski [2] presenting a more general notion of binding groups (namely the "binding groupoid"). Thanks to the methods used in [3], I managed to generalize the Borel–Tits theorem in the context of algebraic groups in the functorial sense, and [2] permitted me to find a formulation for groupoids. The next step of my work will be to determine if it is possible to find a model-theoretic proof of this theorem in more general cases, algebraically as much as model-theoretically. I will present in my talk the results I obtained, and in which way they could eventually be improved.

[1] Armand Borel and Jacques Tits, *Homomorphismes "Abstraits" de groupes algébriques simples*, ***Annals of mathematics***, vol. 97 (1973), pp. 499–571.

[2] Ehud Hrushovski, *Groupoids, imaginaries and internal covers*, `http://arxiv.org/PS_cache/math/pdf/0603/0603413v2.pdf`, 2009.

[3] Moshe Kamensky, *Model theory and the Tannakian formalism*, `http://www.nd.edu/~mkamensk/papers/tannakian.pdf`, 2009.

[4] Bruno Poizat, ***Groupes stables***, Nur al-Mantiq wal-Ma'rifah, 1987.

► HAJIME ISHIHARA AND TAKAKO NEMOTO, *CZF and the notion of set generation*.
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CZF is a system of constructive set theory proposed by Peter Aczel in 1970s'. Although it does not have Powerset Axiom, it contains *Subset Collection Scheme*, defined below, which guarantees, a "large enough" class of subsets of a given set to be a set:

$$\forall a \forall b \exists c \forall u (\forall x \in a \exists y \in b \varphi(x, y, u) \rightarrow \\ \exists d \in c \forall x \in a \exists y \in d \varphi(x, y, u) \wedge \forall y \in d \exists x \in a \varphi(x, y, u)).$$

Many results on CZF with and without additional axioms have been obtained. In these systems, we always face the problem: For a given set, what kind of collection of subsets forms a set? Because some classes of subsets with certain property, e. g., the class of open sets or the class of subgroups, play an important role, the answer to this kind of problems would be quite helpful to reveal mathematical fertility of the systems.

Aczel [1] proposed the following notion of a *set-generation* which guarantees "sufficiently large" collection of subsets of any sets forms a set: A class X is set-generated if there is a *set* y such that

$$y \subseteq X \wedge \forall a \in X(a = \bigcup\{b \in y : b \subseteq a\}).$$

Using this notion, we propose the following *Set Generation Scheme (SGS)*, which guarantees the set-generatedness of a "sufficiently large" collection of subsets of any set:

(SGS_φ) The class $X = \{a \in \mathrm{Pow}(s) : \varphi(a)\}$ is set-generated.

We can show that Subset Collection Scheme is a special case of SGS. We will consider what kind of φ for SGS_φ is proved both in CZF + RDC, Relativized Dependent Choice, and CZF + DC + uREA, union-closed Regular Extension Axiom, and give several important examples of set-generated classes including a construction in [2].

[1] Peter Aczel, *Aspects of general topology in constructive set theory*, ***Annals of Pure and Applied Logic***, vol. 137 (2006), pp. 3–29.

[2] Hajime Ishihara and Erik Palmgren, *Quotient topologies in constructive set theory and type theory*, ***Annals of Pure and Applied Logic***, vol. 141 (2006), pp. 257–265.

► ISTVAN JUHASZ AND MENACHEM MAGIDOR, *On the maximal resolvability of monotonically normal spaces*.
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We continue the earlier work of Juhasz, Soukup, and Szentmiklossy and show that all monotonically normal spaces are maximally resolvable if and only if every uniform ultrafilter is completely decomposable. Hence the existence of a monotonically normal space which is not maximally resolvable is equi-consistent with the existence of a measurable cardinal. We also prove it is consistent (modulo a measurable cardinal) that there is a monotonically normal space in which all non-empty open sets have size $\aleph_\omega$ but is not ω_1-resolvable. It follows from the results of Juhasz, Soukup, and Szentmiklossy that this is best possible.

► ANASTASIYA KARPENKO, *Weak interpolation property over $wK4$*.
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We consider the language which contains at least one constant $\top$ ("truth") or $\perp$ ("false").

A *normal modal logic K* is any set of modal formulas containing all the classical tautologies and the axioms $\Box(A \to B) \to (\Box A \to \Box B)$, $\Diamond A \equiv \neg\Box\neg A$ and closed under the rules of modus ponens, necessitation and substitution. The logics $wK4$ and DL are defined in the following way: $wK4 = K + ((A\&\Box A) \to \Box\Box A)$, $DL = wK4 + (A \to \Box\Diamond A)$. By $NE(L)$ we denote the set of all normal modal logics containing L.

If $\mathbf{p}$ is a list of propositional variables, let $A(\mathbf{p})$ denote a formula whose all variables are in $\mathbf{p}$. Let L be a logic, $\vdash_L$ a consequence relation associated with L. Suppose that $\mathbf{p}, \mathbf{r}$ are disjoint lists of propositional variables. We define the *interpolation property for deducibility* and *weak interpolation property*:

IPD: If $A(\mathbf{p}, \mathbf{q}) \vdash_L B(\mathbf{p}, \mathbf{r})$, then there exist a formula $C(\mathbf{p})$ such that $A(\mathbf{p}, \mathbf{q}) \vdash_L C(\mathbf{p})$ and $C(\mathbf{p}) \vdash_L B(\mathbf{p}, \mathbf{r})$.

WIP: If $A(\mathbf{p},\mathbf{q})$, $B(\mathbf{p},\mathbf{r}) \vdash_L \bot$, then there exist a formula $C(\mathbf{p})$ such that $A(\mathbf{p},\mathbf{q}) \vdash_L C(\mathbf{p})$ and $C(\mathbf{p})$, $B(\mathbf{p},\mathbf{r}) \vdash_L \bot$.

Larisa Maksimova proved that *IPD* and *WIP* are equivalent to some variants of amalgamation. A class V has the *amalgamation property* if it satisfies the condition:

AP: For any $\mathcal{B}, \mathcal{C} \in V$ with a common subalgebra $\mathcal{A}$, there exist an algebra $\mathcal{D}$ in V and monomorphisms $\delta\colon \mathcal{B} \to \mathcal{D}$ and $\varepsilon\colon \mathcal{C} \to \mathcal{D}$ such that $\delta(x) = \varepsilon(x)$ for all $x \in \mathcal{A}$.

It is well known that there is a duality between normal modal logics and varieties of modal algebras. For a given logic L, its associated variety $V(L)$ is determined by identities $A = \top$ for A provable in L.

An algebra is said to be *simple* if it has exactly two congruences.

THEOREM 1 [1]: For any normal modal logic L:

(1) L has IPD iff $V(L)$ has AP;

(2) L has WIP iff the class of finitely generated and simple algebras of $V(L)$ has AP.

The modal algebra V_n^m has $n+m$ atoms $a_1, \dots, a_n, b_1, \dots, b_m$, where

$$\Diamond x = \begin{cases} 1, & \text{if } x = a_i, (1 \le i \le n); \\ \neg x, & \text{if } x = b_j, (1 \le j \le m). \end{cases}$$

THEOREM 3: Every finitely generated finitely indecomposable DL-algebra is simple and isomorphic to V_n^m for some $n + m > 0$.

THEOREM 4: *WIP* and *IPD* are equivalent over *DL*.

THEOREM 5: The interpolation property for deducibility is decidable over *DL*.

THEOREM 6: There are exactly 16 logics with *IPD* in $NE(DL)$.

THEOREM 7: The logic $L \in NE(wK4)$ has *WIP* iff the logic $L + (A \to \Box\Diamond A)$ has *WIP*.

THEOREM 8: The weak interpolation property is decidable over $wK4$.

[1] L. MAKSIMOVA, *Definability and interpolation in non-classical logics*, ***Studia Logica***, vol. 82 (2006), pp. 271–291.

► HIROTAKA KIKYO AND AKITO TSUBOI, *On generic automorphisms and the strict order property*.

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Let T be a complete, model complete theory in a language $\mathcal{L}$, and suppose $M_0 \models T$ and $\sigma_0 \in \mathrm{Aut}_{\mathcal{L}}(M_0)$ are given. Consider the following theory in language $\mathcal{L}(M_0, \sigma)$:

$$T_0 = T + \text{“}\sigma \text{ is an } \mathcal{L}\text{-automorphism”} + \text{“}\sigma \mid M_0 = \sigma\text{”}.$$

We obtained the following:

1. If there is a definable partial order $<$ on M_0, an element $a \in M_0$ and an integer n such that $a < \sigma_0^n(a)$, then T_0 has no model companion.
2. In the following cases, for any $\sigma_0 \in \mathrm{Aut}_{\mathcal{L}}(M_0)$, T_0 has no model companion.
   (a) M_0 is a random graph;
   (b) M_0 has a definable total order;
   (c) M_0 is a full n-ary infinite tree; and
   (d) M_0 is a full infinite branching tree with an infinite depth.
3. There is a theory T with the strict order property such that T_0 has a model companion for some M_0 and σ_0.

[1] H. KIKYO, *Model companions of theories with an automorphism*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 3, pp. 1215–1222.

[2] H. KIKYO and S. SHELAH, *The strict order property and generic automorphisms*, ***The Journal of Symbolic Logic***, vol. 67 (2002), no. 1, pp. 214–216.

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Buss' theory T_2^0, axiomatized by the induction scheme for so-called sharply bounded formulas (in the language of arithmetic extended by symbols for $\log x$, $2^{\log x \cdot \log y}$ and $\lfloor \frac{x}{2} \rfloor$), is one of the bottom levels of the well-known bounded arithmetic hierarchy. T_2^0 was always thought to be a very weak theory, but for a long time there were no independence results separating it from full bounded arithmetic. Recently, in [1], it was shown that T_2^0 does not prove that all non-trivial divisors of powers of 2 are even.

The proof of that result uses methods vaguely similar to those familiar from the study of *IOpen*, the theory axiomatized by induction for open (quantifier-free) formulas in the language of ordered rings. I will discuss how the connection between sharply bounded induction and open induction can be made more explicit. This leads to relatively easy arguments showing that T_2^0 does not prove e.g., the irrationality of $\sqrt{2}$ or Fermat's Last Theorem for exponent 3, while various extensions of T_2^0 do not prove the existence of infinitely many primes.

[1] S. Boughattas and L. A. Kołodziejczyk, *The strength of sharply bounded induction requires MSP*, ***Annals of Pure And Applied Logic***, vol. 161 (2010), no. 4, pp. 504–510.

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We adopt the convention from hypergraph theory and say that a relational structure $\mathcal{M}$ is *l-colourable* if its elements can be coloured with the colours $1, 2, \ldots, l$ in such a way that every relationship of $\mathcal{M}$ contains (at least) two elements of different colours. We say that $\mathcal{M}$ is *uniquely l-colourable* if it is l-colourable and any two l-colourings of it are the same up to permutation of the colours. In [1] it was proved that almost all l-colourable finite undirected graphs are uniquely l-colourable and that the class of l-colourable finite undirected graphs has a (labelled) 0–1 law. We extend this result to finite relational structures.

Let L be a first-order language with finite relational vocabulary. For every positive integer n, let $\mathbf{C}_n$ denote the set of all l-colourable L-structures with universe $[n] = \{1, \ldots, n\}$, and let $\mathbf{C} = \bigcup_{n \in \mathbb{N}} \mathbf{C}_n$. When saying that *almost all* $\mathcal{M} \in \mathbf{C}$ *has property* P we mean that the proportion of $\mathcal{M} \in \mathbf{C}_n$ which have property P approaches 1 as $n \to \infty$. By a *(labelled)* 0–1 *law for* $\mathbf{C}$ we mean that, for every L-sentence φ, either φ or $\neg\varphi$ is true in almost all $\mathcal{M} \in \mathbf{C}$.

Theorem A. (i) There is an L-formula $\xi(x, y)$ such that for almost all $\mathcal{M} \in \mathbf{C}$ the following holds: for every l-colouring $\gamma : M \to [l]$ of $\mathcal{M}$ and all $a, b \in M$, $\gamma(a) = \gamma(b)$ if and only if $\mathcal{M} \models \xi(a, b)$.
(ii) Almost all $\mathcal{M} \in \mathbf{C}$ are uniquely l-colourable.
(iii) $\mathbf{C}$ has a (labelled) 0–1 law.

