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1996 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '96

Donostia-San Sebastián, Spain, July 9–15, 1996

The 1996 European Summer Meeting of the Association was held at the University of the Basque Country, at Donostia (San Sebastián), Spain, on July 9–15, 1996. It was organized by the Institute for Logic, Cognition, Language, and Information (ILCLI) and the Department of Logic and Philosophy of Science of the University of the Basque Country. It was supported by: the Universidad del Pais Vasco/Euskal Herriko Unibertsitatea, the Ministerio de Education y Ciencia (DGCYT), Hezkuntza Saila (Eusko Jaurlaritza), Gipuzkoako Foru Aldundia, and Kutxa Fundazioa.

The Program Committee consisted of K. Ambos Spies (Heidelberg), J. L. Balcázar (Barcelona), J. E. Fenstad (Oslo), D. Israel (Stanford), H. Kamp (Stuttgart), R. Kaye (Birmingham), J. M. Larrazabal (San Sebastián), D. Lascar (Paris, Chair), A. Marcja (Firenze), G. Mints (Stanford), M. Otero (Madrid), S. Ronchi della Rocca (Torino), K. Segerberg (Uppsala), and L. Vega (Madrid).

The Organizing Committee consisted of X. Arrazola (San Sebastián), R. Beneyto (Valencia), B. Carrascal (San Sebastián), K. Korta (San Sebastian), J. M. Larrazabal (San Sebastián, Chair), J. C. Martínez (Barcelona), J. M. Méndez (Salamanca), F. Migura (Victoria), and J. Pérez (Victoria).

The main topics of the meeting were: Model Theory, Proof Theory, Recursion and Complexity, Models of Arithmetic, Logic for Artificial Intelligence, Formal Semantics of Natural Language, and Philosophy of Contemporary Logic. The following twenty-three one-hour lectures were given at the meeting:

N. Asher (University of Texas), *Mathematical foundations of discourse structure*.

M. L. Bonet (Universidad Politécnica de Cataluña), *Complexity of propositional proofs*.

J. Etchemendy (Stanford University), *Heterogeneous logic*.

D. Evans (University of East Anglia), *Group-theoretic methods for classifying finite covers of Omega-categorical structures*.

L. Gordeev (University of Tübingen), *Finite proof theory*.

M. Hyland (University of Cambridge), *Proofs as process*.

V. Kanovei (Moscow TEI), *On external Scott algebras and some other definability results in nonstandard Peano models*.

U. Kohlenbach (Frankfurt University), *Arithmetizing proofs in analysis*.

R. Kossak (City University of New York), *Satisfaction classes and automorphisms of models of PA*.

M. Kummer (Universität Karlsruhe), *A recursion theoretic view of Kolmogorov complexity*.

S. Lempp (University of Wisconsin), *Decidability and undecidability in the enumerable*

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Turing degrees.

B. Nebel (University of Ulm), *Why should we care about computational costs?*.

A. Nesin (Tehran), *A survey of results in groups of finite Morley rank*.

H. J. Ohlbach (Imperial College, London), *Reducing the recursivity in clause sets*.

M. Pentus (Moscow State University), *Free semigroup semantics for Lambek syntactic calculus*.

M. Pinkal (Universität des Saarlandes), *Natural-language semantics with higher-order constraints*.

G. Priest (University of Queensland), *Inconsistent arithmetics and non-Euclidean geometries*.

M. Reynolds (King's College, London), *Intervals and two-dimensional temporal logic*.

E. Sandewall (Linkoping University), *Underlying semantics for logics of actions and change*.

J. Schmerl (University of Connecticut), *Arithmetically saturated models of Peano arithmetic*.

S. Starchenko (Vanderbilt University), *Constructing groups and fields in o-minimal structures*.

G. Takeuti (University of Illinois), *Proof theory and problems of complexity theory*.

L. Torenvliet (University of Amsterdam), *On the structure of complete sets*.

The following six tutorials were given:

A. Berarducci (University of Pisa), *Open induction* (3 hours).

M. Dezani-Ciancaglini (University of Torino), *Intersections types* (3 hours).

M. Hyland (University of Cambridge), *Linear logic* (90 minutes).

J. Lutz (Iowa State University), *Resource-bounded measure* (3 hours).

J. J. C. Meyer (Utrecht University), *Logic for multi-agent systems* (3 hours).

C. Steinhorn (Vassar College, New York), *O-minimality* (3 hours).

Abstracts of invited lectures, tutorials, and contributed talks given by members of the Association for Symbolic Logic follow.

For the Program Committee
DANIEL LASCAR

Abstracts of invited talks

▶ M. LUISA BONET, *Complexity of propositional proofs*.

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We are going to talk about propositional proof systems and their relationship with the field of complexity theory in Computer Science. Proof theorists have defined a number of propositional proofs systems, among them, resolution, sequent calculus, Frege systems (Hilbert style), etc.

In 1974, Cook and Reckhow gave a new definition of propositional proof system, that allowed them to relate the lengths of proof of tautologies, with major questions in complexity theory. They defined a propositional proof system, as a polynomial time function from the set of strings in some alphabet, onto the set of tautologies. This means that a propositional proof system, is one in which we can check that some sequence of symbols is a proof of a tautology, in polynomial time. From this definition, they proved that the existence of a propositional proof system that had "short" proofs for all tautologies, was equivalent to the assertion NP=Co-NP. A corollary of this is that if all propositional proof systems have classes of tautologies that require "long" proofs, then P is not equal to NP. This corollary gave rise to Cook's program. The idea is to take different proof systems, and try to prove superpolynomial lower bounds for them. I.e., for particular proof systems, to find families of tautologies that require superpolynomial length proofs.

After about 20 years of this program, some successes have been obtained. In 1985 Armin Haken proved that the propositional version of the pigeonhole principle requires exponential size resolution proofs. After that superpolynomial lower bounds were proven for the following systems: Bounded-depth Frege (Atjai '88, Pitassi-Impagliazzo-Beame '92, Krajicek-Pudlak-Woods '92), Cutting Planes (Bonet-Pitassi-Raz '94, Pudlak '95, Hakin-Cook '95), and Nullstellensatz proof systems of bounded degree (Beame-Impagliazzo-Krajicek-Pitassi-Pudlak '95).

The previous lower bounds required difficult techniques, and knowledge of fields like algebraic geometry, circuit complexity theory, etc. But still at this stage, nobody knows how to prove lower bounds for sophisticated proof systems like Frege or sequent calculus, and much less, lower bounds for systems more powerful than those. We will talk about different possibilities on how to continue Cook's program. One direction is to try to work on techniques to prove polynomial lower bounds for Frege systems. Even that seems a very difficult task. Another direction, is to explore and maybe define proof systems somewhat less powerful than Frege or sequent calculus, but more sophisticated than the ones we already know how to prove lower bounds for.

► L. GORDEEV, *Finite proof theory*.

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The familiar Gentzen-style proof theory deals