For any number α we call an l-colouring $\gamma : M \to [l]$ of $\mathcal{M}$ *α-rich* if for every $i \in [l]$, $|\gamma^{-1}(i)| \geq \alpha$. The following result is an essential ingredient of the proof of the above theorem:

Theorem B. There is a constant $\mu > 0$ such that the proportion of $\mathcal{M} \in \mathbf{C}_n$ which have a μn-rich l-colouring approaches 1 as $n \to \infty$.

Both theorems still hold if we only consider structures in which all relation symbols are interpreted as irreflexive and symmetric relations.

[1] Ph. G. Kolaitis, H. J. Prömel, and B. L. Rothschild, *K_{l+1}-free graphs: asymptotic structure and a 0–1 law*, ***Transactions of The American Mathematical Society***, vol. 303 (1987), no. 2, pp. 637–671.

► MILOŠ S. KURILIĆ AND ALEKSANDAR PAVLOVIĆ, *Topologies on Boolean algebras generated by sequential convergence*.
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A sequence $x = \langle x_n : n \in \omega \rangle$ in a complete Boolean algebra $\mathbb{B}$ converges to $a \in \mathbb{B}$ algebraically if and only if $\liminf x_n = \limsup x_n = a$ and the algebraic convergence determines the sequential topology τ_s on $\mathbb{B}$ considered in the context of von Neumann's and Maharam's problem By [1], if a sequence $\langle x_n \rangle$ converges to a in the space $\langle \mathbb{B}, \tau_s \rangle$, then $a = a_x = b_x$ where

$$a_x = \bigwedge_{A \in [\omega]^\omega} \bigvee_{B \in [A]^\omega} \bigwedge_{n \in B} x_n, \quad b_x = \bigvee_{A \in [\omega]^\omega} \bigwedge_{B \in [A]^\omega} \bigvee_{n \in B} x_n.$$

Here we investigate the topology $\mathcal{O}_{ab}$ determined by this convergence. For example, we show that it is topological and that the topologies τ_s and $\mathcal{O}_{ab}$ can be different.

[1] M. S. KURILIĆ and A. PAVLOVIĆ, *Aposteriori convergence in complete Boolean algebras with the sequential topology*, ***Annals of Pure and Applied Logic***, vol. 148 (2007), no. 1-3, pp. 49–62.

► MILOŠ S. KURILIĆ AND STEVO TODORČEVIĆ, *Forcing by elementary substructures of the rational line*.
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The partial order $\langle E(\mathbb{Q}), \subset \rangle$, where $E(\mathbb{Q})$ is the collection of subsets of the rational line which are order-isomorphic to $\mathbb{Q}$, is investigated. This poset can be recognized as the set of elementary substructures of the rational line ordered by the relation $\prec$ and it is forcing equivalent to the quotient $P(\mathbb{Q})/\mathcal{I}_{\text{scatt}}$, where $\mathcal{I}_{\text{scatt}}$ is the ideal of scattered subsets of the rational line. Also, the poset $\langle E(\mathbb{Q}), \subset \rangle$ is forcing equivalent to the two-step iteration of the Sacks forcing and $P(\omega)/\text{Fin}$ of the Sacks extension.

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A finitely generated group is said to be *quasi-finitely axiomatizable* (QFA), in the sense of A. Nies (see [3]), if it is characterized among finitely generated groups, up to isomorphism, by a single first-order formula. A group is said to be *prime* if it embeds elementarily into each model of its theory.

In [4], A. Nies asks if there exists a group which is QFA, or possibly finitely generated and prime, and which is simple. The study of R. Thompson's group F gives an example of a group which is not simple, but which is not far from being simple. In fact, its derived subgroup is simple.

Interpreting the arithmetic in a group may be useful in order to prove that this group is QFA, and it will be essential here, because this will allow us to define orders on some of its subgroups. This work is already done in [2] by V. Bardakov and V. Tolstykh, who were aiming to show that the theory of R. Thompson's group F is undecidable. T. Altinel and A. Muranov in [1] generalize this result to other piecewise linear permutations. Thinking about A. Nies's question, they ask if R. Thompson's group F is biinterpretable with the arithmetic.

[1] Tuna Altinel and Alexey Muranov, *Interprétation de l'arithmétique dans certains groupes de permutations affines par morceaux d'un intervalle*, ***Journal de l'Institut de Mathématiques de Jussieu***, vol. 8 (2009), no. 4, pp. 623–652, (in French).

[2] Valery Bardakov and Vladimir Tolstykh, *Interpreting the arithmetic in Thompson's group F*, ***Journal of Pure and Applied Algebra***, vol. 211 (2007), no. 3, pp. 633–637.

[3] André Nies, *Separating classes of groups by first-order sentences*, ***International Journal of Algebra and Computation***, vol. 13 (2003), no. 3, pp. 287–302.

[4] ———, *Comparing quasi-finitely axiomatizable and prime groups*, ***Journal of Group Theory***, vol. 10 (2007), no. 3, pp. 347–302.

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Let T be the elementary theory of a Boolean algebra $\mathfrak{B}$. For every such theory, except one, Yu. L. Ershov in [1] found a finite sequence of unary predicates $P_0, \dots, P_n$ defined by first order formulas such that $\mathfrak{B}$ is strongly computable iff $\mathfrak{B}$ is computable and all predicates $P_0, \dots, P_n$ are also computable.

We consider the following problem: if $S \subseteq \{P_0, \dots, P_n\}$ is a subset, $\mathfrak{B}$ is computable, and all predicates in S are computable in $\mathfrak{B}$, then can we assert that $\mathfrak{B}$ is decidable?

Some partial cases were considered earlier in papers of Goncharov, Odintsov, Vlasov and Alaev. In particular, Alaev obtained an answer for all initial segments S of the form $\{P_0, \dots, P_k\}$. In this paper, an answer for all possible subsets S is presented.

The elementary theory of a Boolean algebra $\mathfrak{B}$ is described by its elementary characteristic. Let $\{\mathrm{E}_n\}_{n\in\omega}$ be the sequence of Ershov–Tarski ideals. Let At_0, Als_0, Atm_0 be predicates which delineate the set of atoms, the ideal of atomless elements, and the ideal of atomic elements, respectively. For all $k \in \omega$, let At_k be a predicate which delineates in every Boolean algebra the set of elements x such that x/E_k is an atom. Predicates Als_k and Atm_k are defined similarly. The above-mentioned sequence $P_0, \dots, P_n$ is $\mathrm{At}_0, \mathrm{Als}_0, \mathrm{Atm}_0, \mathrm{E}_1, \dots, \mathrm{At}_t, \mathrm{Als}_t, \mathrm{Atm}_t, \mathrm{E}_{t+1}$, where t depends on the theory T.

Theorem 1. *Let $n \in \omega$, $\mathfrak{B}$ be a computable Boolean algebra of elementary characteristic $(n, 0, 1)$, $S \subseteq \{P_0, \dots, P_{4n+3}\}$, and all predicates in S be computable in $\mathfrak{B}$. If for every $k < n$, the predicate At_k is in S and at least one of the predicates Als_k and Atm_k is in S, then $\mathfrak{B}$ is decidable; otherwise, it is not necessarily decidable.*

Theorem 2. *Let $n \in \omega$, $\mathfrak{B}$ be a computable Boolean algebra of elementary characteristic $(n, 1, 0)$, $S \subseteq \{P_0, \dots, P_{4n+3}\}$, and all predicates in S be computable in $\mathfrak{B}$. If for every $k < n$, the predicate At_k is in S, and for every $m < n - 1$, at least one of the predicates Als_m and Atm_m is in S, then $\mathfrak{B}$ is decidable; otherwise, it is not necessarily decidable.*

Theorem 3. *Let $n \in \omega$, $\mathfrak{B}$ be a computable Boolean algebra of elementary characteristic $(n, \omega, 0)$, $S \subseteq \{P_0, \dots, P_{4n+3}\}$, and all predicates in S be computable in $\mathfrak{B}$. If for every $k \leq n$, the predicate At_k is in S, and for every $m < n$, at least one of the predicates Als_m and Atm_m is in S, then $\mathfrak{B}$ is decidable; otherwise, it is not necessarily decidable.*

For a Boolean algebra of another elementary characteristic except $(\omega, 0, 0)$, the answer for our problem can be obtained from the theorems 1, 2 and 3. Goncharov showed that there is no finite set of first order formulas that can provide the decidability of a Boolean algebra of elementary characteristic $(\omega, 0, 0)$.

[1] Yu. L. Ershov, *Decidability of the elementary theory of relatively complemented lattices and the theory of filters*, ***Algebra i Logika***, vol. 3 (1964), no. 3, pp. 17–38.

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We present a family of rank functions for use in the analysis of stability in abstract elementary classes with amalgamation, and derive a partial stability spectrum result for tame classes that generalizes a result of [1]. We also extract a partial spectrum result for weakly tame AECs, thanks to the surprise appearance of a notion from the theory of accessible categories. We highlight the connections between these two fields (whose deep affinities have yet to be fully appreciated) and distill AECs down to their category-theoretic essence. Once we begin looking at things through the eyes of a category theorist (using, in particular, ideas from [3]), some very surprising results appear. In particular, using nothing more than the Yoneda embedding, we obtain a peculiar structure theorem for categorical AECs, an equivalence of categories that identifies the large structures in a κ-categorical AEC with sets equipped with an action of the monoid of endomorphisms of the unique structure of cardinality κ.

[1] JOHN BALDWIN, DAVID KUEKER, and MONICA VANDIEREN, *Upward stability transfer for tame abstract elementary classes*, ***Notre Dame Journal of Formal Logic***, vol. 47, no. 2, pp. 291–298.

[2] MICHAEL LIEBERMAN, ***Topological and category-theoretic aspects of abstract elementary classes***, Doctoral dissertation, University of Michigan, 2009.

[3] JIŘÍ ROSICKÝ, *Accessible categories, saturation and categoricity*, ***The Journal of Symbolic Logic*** vol. 62, pp. 891-901.

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Hybrid automata is an automata with not only discrete, but also continuous parameters. They are widely used for modeling of different tasks related to micro-electronics [1].

In the paper we study the model of movement of a point on the two-dimensional torus along two ortogonal families of geodesics. The torus has different areas that have different colors. There are three types of the areas:

- where we cannot change the direction
- where we can change the direction
- where we must change the direction.

Our main point of interest is the reachability problem for this model in both discrete and continuous cases. We reduce it to the reachability problem on the graph.

The algorithm that builds finite graphs for every mentioned model is provided in the paper. We made estimations of the number of edges in the graph as well as estimations of both discrete and continuous tasks. We have proven that graph complexity for the hybrid case doesn't exceed graph complexity for the discrete case, and often the complexity in the discrete case exponentially exceeds the complexity in the hybrid case.

THEOREM.
- *Discrete automata case. Let the torus be painted into m areas which sizes are $h_j \times w_j$ then the amount of complex states won't be less than $12 \sum h_j w_j$ and not greater than $48 \sum h_j w_j$.*
- *Hybrid automata case. Let the torus be painted into m areas then the complexity of the diagram of the hybrid automata won't be greater than $48(2m + 1)^2$.*

[1] R. ALUR and D. L. DILL, *Automata for modeling real-time systems*, ***Lecture Notes in Computer Science***, vol. 443 (1990), pp. 332–335.

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AQI is the theory which posits the existence of outputs of all infinite time Turing machine computations, whether writable, eventually writable, or accidentally writable. Philip

Welch [2] showed that the least ordinal satisfying AQI is less than the least ordinal satisfying Σ^0_3 Determinacy. We extend his argument to show that the ordinals writable by the much more powerful feedback ITTMs of [1] are also less than the least model of Σ^0_3 Determinacy.

[1] ROBERT LUBARSKY, *ITTMs with feedback*, ***Ways of proof theory***, (Ralf Schindler, editor), Ontos Verlag, pp. 341–353, 2010.

[2] PHILIP WELCH, *Weak systems of determinacy and arithmetical quasi-inductive definitions*, ***The Journal of Symbolic Logic***, vol. 76 (2011), pp. 418–436.

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We prove that the joint consistency property is equivalent to the weak interpolation property in extensions of Johansson's minimal logic J. We find some algebraic criteria for validity of WIP in extensions of the minimal logic.

The weak interpolation property WIP in a logic L is defined as follows:

WIP. If $A, B \vdash_L \bot$, then there exists a formula C such that $A \vdash_L C$ and $B \vdash_L \neg C$, and all non-logical symbols of C are in both A and B.

Let L be any axiomatic extension of the minimal logic. An *open L-theory of a language* $\mathcal{L}$ is a set $T \subseteq \mathcal{L}$ closed with respect to $\vdash_L$. An open L-theory T is *consistent* if it does not contain $\bot$. An L-theory is *complete in* $\mathcal{L}$ if either $A \in T$ or $\neg A \in T$ for any sentence A of $\mathcal{L}$. We define *the joint consistency property*:

JCP. Let T_1 and T_2 be two L-consistent open L-theories in the languages $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, and $T_1 \cap T_2$ is complete in the common language $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$. Then $T_1 \cup T_2$ is L-consistent.

THEOREM 1. *For any (predicate or propositional) extension L of the minimal logic, WIP and JCP are equivalent.*

The language of J contains $\&, \vee, \rightarrow, \bot$ as primitive; negation is defined by $\neg A = A \rightarrow \bot$. The logic J can be given by the calculus, which has the same axiom schemes as the positive intuitionistic calculus Int^+, and the only rule of inference is modus ponens. By a J-*logic* we mean an arbitrary set of formulas containing all the axioms of J and closed under modus ponens and substitution rules. We denote $\mathrm{Int} = \mathrm{J} + (\bot \rightarrow p)$, $\mathrm{Neg} = \mathrm{J} + \bot$, $\mathrm{Gl} = \mathrm{J} + (p \vee (p \rightarrow \bot)) = \mathrm{J} + (p \vee \neg p)$. A J-logic is *superintuitionistic* if it contains the intuitionistic logic Int, and *negative* if contains Neg.

The problem of weak interpolation over J is reducible to the same problem over Gl.

THEOREM 2. *For any* J*-logic L, L has WIP if and only if $L + (p \vee \neg p)$ has WIP.*

In particular, all superintuitionistic logics possess WIP.

We define *the weak amalgamation property* WAPJ: For any $\mathbf{B}, \mathbf{C} \in V$ with a common subalgebra $\mathbf{A}$ there are $\mathbf{D} \in V$ and homomorphisms $g\colon \mathbf{B} \rightarrow \mathbf{D}$, $h\colon \mathbf{C} \rightarrow \mathbf{D}$ such that $g(x) = h(x)$ for all $x \in \boldsymbol{A}$ and $\bot \neq \top$ in $\mathbf{D}$, whenever $\bot \neq \top$ in $\mathbf{A}$.

With any J-logic a variety $V(L)$ of J-algebras is associated. We find an algebraic equivalent of WIP.

THEOREM 3. *Let L be a* J*-logic. Then L has WIP iff $V(L)$ has WAPJ.*

THEOREM 4. *There exist a continuum of* Gl*-logics with WIP and a continuum of* Gl*-logics without WIP.*

A J-algebra $\mathbf{A}$ is *negative* if $\bot = \top$ in $\mathbf{A}$. For a negative algebra $\mathbf{A}$ we define a J-algebra $\mathbf{A}^\Lambda$ obtained from $\mathbf{A}$ by adding a new greatest element $\top$; $\bot_{\mathbf{A}^\Lambda} = \bot_{\mathbf{A}}$. Let $\Lambda(L) = \{\mathbf{A}^\Lambda \mid \mathbf{A}^\Lambda \in V(L)\}$.

THEOREM 5. *A* J*-logic L has WIP iff the class $\Lambda(L)$ is either empty or has the amalgamation property.*

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We introduce the notion of an invariantly universal pair (S, E) where S is an analytic quasi-order and $E \subseteq S \cap S^{-1}$ is an analytic equivalence relation on some Polish space X. This means that for any analytic quasi-order R there is a Borel $B \subseteq X$ invariant under E such that R is Borel equivalent to the restriction of S to B.

The notion is inspired by a result of S. Friedman and L. Motto Ros: the pair consisting of the quasi-order of embeddability and the equivalence relation of isomorphism on graphs with vertex set ω is invariantly universal.

We find a sufficient condition for invariant universality, and we use it to show that the phenomenon of invariant universality is widespread. In fact it occurs for a great number of complete analytic quasi-orders, arising in different areas of mathematics, when they are paired with natural equivalence relations. For example, in the space of separable Banach spaces, the pair consisting of linear isometric embeddability and linear isometry is invariantly universal.

This is joint work with Riccardo Camerlo and Luca Motto Ros.

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A computable structure $\mathcal{A}$ is *computably categorical* if for every computable $\mathcal{B}$ classically isomorphic to $\mathcal{A}$, there exists a computable isomorphism from $\mathcal{A}$ onto $\mathcal{B}$. For many classes of computable structures, structural characterizations of this property have been established. We start to do the same for fields, by considering the case of an algebraic field F with a splitting algorithm, i.e., such that the set of irreducible polynomials in $F[X]$ is computable. We prove that such a field is computably categorical if and only if its orbit relation

$$G = \{\langle a, b\rangle : \exists \psi \in \mathrm{Aut}(F) \text{ with } \psi(a) = b\}$$

is computable.

With F algebraic, the orbit relation G can also be defined by an $\forall\exists$ formula in the language of fields. Ostensibly, the condition given above is therefore Σ^0_4, but since F is assumed to have a splitting algorithm, computability of G is in fact Σ^0_3.

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Sz. Mikulasz proved that algebras of binary relations whose similarity type includes intersection, composition, converse negation and the identity constant form a non-finitely axiomatizable quasivariety [2]. It is known that adding union or converse into the similarity type leads to such result. One can investigate a class of algebras whose similarity type has been motivated by relevance logic. Mikulasz is interested in logic R and look from this point of view at the class of relation algebras and representable relation algebras. My aim is to examine the wider class of relevance logic including the weaker systems and their algebras. Generally one can say, that the class of algebras of binary relations with set-theoretic operations of intersection, composition, left and right residuals computed in a transitive relation is finitely axiomatizable [3]. The subreduct to intersection and composition is finitely axiomatizable as well, but if intersection is replaced by either union or converse it leads to non-finite axiomatization.

[1] Benekikt Loewe, Boris Piwinger, and Thoralf Raesch, editors, ***Classical and new paradigms of computation and their complexity hierarchies***, Springer, 2004.

[2] Szabolcs Mikuls, *Axiomatizability of algebras of binary relations*, in [1].

[3] ———, *Algebras of relations and relevance logic*, ***Journal of Logic and Computation***, vol. 19 (2009), no. 2, pp. 305–321.

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The *depth* of a BA A is the supremum of the cardinalities of subsets of A well-ordered by the Boolean ordering; it is denoted by $\mathrm{depth}(A)$. Further,

$$\mathrm{depth}_{\mathrm{Hr}}(A) = \{(\kappa,\lambda)\colon \text{there is a homomorphic image } B \text{ of } A \text{ such that } |B| = \lambda \text{ and } \mathrm{depth}(B) = \kappa\}.$$

Theorem 1. *Suppose that* $\omega \le \rho \le \kappa$. *Let* $A = \langle[\kappa]^{\le\rho}\rangle_{\mathbf{P}(\kappa)}$. *Let*

$$S = \{(\mu,\nu)\colon \omega \le \mu \le \nu \le 2^\rho, \nu^\omega = \nu\};$$
$$T = \{(\mu,\nu^\rho)\colon \rho^+ \le \mu \le 2^\rho, 2^\rho < \nu^\rho, \nu \le \kappa\}.$$

Then $\mathrm{depth}_{\mathrm{Hr}}(A) = S \cup T$.

Theorem 2. *Suppose that* λ *is an uncountable regular limit cardinal and* $\lambda \le \kappa$. *Let* $A = \langle[\kappa]^{<\lambda}\rangle_{\mathbf{P}(\kappa)}$. *Define*

$$S = \{(\mu,\nu)\colon \omega \le \mu \le \nu < 2^{<\lambda}, \nu^\omega = \nu\};$$
$$T = \{(\mu,\nu^{<\lambda})\colon \lambda \le \mu \le 2^{<\lambda} \le \nu \le \kappa\};$$
$$U = \{(\mu,2^{<\lambda})\colon \omega \le \mu \le 2^{<\lambda}\}.$$

Then:
(i) $S \cup T \subseteq \mathrm{depth}_{\mathrm{Hr}}(A)$;
(ii) *If there is a* $\rho < \lambda$ *such that* $2^\rho = 2^{<\lambda}$, *then* $\mathrm{depth}_{\mathrm{Hr}}(A) = S \cup T \cup U$.
(iii) *If* $2^\rho < 2^{<\lambda}$ *for all* $\rho < \lambda$, *then* $\mathrm{depth}_{\mathrm{Hr}}(A) = S \cup T$.

► VLADISLAV NENCHEV, *Logics for stable and unstable mereological relations*.
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This paper presents predicate and modal logics based on relational structures with stable and unstable versions of three well-known mereological relations: *part-of*, *overlap* and its dual *underlap* (denoted respectively by $\le$, O and U). They can be defined in non-degenerate Boolean algebra $(B, \le, 0, 1, ., +, *)$ as follows: $x \le y$ is the Boolean ordering, $x\mathsf{O}y$ iff $x.y \ne 0$ and $x\mathsf{U}y$ iff $x^*.y^* \ne 0$ iff $x + y \ne 1$ (see [1] for details about mereology). These three relations can be axiomatized by a set of universal first-order sentences and in this form they represent the notion of *mereological structure* (described in [2] in another terminology). A natural way to define the stable and unstable counterparts of the mereological relations will be over the vectors from the cartesian product of a set of mereological structures. Let

$(W_i, \leq_i, \mathsf{O}_i, \mathsf{U}_i)$ be a mereological structure for every i in a given set of indices. Then the stable and unstable relations can be defined as follows: $x \leq^\forall y$ iif $(\forall i \in I)(x_i \leq_i y_i)$, $x \leq^\exists y$ iif $(\exists i \in I)(x_i \leq_i y_i)$ and similarly for $O^\forall$, $O^\exists$, $U^\forall$ and $U^\exists$, where x_i is the i-th coordinate of the vector x.

This semantics presents a development in the area of alternative theories of space and time, started by Whitehead (see [3]). He intended to use mereology to build a new, point-free theory of space, eventually adding more relations of topological nature, like *contact*. Namely the use of mereological structures in the above definition marks the alternative approach to space. Considering time we may think of the set of mereological structures as a collection of instances of a changing structure, with each instance representing the state of the structure in different moment in time. Thus the structures with stable and unstable relations become dynamic versions of the static mereological structures. And although we don't have any relation between the time moments (meaning that the time isn't structured) we may define simple temporal properties like *stability* and *unstability*. In this way we take the alternative approach to time.

The paper continues with axiomatization of the newly defined six relations with appropriate universal first-order sentences that are true in the above semantics. In this way we obtain *abstract structures* for these relations for which we prove representation theorem applying a generalization of the Stone-like techniques for Boolean algebras. We prove completeness theorem for the quantifier-free fragment of the corresponding first-order logic and its decidability and show that its satisfiability problem is in NP. We use the abstract structures as semantic base of a polymodal logic for which we provide complete axiomatization and a decidability proof via filtration. The paper is based on the author's master thesis, prepared under the guidance of Prof. Dimiter Vakarelov.

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[2] D. Vakarelov, *Logical analysis of positive and negative similarity relations in property systems*, ***First world conference on the Fundamentals of Artificial Intelligence*** (Paris, France), (Mishel De Glas and Dov Gabbay, editors), 1991, pp. 491–499.

[3] A. N. Whitehead, ***Process and reality***, MacMillan, New York, 1929.

► STELA K. NIKOLOVA, *Uniform Turing admissibility over an arbitrary abstract structure*.
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A typical "external" approach to computability over an abstract domain consists in the following: Given an abstract (partial) structure $\mathcal{A}$, we first introduce an *enumeration* of $\mathcal{A}$, which is an ordered pair $(\kappa, \mathcal{B})$, where $\mathcal{B}$ is a structure over (some subset of) $\mathbb{N}$ and κ is a strong homomorphism from $\mathcal{B}$ to $\mathcal{A}$. Then a partial function f in $|\mathcal{A}|$ is said to be *admissible*, if for every enumeration $(\kappa, \mathcal{B})$ there exists a partial function g in N, computable (in some sense) in $\mathcal{B}$, such that the equality

$$\kappa(g(x_1, \dots x_n)) \simeq f(\kappa(x_1), \dots, \kappa(x_n))$$

holds for every $x_1, \dots, x_n$ in $|\mathcal{B}|$.

Clearly this approach requires countability of $|\mathcal{A}|$. However certain notion of admissibility over uncountable domains is still possible. It is based on the observation that during a single computation only finitely many elements of $|\mathcal{A}|$ could be processed, so we do not need to enumerate the whole data domain, but only a suitable countable part of it—$\mathcal{A}(\bar{s}, \beta)$, which depends on the input $\bar{s}$ and—in the case of non-deterministic algorithms—on some (possibly) infinite sequence $\beta \in |\mathcal{A}|^{\mathbb{N}}$.

We say that the functional $F\colon |\mathcal{A}|^n \times |\mathcal{A}|^{\mathbb{N}} \to |\mathcal{A}|$ is *uniformly Turing admissible in* $\mathcal{A}$, if there exists a Turing operator Γ, such that for every $\bar{s} \in |\mathcal{A}|^n$ and $\beta \in |\mathcal{A}|^{\mathbb{N}}$ and every enumeration

$(\kappa, \mathcal{B})$ of $\mathcal{A}(\bar{s}, \beta)$ the equality

$$\kappa(\Gamma(\mathcal{B}, \alpha)(x_1, \ldots, x_n)) \simeq F(s_1, \ldots, s_n, \beta)$$

holds for every $\bar{x} \in |\mathcal{B}|^n$ and $\alpha \in |\mathcal{B}|^{\mathbb{N}}$, such that $\kappa(\bar{x}) = \bar{s}$ and $\kappa(\alpha) = \beta$.

A multiple valued function f is said to be *uniformly Turing admissible in* $\mathcal{A}$, if there exists an uniformly Turing admissible functional F, such that for every $\bar{s} \in |\mathcal{A}|^n$: $t \in f(\bar{s}) \iff \exists\beta(F(\bar{s}, \beta) \simeq t)$. We obtain an explicit syntactical description of the uniformly Turing admissible functionals and as a consequence—characterization of the admissible functions.

► FRANCIS OGER, *Tilings and associated relational structures*.
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We fix a locally compact metric space E and a group G of isometries of E. *Tiles* are compact subsets of E. A *tiling* is a covering of E by tiles. Isomorphism between sets of tiles is defined modulo G.

A *tiling system* is a nonempty set Δ of *local rules*, each of them specifying a finite set of possible configurations for the tiles within some given distance of one of them. A Δ-*tiling* (resp. a Δ-*patch*) is a tiling (resp. a set of tiles) which satisfies Δ.

With appropriates hypotheses on E, G, Δ, we have a finite relational language L_Δ such that Δ-tilings can be interpreted as L_Δ-structures: Isomorphism (resp. local isomorphism) for Δ-tilings is equivalent to isomorphism (resp. local isomorphism, or elementary equivalence) for associated L_Δ-structures. The distances between tiles in a Δ-tiling and in the associated L_Δ-structure are equivalent.

We show that any L_Δ-structure is associated to a Δ-tiling if, for an integer r depending on Δ, each substructure of radius r is associated to a Δ-patch.

We are especially interested in the tiling systems Δ which are *aperiodic* in the following sense: no Δ-tiling is periodic, but each Δ-tiling *satisfies the local isomorphism property*, i.e., each bounded subpatch appears everywhere in the tiling. In the best cases, such as Penrose tilings and Robinson tilings, we show that the tiling has a superstable finitely axiomatizable theory. We prove that paperfolding sequences indexed by $\mathbb{Z}$ and "regular" coverings of $\mathbb{R}^2$ by sets of curves associated to such sequences are also aperiodic.

Concerning uniformly locally finite structures on finite relational languages, and in particular tilings, which satisfy the local isomorphism property, we characterize those which are elementarily equivalent to structures without nontrivial automorphism.

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Henselian valued fields of residual characteristic 0 satisfy the *Ax–Kochen–Ershov* (A–K–E) principle, which roughly means that the theory of a such field is encoded in its value group and its residue field. For many classes of valued fields of characteristic $p > 0$, it is not known whether they satisfy the A–K–E principle, while their natural analogues in characteristic 0 do. To prove an A–K–E type result one has to get sufficient information about $v(P(x))$ from $v(x)$, where P is a polynomial. But in general $v(x)$ does not determine $v(P(x))$: x may create a *jump*.

A polynomial over a field K of characteristic $p > 0$ is said to be *additive* if it is additive as a map on the algebraic closure of K. Additive polynomials are of the form $\sum_{i=0}^{n} a_i T^{p^i}$ and they form a right (or left) Euclidean Ore ring under addition and composition, acting naturally on K. In this talk I will define *valued modules* over such rings and a *linear Hensel Property* in such structures. Then I will explain how this allows a way of replacing jump points by "regular ones" which yields A–K–E type results for some classes of valued modules.

[1] Pillar Dellunde, Françoise Delon, and Françoise Point, *The theory of modules of separably closed fields 1*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 997–1015.

[2] Franz-Viktor Kuhlmann, *Additive polynomials and their role in the model theory of valued fields*, ***Logic in Tehran***, Lecture Notes in Logic 26, Association for Symbolic Logic, 2006, pp. 160–203.

► SERGEY OSPICHEV, *Friedberg numberings in Ershov's hierarchy*.
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In this work are considered computable numberings [4] of families from various classes Σ_a^{-1} in Ershov's hierarchy [1], where a is constructive ordinal. In [2] are considered open problems of numberings theory. One of questions is about computable Friedberg numberings of Σ_a^{-1}-sets. In [3] was constructed example of Σ_n^{-1}-computable Friedberg numbering of family of all Σ_n^{-1}-sets, $n \in \omega$.

In work is proved

Theorem 1. *There is Σ_a^{-1}-computable Friedberg numbering of family of all Σ_a^{-1}-sets.*

Also, was constructed a Σ_a^{-1}-computable numbering of family of all Δ_a^{-1}-sets. And using this fact, is proved

Corollary 2. *There is Σ_a^{-1}-computable Friedberg numbering of family of all Δ_a^{-1}-sets.*

[1] M. M. Arslanov, ***Ershov hierarchy***, Kazan, Kazan State University, 2007.

[2] S. S. Goncharov and S. Badaev, *Theory of numberings, open problems*, ***Contemporary Mathematics***, vol. 257, pp. 23–38.

[3] S. S. Goncharov, S. Lempp, and D. R. Solomon, *Friedberg numberings of families of n-computably enumerable sets*, ***Algebra and Logic***, vol. 41, no. 2, pp. 143–154.

[4] S. S. Goncharov and A. Sorbi, *Computable numberings and nontrivial Rogers semilattices*, ***Algebra and Logic***, vol. 36, no. 6, pp. 621–641.

► KOUSHIK PAL, *Model theory of multiplicative valued difference fields*.
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A *difference field* (K, σ) is a field K with a distinguished automorphism $\sigma : K \to K$. A *valued difference field* (K, σ, v) is a difference field (K, σ) with a valuation $v : K \to \Gamma \cup \{\infty\}$, where Γ is an ordered abelian group, and ∞ is a new element which satisfies:

$$\begin{aligned} &1.\infty > \gamma && \forall \gamma \in \Gamma \\ &2.\infty + \gamma = \gamma + \infty = \infty + \infty = \infty && \forall \gamma \in \Gamma. \end{aligned} \tag{1}$$

The theory of such a structure depends on how the automorphism σ interacts with the valuation function v. If (K, σ, v) satisfies $v(\sigma(x)) = v(x)\ \forall x \in K$, the valued difference field is called *isometric*. The model theory of isometric valued difference fields have been extensively studied by Thomas Scanlon [3], and Luc Bélair, Angus Macintyre and Thomas Scanlon [2]. If (K, σ, v) satisfies $v(\sigma(x)) > nv(x)$ for all $n \in \mathbb{N}$ and for all $x \in K^\times$ such that $v(x) > 0$, then the valued difference field is called *contractive*. The model theory of such structures has been studied by Salih Azgin [1]. I am going to talk about a more general case, which incorporates the above two cases, and which we call *multiplicative*. A multiplicative valued difference field satisfies $v(\sigma(x)) = qv(x)$, where $q\ (> 0)$ is interpreted as an element of a real-closed field, and its type is determined by rational cuts. For example, q could be 2, i.e., $v(\sigma(x)) = 2v(x)$. I will give axiomatization for such theory, prove an Ax–Kochen–Ershov kind of result and show that the theory admits relative quantifier elimination.

[1] Salih Azgin, *Valued fields with contractive automorphism and Kaplansky fields*, submitted, May 2009, http://www.math.mcmaster.ca/~sazgin/vdca.pdf.

[2] Luc Bélair, Angus Macintyre, and Thomas Scanlon, *Model theory of the Frobenius on the Witt vectors*, ***American Journal of Mathematics***, vol. 129 (2007), no. 3, pp. 665–721.

[3] Thomas Scanlon, *A model complete theory of valued D-fields*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 4, pp. 1758-1784.

► ANDREA PEDEFERRI, *Formalism or pluralism; which better describes mathematical practice?*
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There are three parts to the paper. In the first, we give a characterization of the philosophical position in mathematics called *Formalism*. In the second part we look at some reports by mathematicians about their work and at some proofs in mathematics. We find that the actual practice of mathematics does not fit well with the Formalist account. In the third part we show how Pluralism preserves some insights of Formalism, but also provides a fuller account which better fits the examples.

The parts of Formalism which are retained in Pluralism are (1) that there is no philosophical commitment to a unique mathematical ontology, (2) that the notion of mathematical truth is also not absolute. Instead, we have *truth given a theory*. For both the Formalist and the Pluralist, ontology and truth are relative terms—relative to the systems being used at the moment. This echoes Shapiro's Structuralism. (3) The Pluralist shares with Formalism the emphasis on the symbols and their manipulation. Ideally this is quite rigid. Our examples show that mathematicians sometimes deviate from the ideal. The Pluralist parts company with the Formalist over what to make of this observation.

The Formalist would hold us to a standard of perfect manipulation; the Pluralist allows deviance. This does not mean that *anything goes* under Pluralism. On the contrary, the ideal of perfect formal proof is known and appreciated. Deviance is permitted, but it is used to force us to be explicit about our deviance from the norm. In other words, rather than hold mathematicians to a rigid standard of rigor, we use the standard of rigor to make us aware of deviance. This is simply practical. In fact, it is part and parcel of the creativity of mathematics.

► MICHAEL ROIZNER, *Elementary equivalence of the automorphism groups of Abelian p-groups*.
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In the paper we consider elementary properties (i. e., properties expressible in first order terms) of the automorphisms group of Abelian p-groups.

In [1] E. I. Bunina and A. V. Mikhalev discovered connection of second order properties of an Abelian p-group with first order properties of its endomorphism ring (the analogue of Baer–Kaplansky Theorem for elementary equivalence).

Namely, it is proved in [1] that the endomorphism rings of Abelian p-groups are elementary equivalent if and only if the corresponding groups have some special limitations of the second order theories equal.

In this paper we discover the connection of the second order properties of an Abelian p-group and first order group properties of its automorphism group, for the case $p > 2$. The connection of isomorphisms of Abelian p-groups and isomorphisms of their automorphism groups was considered by Leptin and Liebert ([2], [3]).

We prove the following theorem.

Theorem 1. *Let A, C be p-groups, $p \geq 3$. If* $\operatorname{Aut} A \equiv \operatorname{Aut} C$, *then the basic subgroups and the divisible parts of the groups A and C are equivalent in second order logic.*

[1] E. I. Bunina and A. V. Mikhalev, *Elementary equivalence of the endomorphism rings of Abelian p-groups*, ***Journal of Mathematical Sciences***, vol. 6 (2006), no. 137, pp. 5212–5274.

[2] H. Leptin, *Abelsche p-Gruppen und ihre Automorphismengruppen*, ***Mathematische Zeitschrift***, vol. 73 (1960), no. 3, pp. 235–253.

[3] W. Liebert, *Isomorphic automorphism groups of primary Abelian groups II*, ***Contemprorary Mathematics***, vol. 87 (1989), pp. 51–59.

► GÁBOR SÁGI, *Vaught's conjecture from the perspective of algebraic logic*.
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For a first order theory T and a cardinal κ the cardinality of pairwise non-isomorphic models of T of cardinality κ is denoted by $I(T, \kappa)$. Vaught's conjecture claims, that

$$I(T, \aleph_0) > \aleph_0 \text{ implies } I(T, \aleph_0) = 2^{\aleph_0},$$

that is, if T has uncountably many countable, pairwise non-isomorphic models, then T has continuum many countable, pairwise non-isomorphic models.

During the last decades, a considerable effort has been made to study Vaught's conjecture. We will discuss some recently obtained results related to some special cases, generalizations and modifications of Vaught's conjecture. These results were obtained by applying some tools of algebraic logic, such as the representation theory of locally finite dimensional cylindric algebras. Connections with group theory will also be discussed.

► GÁBOR SÁGI AND ZALÁN GYENIS, *Morley's theorem in the finite*.
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By a celebrated theorem of Morley, a theory T is $\aleph_1$-categorical if and only if it is κ-categorical for all uncountable κ. In this talk we survey the first steps towards extending Morley's categoricity theorem "to the finite". In more detail, we are presenting conditions, implying that certain finite subsets of certain $\aleph_1$-categorical T have at most one n-element model for each natural number $n \in \omega$, counting up to isomorphism, of course. Our techniques involve stability theoretical methods as well as the method of extending decomposable partial elementary mappings to decomposable isomorphisms between certain ultraproducts of finite structures.

► SAM SANDERS, *A copy of several Reverse Mathematics*.
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Reverse Mathematics is a program in foundations of mathematics initiated by Friedman ([1, 2]) and developed extensively by Simpson ([4]). Its aim is to determine which minimal axioms prove theorems of ordinary mathematics. Nonstandard methods have played an important role in this program ([3, 5]). We are interested in Reverse Mathematics where equality is replaced by the nonstandard relation $\approx$, i.e., equality up to infinitesimals. We obtain a 'copy' of Reverse Mathematics for WKL_0 in a weak system of nonstandard arithmetic. Surprisingly, the same system is also a 'copy' of Constructive Reverse Mathematics. Our results have applications in Physics and the Philosophy of Science.

[1] Harvey Friedman, *Some systems of second order arithmetic and their use*, ***Proceedings of the International Congress of Mathematicians*** (Vancouver, B.C., 1974), vol. 1, Canadian Mathematical Congress, 1975, pp. 235–242.

[2] ———, *Systems of second order arithmetic with restricted induction I & II (Abstracts)*, ***The Journal of Symbolic Logic***, vol. 41 (1976), pp. 557–559.

[3] H. JEROME KEISLER, *Nonstandard arithmetic and reverse mathematics*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), no. 1, pp. 100–125.

[4] STEPHEN G. SIMPSON, ***Subsystems of second order arithmetic***, Perspectives in Mathematical Logic, Springer–Verlag, Berlin, 1999.

[5] KAZUYUKI TANAKA, *The self-embedding theorem of WKL0 and a non-standard method*, ***Annals of Pure and Applied Logic***, vol. 84 (1997), no. 1, pp. 41–49.

► DENIS I. SAVELIEV, *On a question of Mycielski*.
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Mycielski asked [1] whether all subsets of the space $\kappa^{\mathrm{cf}\,\kappa}$ with its lexicographic order topology that are constructible from $\kappa^{\mathrm{cf}\,\kappa}$ may have the properties that are immediate analogs of the Baire and the perfect set properties, for all κ. He asked also [2] more generally about appropriate descriptive set theories for these spaces. We derive some consequences of Mycielski's proposal, e.g., by showing that then the universe is not constructible from any set and contains a certain portion of large cardinals. We show also that, under some assumptions on κ, most of concepts and results of classical descriptive set theory have immediate analogs for the space $\kappa^{\mathrm{cf}\,\kappa}$.

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[2] ———, A personal communication, 2004.

► PETER SCHROEDER-HEISTER, *An alternative implication-left schema for the sequent calculus*.
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As an alternative to Gentzen's schema $(\to\mathrm{L})$ for the introduction of implication on the left side of the sequent sign in the intuitionistic sequent calculus **LJ** we propose the schema $(\to\mathrm{L})^\circ$:

$$(\to\mathrm{L})\ \frac{\Gamma\vdash A\ \ \Delta, B\vdash C}{\Gamma,\Delta, A\to B\vdash C}\qquad (\to\mathrm{L})^\circ\ \frac{\Gamma\vdash A}{\Gamma, A\to B\vdash B}$$

In the absence of cut, $(\to\mathrm{L})^\circ$ is weaker than $(\to\mathrm{L})$. In the system based on $(\to\mathrm{L})^\circ$, cut is admissible except for cuts whose left premiss is the conclusion of $(\to\mathrm{L})^\circ$, i.e., cuts of the following restricted form:

$$\begin{array}{c} (\to\mathrm{L})^\circ\ \dfrac{\vdots}{\Gamma\vdash A}\qquad \begin{array}{c}\vdots\\ A,\Delta\vdash C\end{array} \\ \text{(cut)}\ \overline{\qquad\qquad\Gamma,\Delta\vdash C\qquad\qquad} \end{array}$$

Using cut in this restricted form, $(\to\mathrm{L})$ and $(\to\mathrm{L})^\circ$ can be shown to be equivalent. Unlike full cut, applications of restricted cut do not compromise the subformula property and are harmless in this sense. Philosophically, $(\to\mathrm{L})^\circ$ is motivated by the interpretation of implications as rules [1, 2] and can be viewed as a direct translation of *modus ponens* into the sequent calculus.

[1] P. SCHROEDER-HEISTER, *Generalized elimination inferences, higher-level rules, and the implications-as-rules interpretation of the sequent calculus*, ***Advances in natural deduction*** (E. H. Haeusler, L. C. Pereira and V. de Paiva, editors), 2010.

[2] ———, *Implications-as-rules vs. implications-as-links: An alternative implication-left schema for the sequent calculus*, ***Journal of Philosophical Logic***, in print.

► LYNN SCOW, *The property "T witnesses (T', Γ)-reduction" as a tool to characterize classes of theories, T.*
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We introduce new notation to talk about a notion of generalized indiscernible sequence from Shelah's Classification Theory, Chapter VII. Say that a *T'-indiscernible in T* is a sequence of parameters $(\overline{a}_i : i \in I)$ in a model $M \vDash T$, where I is a model of T', weakly-saturated for finite quantifier-free types and any tuples $\overline{i}, \overline{j}$ with the same quantifier-free type in I index parameters $\overline{a}_{\overline{i}}, \overline{a}_{\overline{j}}$ with the same complete type in M. For a subset Γ of $L_{T'}$-formulas, we say $(\overline{a}_i : i \in I)$ is *Γ-indiscernible*, if it suffices for the two tuples $\overline{i}, \overline{j}$ to agree on the quantifier-free formulas from Γ in order for them to index parameters realizing the same complete type in M. Fix an L'-theory T', and a set of L'-formulas, Γ. We can now state a property of L-theories T: namely that any T'-indiscernible in T must be Γ-indiscernible. Abbreviate this property as, "T witnesses (T', Γ)-reduction." We will explore the different stability-theoretic classes of theories T that can be characterized by instances of this property.

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Let A and B be disjoint sets of vertices in a graph G. *Menger's theorem* states that there is a collection of disjoint paths P, each starting in A and ending in B, and a selection C of one vertex from each path in P such that removing the vertices C from the graph G disconnects A from B. Menger's theorem is a classical result for finite graphs, is proved for arbitrary countable graphs in [1], and is proved for arbitrary infinite graphs in [2]. The special case of Menger's theorem restricted to bipartite graphs is called *König's duality theorem* which, in the countable case, is equivalent to ATR_0 over RCA_0 [3], [4]. We provide a proof of Menger's theorem for countable graphs from the axioms of Π^1_1-CA_0. This proof uses several metamathematical techniques, most notably that of countable coded β-models. By general considerations, Menger's theorem cannot be equivalent to Π^1_1-CA_0 over RCA_0. Thus Menger's theorem is at least as strong as ATR_0 and is strictly weaker than Π^1_1-CA_0.

[1] Ron Aharoni, *Menger's theorem for countable graphs*, ***Journal of Combinatorial Theory, Series B***, vol. 43, no. 3, pp. 303–313.

[2] Ron Aharoni and Eli Berger, *Menger's theorem for infinite graphs*, ***Inventiones Mathematicae***, vol. 176, no. 1, pp. 1–62.

[3] Ron Aharoni, Menachem Magidor, and Richard A. Shore, *On the strength of König's duality theorem for infinite bipartite graphs*, ***Journal of Combinatorial Theory, Series B***, vol. 54, no. 2, pp. 257–290.

[4] Stephen G. Simpson, *On the strength of König's duality theorem for countable bipartite graphs*, ***The Journal of Symbolic Logic***, vol. 59, no. 1, pp. 113–123.

► TEODOR J. STĘPIEŃ AND ŁUKASZ T. STĘPIEŃ, *Atomic entailment and classical entailment.*
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1. *Terminology.* Let C, N, A, K, E denote the connectives of implication, negation, disjunction, conjuction and equivalence. S_1 is the set of all well-formed formulas of predicate calculi. $P^n_k(t_1, \dots, t_n)$ is a simple formula and *Smp* is the set of all simple formulas and $\bigwedge x_k, \bigvee x_k$ are quantifiers. $P_1(\phi)$ denotes the set of all predicate letters occuring in ϕ and $vf(\phi)$ denotes the set of all free variables occuring in $\phi(\phi \in S_1)$. $\Rightarrow, \Leftrightarrow, \&, \exists, \forall$ are metalogical symbols. r^1_* denotes the substitution rule for simple formulas. $\langle\{\alpha\}, \beta\rangle \in r^1_* \Leftrightarrow \beta = h^e(\alpha)$ for some endomorphism $h^e : S_1 \to S_1$, which is an extension of the function $e : Smp \to S_1$

($e \in \epsilon_*^1$) (for details see [1]). r_0^1 denotes Modus Ponens for predicate calculi. r_+ denotes the generalization rule. $R_{0+} = \{r_0^1, r_+\}$. For every $\alpha \in S_1$, $\alpha^* = \bigvee x_1, \dots, \bigvee x_k N\alpha$, if $vf(\alpha) = \{x_1, \dots, x_k\}$ and $\alpha^* = N\alpha$, if $vf(\alpha) = \emptyset$. For every $\phi \in S_1$, $\bigwedge \phi = \bigwedge x_1 \cdots \bigwedge x_k \phi$, if $vf(\phi) = \{x_1, \dots, x_k\}$. $Cn_1(R, X)$ is the smallest subset of S_1, containing X and closed under the rules R. The couple $\langle R, X\rangle$ is called a system. $Syst \cap A_1$ denotes the class of all systems $\langle R, X\rangle$ based on atomic entailment and $Syst \cap C_1$ denotes the class of all systems $\langle R, X\rangle$ based on classical entailment. $\phi \left|\frac{A_1}{\text{R,X}}\right. \psi$ denotes that ψ results atomically from ϕ on the ground of the system $\langle R, X\rangle$ and $\phi \left|\frac{C_1}{\text{R,X}}\right. \psi$ denotes that ψ results classically from ϕ on the ground of the system $\langle R, X\rangle$. L_2 is the set of all formulas valid in the classical predicate calculus.

2. *Atomic entailment.*

Definition 2.1. Let $Cn_1(R, X) = L \neq \emptyset$ and $\phi, \psi \in S_1$. Then $\phi \left|\frac{A_1}{\text{R,X}}\right. \psi$ iff the following conditions are satisfied:

(1) $(\forall e \in \epsilon_*^1)[h^e(\bigwedge \phi) \in L \Rightarrow h^e(\psi) \in L \;\&\; P_1(h^e(\bigwedge \phi)) \subseteq P_1(h^e(\psi))]$

(2) $(\forall e \in \epsilon_*^1)[h^e(CC\psi^*\phi^*\phi^*) \in L \Rightarrow h^e(\phi^*) \in L \;\&\; P_1(h^e(\psi^*)) \subseteq P_1(h^e(\phi^*))]$

Definition 2.2. $\langle R, X\rangle \in Syst \cap A_1$ iff the following condition is satisfied:

$$(\forall \phi, \psi \in S_1)\left[C \bigwedge \phi\psi \in Cn_1(R, X) \Leftrightarrow \phi \left|\frac{A_1}{\text{R,X}}\right. \psi\right]$$

3. *Classical entailment.*

Definition 3.1. Let $Cn_1(R, X) = L \neq \emptyset$ and $\phi, \psi \in S_1$. Then $\phi \left|\frac{C_1}{\text{R,X}}\right. \psi$ iff the following conditions are satisfied:

(1) $(\forall e \in \epsilon_*^1)[h^e(\bigwedge \phi) \in L \Rightarrow h^e(\psi) \in L]$

(2) $(\forall e \in \epsilon_*^1)[h^e(CC\psi^*\phi^*\phi^*) \in L \Rightarrow h^e(\phi^*) \in L]$

Definition 3.2. $\langle R, X\rangle \in Syst \cap C_1$ iff the following condition is satisfied:

$$(\forall \phi, \psi \in S_1)\left[C \bigwedge \phi\psi \in Cn_1(R, X) \Leftrightarrow \phi \left|\frac{C_1}{\text{R,X}}\right. \psi\right]$$

4. *The main results.*

Theorem 1. $(\exists L_D \subseteq L_2)\langle R_{0+}, L_D\rangle \in Syst \cap A_1$

Theorem 2. $\langle R_{0+}, L_2\rangle \in Syst \cap C_1$

Remark. Peano's Arithmetic System and all the well-known set theories can be based on the system $\langle R_{0+}, L_D\rangle$. Thus, $\langle R_{0+}, L_D\rangle$ is a non-classical logic, which is enough for the formalization of the classical mathematics and the paradoxes of material implication do not occur in the system $\langle R_{0+}, L_D\rangle$.

[1] W. A. Pogorzelski, ***The classical calculus of quantifiers***, PWN, 1981.

▶ ALEXEY STUKACHEV, *Effective presentations of uncountable structures*.
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We consider computability on abstract structures via the formalism based on the notion of Σ-definability in admissible sets and, in particular, in HF-superstructures (see [1]). Providing a natural generalization of the classical computable model theory, this approach can be used for measuring the complexity of abstract structures, no matter countable or not.

Concerning the conjecture stated by Yu.L. Ershov in [2], we study some (local) properties of Σ-degrees of uncountable structures. Namely, we prove that any c-simple theory with an additional property of discreteness has an uncountable model Σ-definable in $\mathbb{HF}(\mathfrak{L})$, where

$\mathfrak{L}$ is a dense linear order. As a corollary, this is true for any submodel complete c-simple theory with a finite signature, while in the class of submodel complete c-simple theories with infinite signatures there is a counterexample [3].

[1] YU.L. ERSHOV, ***Definability and computability***, Plenum, New York, 1996.

[2] ———, *Σ-definability of algebraic structures*, ***Handbook of recursive mathematics***, (Yu.L. Ershov, S. S. Goncharov, A. Nerode, and J. B. Remmel, editors), Elsevier, Amsterdam, 1998, pp. 235–260.

[3] A. I. STUKACHEV, *Σ-definability in hereditary finite superstructures and pairs of models*, ***Algebra and Logic***, vol. 46 (2004), pp. 258–270.

[4] ———, *Sigma-definability of uncountable models of c-simple theories*, ***Siberian Mathematical Journal***, to appear.

► DARIUSZ SUROWIK, *On temporal epistemic logic*.
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We would like to discuss some temporal epistemic logic and to consider some philosophical problems in the language of this logic. Similar systems were discussed in [1] and [2]. Time and knowledge are strongly related. Temporal reasoning and knowledge representation is getting more attention in recent years. To formalize considerations on knowledge changing in time we use some combined logic. The natural way to construct of this kind od logic is to insert epistemic logic in a temporal framework. We add to an existing propositional system two sets of specific operators. One set of specific operators is the set of temporal operators, which we use to describe temporal interactions. The basic language in which the the agent can describe his knowledge is propositional epistemic logic. Then the other set of specific operators is the set of epistemic operators to describe the agent's knowledge. We construct multi-modal system combining tense and knowledge operators. Reasoning can be seen as an activity of an agent taking place in time. It is a stepwise process. The agent starts with a set of initial facts to which it applies some rules to arrive at a new state, in which he has more knowledge. In this new state, the agent may apply rules to arrive at a next state. We assume that semantic time is linear, discrete and has a starting point (time is isomorphic to the set of natural numbers). Moreover, we assume that the agent may perform positive and negative introspection, then knowledge is modeled as an S5-like modality. We construct sound and complete system of temporal epistemic logic. To prove of the completeness of our system we use the method described in [3].

[1] ERICA CALARDO and VLADIMIR RYBAKOV, *An axiomatisation for the multi-modal logic of knowledge and linear time LTK*, ***Logic Journal of the IGPL***, vol. 15 (2007), no. 3, pp. 239–254.

[2] JOERI ENGELFRIET, *Minimal temporal epistemic logic*, ***Notre Dame Journal of Formal Logic***, vol. 37 (1996), no. 2, pp. 233–259.

[3] MARCELO FINGER and DOV M. GABBAY, *Adding a temporal dimension to a logic system*, ***Journal of Logic, Language and Information***, vol. 1 (1992), no. 3, pp. 203–233.

► MARGARET E. M. THOMAS, *The density of rational points on certain Pfaffian surfaces*.
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In this talk, we shall outline some results relating to the problem of bounding the density of rational points on sets definable in o-minimal expansions of the real field. We shall focus on a conjecture of Wilkie about sets definable in the real exponential field and review the progress that has so far been made towards this. In particular, we shall present some results in this direction for certain surfaces; these results are joint work with Gareth O. Jones.

► TINKO TINCHEV AND MITKO YANCHEV, *Generalized graded modal logic: tree-like models, axiomatization and decidability*.
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The graded modal language is a propositional language with a set of unary modal operators $\Diamond_n$ for any natural number $n \geq 0$. In a Kripke model $\mathcal{M}$ over a frame $\mathcal{F} = (W, R)$ a formula $\Diamond_n \varphi$ is true at $x \in W$ iff the set $R_\varphi(x) = \{y \mid xRy \text{ and } \varphi \text{ is true at } y\}$ has more than n elements. In this way $\Diamond_n$ has quantitative meaning.

We consider an extension of this 'counting' language—generalized graded modal language —adding unary 'majority' operators M^r for any rational r, $0 < r < 1$, which operators have, in some sense, qualitative meaning. We consider a semantics for the generalized graded modal language over the class of Kripke frames, satisfying the following condition: the set $R(x) = \{y \mid xRy\}$ is finite for any $x \in W$. We say that the formula $M^r\varphi$ is true at $x \in W$ iff $|R_\varphi(x)|/|R(x)| > r$.

Next we explore a connection between satisfiability of a formula and the existence of an integer solution of a system of linear inequalities with integer coefficients. In particular, we obtain that the satisfiability is equivalent to the satisfiability in the class of finite trees. We present a quite natural for the 'tree' semantics formal system with an infinite rule of inference and using a technique, based on the connection, stated above, we prove for that system both decidability of the satisfiability problem and completeness with respect to the class of all frames.

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In [1] a general abstract analysis of the torsionfree part of the Ziegler spectrum of a group ring RG, with R a Dedekind domain and G a finite group, is provided. Here we refine those results and we describe the torsionfree part of the spectrum, both the points and the topology, over the integral group ring of the Klein four group—a "tame" ring. For instance, we show that the Cantor–Bendixson rank of this space is 3.

This is a joint work with Gena Puninski (Moscow).

[1] M. Prest, A. Marcja, and C. Toffalori, *The torsionfree part of the Ziegler spectrum of RG when R is a Dedekind domain and G is a finite group*, ***The Journal of Symbolic Logic***, vol. 67 (2002), no. 3, pp. 1126–1140.

► TOSHIMICHI USUBA, *Unbranched property on singular cadinals*.
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We introduce a new combinatorial principle UB_λ, UnBranched property, for a singular cardinal λ. UB_λ is the principle asserting that there exists a function $f\colon [\lambda^+]^{<\omega} \to \lambda^+$ such that for every f-closed $x, y \subseteq \lambda^+$, if $x \cap \lambda = y \cap \lambda$, $\sup(x \cap \lambda) = \lambda$, and $\sup(x) \leq \sup(y)$ then $x \subseteq y$. We show that UB_λ is implied by the principle ADS_λ. ADS_λ is known as a very weak principle ([1]), but is inconsistent with some large cardinals. However we also show that UB_λ is consistent with almost all large cardinals and large cardinal properties: For instance, UB_λ is consistent with the existence of supercompact cardinals below λ and with Martin's Maximum. We observe some applications of UB_λ such as: (1) $\mathrm{UB}_{\omega_\omega}$ refutes $\langle \aleph_{\omega+1}, \aleph_\omega \rangle \twoheadrightarrow \langle \aleph_2, \aleph_1 \rangle$ (but $\mathrm{UB}_{\omega_\omega}$ is consistent with $\langle \aleph_{\omega+1}, \aleph_\omega \rangle \twoheadrightarrow \langle \aleph_1, \aleph_0 \rangle$). (2) UB_λ implies a kind of weak covering lemma. (3) UB_λ implies that for a regular κ with $\mathrm{cf}(\lambda) < \kappa$, every normal ideal over $P_\kappa\lambda$ which has no λ^+ disjoint positive sets is λ^+-saturated.

[1] J. Cummings, M. Foreman, and M. Magidor, *Squares, scales and stationary reflection*, ***Journal of Mathematical Logic***, vol. 1 (2001), no. 1, pp. 35–98.

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The notion of *Turing degree spectrum* of a countable structure $\mathfrak{A}$ is introduced by Richter as the set of all Turing degrees of the presentations of $\mathfrak{A}$. Soskov [1] initiated the study of the properties of the degree spectrum in $\mathcal{D}_e$, the upper semi-lattice of the enumeration degrees. He considered *all* enumerations of the structure, not only the injective ones. The benefit of this definition is that every degree spectrum is upwards closed with respect to total enumeration degrees.

The *co-spectrum* of $\mathfrak{A}$ is the set of all lower bounds of the elements of the degree spectrum of $\mathfrak{A}$. It turns out that its elements can be characterized in terms of a forcing relation and Σ_k^+ formulae, a kind of computable infinitary formulae. The degree spectra behave with respect to their co-spectra very much like the cones of the enumeration degrees $\{\mathbf{x} \mid \mathbf{x} \geq \mathbf{a}\}$ behave with respect to the ideals $\{\mathbf{x} \mid \mathbf{x} \leq \mathbf{a}\}$.

We study this notion under the ω-enumeration reducibility. It is a uniform reducibility between sequences of sets of natural numbers, introduced and studied in [2, 3]. In [4] A. Soskova studies the ω-degree spectrum of a structure. She showed that some properties of the degree spectrum such as the minimal pair theorem and the existence of quasi-minimal degree remain true for the ω-degree spectrum.

The ω-degree co-spectrum is the set of ω-enumeration degrees which are lower bounds of the elements of the ω-degree spectrum. We are interested how the ω-degree co-spectrum differs from the co-spectrum in $\mathcal{D}_e$. A notable difference is that every countable ideal in $\mathcal{D}_e$ is a co-spectrum of a structure, but we show that this does not hold for ω-degree co-spectra.

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[1] I. N. SOSKOV, *Degree spectra and co-spectra of structures*, ***Annuaire de l'Universite de Sofia***, vol. 96 (2004), pp. 45–68

[2] ———, *The ω-enumeration degrees*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 6, pp. 1193–1214

[3] I. N. SOSKOV and B. KOVACHEV, *Uniform regular enumerations*, ***Mathematical Structures in Computer Science***, vol. 16 (2006), no. 5, pp. 901–924

[4] A. A. SOSKOVA, *Omega degree spectra*, ***Logic and theory of algorithms*** (A. Beckmann, C. Dimitracopoulos and B. Loewe, editors), Lecture Notes in Computer Science, 5028, 2008, pp. 544–553

► PAULO S. VELOSO, LUIZ CARLOS PEREIRA, AND EDWARD H. HAEUSLER, ON WHAT THERE MUST BE*: existential validities and some related riddles*.
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Logic should not have purely existential theorems: the only existential sentences that are valid are those with valid universal analogues. One often hears this slogan, as well as as some explanations for it.

An intuitive explanation for this slogan can be roughly as follows. Consider an existential assertion, such as "there exist borogroves", or "something is a borogrove". Such an assertion may perhaps be derived from some hypotheses. But, could it be proved, from no hypothesis whatsoever? In this context, one has absolutely no idea about what 'borogrove' is supposed to mean. Note that a definition of 'borogrove' or other elucidations will not be of much help here. First, they would be counted as hypotheses. Second, if one explains 'borogroves' as, say, "those things that are nimsy", one is bringing 'nimsy' into the picture, and we are back to the same question. So, one can appeal only to logical principles. How would such a proof look like?

Also, when "something is a borogrove" happens to be true, one senses a certain asymmetry, distinguishing borogroves from non-borogroves. Again, this might be quite reasonable if we had some further information about these concepts. But, could this be true on logical grounds only? One feels that the above asymmetry runs counter the logical spirit, especially if accidental. This asymmetry would be inevitable in case everything had to be a borogrove.

Here, we show that every existential validity has a simple universal analogue, which is also valid. We also characterize existential and universal validities in terms of tautologies: we show how to transform existential and universal sentences to propositional counterparts, which will be tautologous exactly when the originating sentences are valid.

A version of the preceding slogan is that logic makes no existential commitments. Well, in a sense, it does make. One usually assumes that constant symbols denote elements of the universe; so the existential sentence $\exists \mathrm{x}\, \mathrm{x} \doteq \mathrm{c}$ is valid. An assertion like "Pegasus is a winged horse" may be paraphrased as "something identical to Pegasus is a winged horse". Similarly, function symbols represent total functions on the universe; so the existential sentence $\exists \mathrm{x} \exists \mathrm{y}\, \mathrm{y} \doteq \mathrm{f}(\mathrm{x})$ is valid. One may perhaps detect another existential commitment in the fact that logic usually considers only nonempty universes, which renders valid the existential sentence $\exists \mathrm{x}\, \mathrm{x} \doteq \mathrm{x}$. It is this commitment that allows one to inferr $\exists \mathrm{x}\, \varphi$ from $\forall \mathrm{x}\, \varphi$. We shall not exclude these cases.

Our main results will provide characterizations for universal-existential first-order consequences in terms of propositional reductions, universal versions, universal connections and bounded models.

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If a set X is recursively enumerable in $Y <_T X$ then X is *recursively enumerable above* or *r.e.a.* Y. Duely, if $\mathbf{x}, \mathbf{y}$ are the Turing degrees of X, Y repsectively then $\mathbf{x}$ is r.e.a. $\mathbf{y}$, and $\mathbf{x}$ is an r.e.a. degree. Relative (recursive) enumerability is almost everywhere in the Turing degrees. Interestingly, it is a property closely related to arithmetic Cohen generics. For example, Jockusch [3] showed that every 1-generic is r.e.a.. Later, Kumabe [4] significantly improved this theorem by showing that every n-generic is r.e.a. another n-generic. On the other hand, Shore proved that every r.e.a. degree bounds a 1-generic (see [5, VI.3.9]).

Recently Ambos-Spies et al. discovered more connections between relative enumerability and 1-genericity in [1]. They showed that if a degree $\mathbf{d}$ is above some non-GL_2 degree then $\mathbf{d}$ is r.e.a. a 1-generic. This theorem was later reproved by Cai and Shore [2] using a more direct approach and an elegant forcing notion.

Ambos-Spies et al. [1] also showed that if $\mathbf{d}$ is r.e.a. then it is recursive enumerable in a 1-generic $\mathbf{g}_0$. Moreover, for such a degree, there exists another 1-generic $\mathbf{g}_1 < \mathbf{d}$ with $\mathbf{d} \leq \mathbf{g}_1'$. So they asked whether $\mathbf{d}$ is necessarily r.e.a. some 1-generic.

Recently we gave a positive answer in [6], which needs some deep results from the theory of recursively enumerable degrees.

[1] KLAUS AMBOS-SPIES, DECHENG DING, WEI WANG, and LIANG YU, *Bounding non-GL_2 and R.E.A.*, ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 3, pp. 989–1000.

[2] MINGZHONG CAI and RICHARD SHORE, *Domination, forcing, array non-recursiveness and relative recursive enumerability*, to appear.

[3] Carl G. Jockusch, Jr., *Degrees of generic sets*, ***Recursion theory: its generalizations and applications, Proceedings of Logic Colloquium '79*** (Leeds), (F. R. Drake and S. S. Wainer, editors), Cambridge University Press, 1980, pp. 119–139.

[4] Masahiro Kumabe, *Relative recursive enumerability of generic degrees*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 1075–1084.

[5] Robert I. Soare, ***Recursively enumerable sets and degrees***, Perspectives in Mathematical Logic, Omega Series, Springer–Verlag, Heidelberg, 1987.

[6] Wei Wang, *Relative enumerability and 1-genericity*, to appear.

► MARS YAMALEEV, *Splitting with avoiding cones in Turing degrees*.
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A Turing degree **a** is splittable in a class of degrees $\mathcal{C}$ avoiding an upper cone of a degree **d** if there exist degrees $\mathbf{x}_0, \mathbf{x}_1 \in \mathcal{C}$ such that $\mathbf{a} = \mathbf{x}_0 \cup \mathbf{x}_1$, $\mathbf{x}_i < \mathbf{a}$ and $\mathbf{d} \not\leq \mathbf{x}_i$ for $i = 0, 1$. The following theorem presents sufficient conditions for a properly 3-computably enumerable (3-c.e.) degree to be splittable in $\mathbf{D}_3$ avoiding the upper cone of another properly 3-c.e. degree.

Theorem 1. *Let* **a** *and* **d** *be properly* 3*-c.e. degrees such that* $\mathbf{0} < \mathbf{d} < \mathbf{a}$ *and there are no* 2*-c.e. degrees between* **a** *and* **d**. *Then the degree* **a** *is splittable in* $\mathbf{D}_3$ *avoiding the upper cone of* **d**.

Theorem 1 holds when **d** is a $\mathbf{\Delta}^0_2$-degree, which does not contain 2-c.e. sets. Theorem 2 states that there exists a whole interval such that all 2-c.e. degrees from this interval are splittable avoiding the upper cone of some c.e. degree below them.

Theorem 2. *There exist c.e. degrees* $\mathbf{b} < \mathbf{a}$ *such that any* 2*-c.e. degree* **d**, *such that* $\mathbf{b} \leq \mathbf{d} \leq \mathbf{a}$, *is splittable in* $\mathbf{D}_2$ *avoiding the upper cone of* **b**.

► IGOR YANOVICH, *Evaluation tree languages*.
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Formulas of standard modal systems like **K** are evaluated in a Kripke model locally, at a point. More expressive hybrid languages [1] also involve evaluation at a point, and the additional expressive power comes from the operators allowing to "remember" certain points in a model ($\downarrow$) and/or jump to points without using accessibility relations (e.g., $\exists$). Evaluation tree (**ET**) languages add expressive power in a different way.

An **ET** formula is evaluated not at a point, but at a tree of points, built in such a way as to include all and only points used while checking the formula. E.g., p in $\Diamond p$ will be evaluated at a 2-level tree with the root w and leaves v s.t. wRv. Non-modal connectives and $\Diamond$ are defined so that formulas including only them are equipotent in a **ET** system and in its underlying standard modal logic. But since trees contain more information than single evaluation points, it is possible to define **ET** operators which cannot be expressed in standard modal languages.

Any **ET** system with operators using the tree but otherwise not allowed to perform jumps within the model is obviously invariant for generated submodels, but possibly not for bisimulations. That puts **ET** languages into the same expressivity region with $\mathbf{HL}(\downarrow)$ and $\mathbf{HL}(\downarrow, @)$. We compare the expressivity of different **ET** and **HL** systems, and discuss the fine cartography of that region, using both the standard notion of expressive power at the level of formulas and a new notion of expressive power at the level of subformulas.

[1] Patrick Blackburn and Jerry Seligman, *Hybrid languages*, ***Journal of Logic, Language and Information***, vol. 4 (1995), pp. 251–272.

▶ KEITA YOKOYAMA AND KOJIRO HIGUCHI, *A nonstandard counterpart of DNR*.
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In [3], Keisler introduced nonstandard counterparts of big five systems RCA_0, WKL_0, ACA_0, ATR_0 and Π^1_1-CA_0 by using the standard part principle and other axioms of nonstandard arithmetic. Similarly, WWKL_0 is characterized by a certain property for Loeb measure (LMP) in [4]. In this talk, we will give a nonstandard counterpart of the DNR axiom. The DNR axiom is the following statement: for any set $A \subseteq \mathbb{N}$ there exists a function $f\colon \mathbb{N} \to \mathbb{N}$ such that for any $e \in \mathbb{N}$ $f(e) \neq \varphi^A_e(e)$, *i.e.*, f is diagonally non-recursive relative to A. In Reverse Mathematics program, Giusto and Simpson [2] showed that a certain version of the Tietze extension theorem implies DNR, and Ambos-Spies, Kjos-Hanssen, Lempp and Slaman [1] showed $\mathrm{DNR} < \mathrm{WWKL}_0$. For the nonstandard characterization of DNR, we will define a nonstandard axiom DNR^*. Then, we will show that DNR^* is a conservative extension of DNR but it is purely nonstandard over a base system of nonstandard second-order arithmetic.

[1] Klaus Ambos-Spies, Bjørn Kjos-Hanssen, Steffen Lempp, and Theodore A. Slaman, *Comparing DNR and WWKL*, ***The Journal of Symbolic Logic***, vol. 69 (2004), no. 4, pp. 1089–1104.

[2] Mariagnese Giusto and Stephen G. Simpson, *Located sets and reverse mathematics*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 3, pp. 1451–1480.

[3] H. Jerome Keisler, *Nonstandard arithmetic and reverse mathematics*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), no. 1, pp. 100–125.

[4] Stephen G. Simpson and Keita Yokoyama, *A non-standard counterpart of WWKL*, preprint.

Abstracts of talks presented by title

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This expository and pedagogical lecture formalizes examples in first-order language interpreted in the natural numbers. For convenience we use ordinary variable-enhanced English: the English sentence schema 'every number x is such that $\mathrm{P}(x)$' translates the first-order schema '$\forall x\, \mathrm{P}(x)$'; 'every number' translates the quantifier '$\forall$' applied to numbers, or 'number x' translates the number variable 'x' following '$\forall$'.

The *Aristotelian Existential Import Schema*, *AEIS*, has as instances every one-premise argument whose premise is a universal sentence and whose conclusion is *the* corresponding existential obtained from the universal by replacing the initial universal quantifier by the existential.

$$\frac{\text{Every number } x \text{ is such that } \mathrm{P}(x).}{\text{Some number } x \text{ is such that } \mathrm{P}(x).}$$

$$\frac{\forall x\, \mathrm{P}(x)}{\exists x\, \mathrm{P}(x)}$$

The *Boolean Existential Import Schema*, *BEIS*, has as its instances every one-premise argument whose premise is a universalized conditional and whose conclusion is *the* corresponding existentialized conjunction obtained from the universalized conditional by replacing the initial universal quantifier by the existential *and* replacing the conditional connective by the conjunction sign.

$$\frac{\text{Every number } x \text{ is such that (if } \mathrm{A}(x)\text{, then } \mathrm{C}(x)).}{\text{Some number } x \text{ is such that } (\mathrm{A}(x) \text{ and } \mathrm{C}(x)).}$$

$$\frac{\forall x\, (\mathrm{A}(x) \to \mathrm{C}(x))}{\exists x\, (\mathrm{A}(x) \ \& \ \mathrm{C}(x))}$$

Every instance of AEIS is valid; AEIS is *panvalid*. Some but not every instance of BEIS is invalid; BEIS is *neutrovalid*, not *paninvalid*. An instance of BEIS is valid iff the existentialization $\exists x\, \mathrm{A}(x)$ of the antecedent condition $\mathrm{A}(x)$ is tautological. This lecture reviews the treatment and non-treatment of these phenomena in logic textbooks—Tarski, Quine, Lemmon, Goldfarb, etc.—while sharpening discussions in this BULLETIN, vol. 11 (2005), p. 460, vol. 11 (2005), pp. 554–555, vol. 12 (2006), pp. 219–240, and vol. 13 (2007), pp. 143–144. Acknowledgements: Arthur Bierman, Joel Friedman, Calvin Jongsma, Corey McGrath, Joaquin Miller, Frango Nabrasa, and Jan von Plato.

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Kurt Ebbinghaus's 1962 85-page German doctoral dissertation [1], supervised by the physicist-philosopher Carl von Weizsäcker, deserves more attention from historians of logic. It made some early steps away from the then-revered, revolutionary 1951 *Łukasiewicz axiomatized universal truths model LAUT* described in [2]. It also made steps toward the now widely-accepted 1972 *Buffalo–Cambridge natural-deduction model BCND* described in [3]. Further, Ebbinghaus took steps contradicting BCND.

Tradition takes a syllogism to be a *valid* premise-conclusion argument—completely determined by its premises and conclusion—all *proper propositions*, true or false, and containing concrete terms: *swan*, *animal*, *snow*, etc. LAUT takes syllogisms to be universally quantified abstract mathematical *truths* devoid of concrete terms. Transitivity of Aristolelian term-inclusion is an axiom of LAUT. BCND takes syllogisms to be deductions—chains of *reasoning*, both direct and indirect—whose premises, intermediate lines, and conclusions following *deduction rules* are all proper propositions. Ebbinghaus takes syllogisms to be deduction rules whose "terms" are abstract uninterpreted formal variables.

BCND is a three-part system—*language*, *deduction system*, and *semantic system*; only the first two parts are purely formal or string-theoretic. Ebbinghaus explicitly insists that his be an entirely formal model (*formales Modell*); he emphatically excludes the concepts of truth and falsity from any role in his model—resonating with an anti-semantic view briefly endorsed by Łukasiewicz in the 1920s but reputiated well before 1951. We describe Ebbinghaus's model and his interpretation of Aristotle, and we compare both with those of LAUT and BCND.

[1] K. EBBINGHAUS, ***Ein formales Modell der Syllogistik des Aristoteles***, Vandenhoeck & Ruprecht, Göttingen, 1964.

[2] J. ŁUKASIEWICZ, ***Aristotle's syllogistic***, Oxford University Press, Oxford, 1951.

[3] R. SMITH, ***Aristotle's Prior Analytics***, Hackett, Indianapolis, 1989.

► JOHN CORCORAN AND MICHAEL PRENTICE, *Mazur's semantic problem*.
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We discuss the following semantic question—apparently about the denotation of the 9-character expression '5 mod 691' in traditional arithmetic—raised by Harvard mathematician

Barry Mazur ([1] page 223).

"Is 5 mod 691 to be thought of as a symbol, or as a stand in for any number that has remainder 5 when divided by 691, or should we take the tack that it (i.e., "5 mod 691") is the equivalence class of all integers that are congruent to 5 mod 691?"

The expression '5 mod 691' is not in subject position in the true sentence '1387 is congruent to 5 mod 691'. If '5 mod 691'—as used in ordinary arithmetic literature—denotes an entity, one would expect the expression to occur as subject in its sentences, but we know of no sentences such as '5 mod 691 is congruent to 1387' using the problematic expression as subject.

Consider the following statement:

1387 is congruent to 5 mod 691

Could this statement be using a three-place relation

x is congruent to y modulo z

to interrelate three numbers: 1387, 5, and 691? If so, thinking that '5 modulo 691' is the object of the sentence—with '1387' as subject and 'is congruent to' as verb—is a *segmentation fallacy* related to *hypostasis* or *reification*: taking a non-denoting sentence fragment to be denoting an entity. Perhaps the "solution" to Mazur's problem is that there is no problem: perhaps he asked a pseudo-question. This article inquires how such issues can be objectively decided.

[1] B. Gold and R. Simons, ***Proof and other dilemmas: Mathematics and philosophy***, MAA, Washington DC, 2008.

► DON FAUST, *Full opposition symmetric evidence logic*.
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Full Opposition Symmetric Evidence Logic (FOS-EL) further explicates the concept of negation with machinery for positivity and negativity, extending explication given by Evidence Logic (*The concept of negation*, ***International Journal of Intelligent Systems***, vol. 15 (2000), pp. 477–493). Thereby FOS-EL can address, through analysis of axiomatizable extensions differentially realizing epistemological aspects of the concept of negation, questions of negative facts long considered by Russell and others.

FOS-EL is equipped, for each predicate symbol R, with atomic formulas built over the four associated predicate symbols confirmatory positive, refutatory positive, confirmatory negative, and refutatory negative, annotated with evidence values taken from an Evidence Space $E(n) = \{i/(n-1) \colon i = 1, \dots, n-1\}$.

In addition to the above-mentioned analysis of the concept of negation, the Boolean Sentence Algebras of the various FOS-EL languages are derived.

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Definition. Let T is uncountably but not totally categorical theory. Let us define the spectrum of constructive models of T by $SCM(T) = \{\kappa \leq \omega \mid \mathcal{M}_\kappa \textit{ is constructive}\}$.

Definition. The model is called $n-$constructive if its Σ_n-diagram is computable.

Spectrum of constructive models of uncountable categorical theories only consisting from prime models was founded with the aid of the example of the prime model constructed in [3] for finite relational binary language and the example of the prime model constructed in [1] for infinite language. The other spectrums of computability were founded in [2] for infinite languages. The question remains opened concerning the possibility of obtention of the same results (as [2]) for the final languages.

We investigate the complexity of the prime model $\mathcal{M}_0$ has been constructed in [3] by an finitely generated group G and theory T of this model is uncountably categorical, where $SCM(T) = \{0\}$.

First we prove that applying the Marker's operators ($\forall-, \exists-$exstention) n times, for any $n > 0$, to prime model $\mathcal{M}_0$ holds:

THEOREM 1. *The model* $(\mathcal{M}_0)_{(\forall\exists)^n}$ *is* $n-$*computable, but not* $(n+k)-$*computable for any* $k > 0$ *and* $n > 0$.

Now we can apply theorem 1 to prove the following result of this paper:

THEOREM 2. *If model* $(\mathcal{M}_0)_{(\forall\exists)^{n+1}}$ *is* $(n+1)-$*computable, then group G has a solvable word problem*.

[1] S. S. GONCHAROV, *Constructive models of* ω_1-*categorical theories*, ***Mathematical Notes***, vol. 23 (1978), no. 6, pp. 486–487.

[2] BAKHADYR KHOUSSAINOV, *Computable models of theories with few models*, ***Notre Dame Journal of Formal Logic***, vol. 38 (1997), no. 2, pp. 165–178.

[3] STEFFEN LEMPP, BERNARD HERWIG, and MARTIN ZIEGLER, *Constructive models of uncountably categorical theories*, ***Proceedings of the American Mathematical Society***, vol. 127 (1999), no. 12, pp. 165–178.

► JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Relevance logics, variable-sharing property and the "mingle" axiom*.
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As it is known, a logic S has the "variable sharing property" (vsp) if in any theorem of S of the form $A \to B$, A and B share at least one propositional variable. According to Anderson and Belnap, the vsp is a necessary property of any relevance logic (cf. [1]). On the other hand, the "mingle" axiom (M) is the schema $A \to (A \to A)$.

Anderson and Belnap "don't like mingle" ([1, §29.5]) because if M is added to Relevance logic R, the result ("R-Mingle") does not have the vsp.

The aim of this paper is to show that: (1) there are logics with the vsp that can hardly be considered relevance logics in the intuitive sense of the term; (2) relevance and mingle are in general compatible.

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[1] A. R. ANDERSON and N. D. BELNAP, JR., ***Entailment. The logic of relevance and necessity***, vol. 1, Princeton University Press, 1975.

► CARLOS A. OLLER, *Parry's analytic implication and genuine rivalry between logics*.
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This paper presents an argument, inspired in a remark by Susan Haack and related to an argument defended by Francesco Paoli, against the thesis that maintains that no genuine rivalry between logics is possible. We argue that a logic whose propositional connectives are characterized by the standard truth-functional semantics of classical logic, but whose consequence relation has non-standard features, constitutes an example of genuine deviance which does not boil down to a change in the meaning of the connectives. If Parry's analytic implication is understood as a consequence relation, the resulting paraconsistent logic constitutes an example of such a genuine rivalry with classical logic.

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Let S be a logic and T a theory built upon S. T is *w-consistent* (consistent in a weak sense) if T does not contain the negation of a theorem of S. On the other hand, a logic S is *constructive* if it is endowed with an intuitionistic type negation. Finally, S is *basic* for a certain notion of consistency if it is the minimal logic adequate to this notion (cf. [1]).

In [1], the basic constructive logic in the Routley–Meyer type ternary relational semantics without a set of designated points is defined, along with its appropriate extensions (i.e., the extensions of this basic logic which are adequate to w-consistency).

The aim of this paper is to carry out a similar research in the Routley–Meyer type relational semantics *with* a set of designated points; that is, in the semantics specifically designed for interpreting relevance logics. Surprisingly, the basic logic and, consequently, all its extensions are not relevance logics.

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[1] G. Robles and J. M. Méndez, *The basic constructive logic for a weak sense of consistency*, ***Journal of Logic, Language and Information***, vol. 17 (2008), no. 1, pp. 89–107.

► STANISLAV SPERANSKY, *On undecidability of the property of maximal specificity*.
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Let μ be a probability over formulas in a fixed propositional language $\mathcal{L}$; e.g., as in [2]. The following definitions capture the informal idea presented by Hempel [1] and are strongly related to inductive-statistical approach to predictions/explanations [3, 4].

Assume $\mathtt{Fact}_{\mathrm{o}}^{*}$ to consist of all literals allowing check in practice, they're permitted to appear in premises of rules [3]. We denote by Rule^{μ} the set of all rules for which conditional probability is determined; $\mathsf{VeR}^{*} \leftrightharpoons \{R \in \mathrm{Rule}^{\mu} \mid \mathtt{Body}\,[R] \subseteq \mathtt{Fact}_{\mathrm{o}}^{*}\}$, where $\mathtt{Body}\,[\cdot]$ stands for a set of literals in the premise of a relevant rule. Then we introduce binary *generality relation* on the set of rules: $R \succ R'$ iff they have the same headings and $\mathtt{Body}\,[R] \subset \mathtt{Body}\,[R']$. A rule $R \in \mathsf{VeR}^{*}$ is called *maximally specific*, or *ms-rule*, iff there is no $R' \in \mathsf{VeR}^{*}$ such that $R \succ R'$ and for conditional probabilities $\mu\,(R) < \mu\,(R')$. An ms-rule R is called an *ms-law* iff there is no ms-rule R' such that $R' \succ R$. The sets of ms-rules and ms-laws are denoted by MSR^{μ} and MSL^{μ} correspondingly. Finally, $R \in \mathsf{VeR}^{*}$ is *improvable* (write "$R \in \mathsf{ImR}^{\mu}$") iff it doesn't belong to MSR^{μ}.

Let γ be a Godel numbering for all well-formed formulas in $\mathcal{L}$, and $\varkappa$ be an analogous encoding for $\{r \in \mathbb{Q} \mid r \geqslant 0\}$. Certainly γ can also be extended to rules. Hereinafter we suppose that $\texttt{Fact}_\text{o}^*$ is a computable set. Denote by $\mathfrak{P}_\mathsf{c}$ the set of all computable rational-valued probability measures (they are computable functions transforming codes of formulas into codes of rational numbers and possessing the certain additive properties).

THEOREM 1. *There exists $\mu \in \mathfrak{P}_\mathsf{c}$ such that ImR^μ is an m-universal set (and so is not computable, i.e., the problem "$R \in \mathsf{ImR}^\mu$?" is unsolvable); as a corollary, for this μ the sets MSR^μ and MSL^μ are not computably enumerable.*

Moreover, there is no universal algorithm to recognize the situation described above.

THEOREM 2. *Let Ξ be the set of all measures $\mu \in \mathfrak{P}_\mathsf{c}$ such that ImR^μ is computable. Then the problem "$\mu \in \Xi$?", where $\mu \in \mathfrak{P}_\mathsf{c}$, is unsolvable (we presuppose Kleene numbering of computable functions).*

[1] C. HEMPEL, *Maximal specificity and lawlikeness in probabilistic explanation*, ***Philosophy of Science***, vol. 35 (1968), no. 2, pp. 116–133.

[2] D. SCOTT and P. KRAUSS, *Assigning probabilities to logical formulas*, ***Aspects of inductive logic***, (J. Hintikka et al, eds.), North-Holland Publishing, 1966, 219–264.

[3] S. SMERDOV and E. VITYAEV, *Probability, logic & learning synthesis: formalizing prediction concept*, ***Siberian Electronic Mathematical Reports***, vol. 9, 340–365.

[4] E. VITYAEV, *The logic of prediction*, ***Mathematical Logic in Asia 2005, Proceedings of the 9th Asian Logic Conference***, (S. Goncharov et al., editors), World Scientific Publisher, 2006, pp. 263–276.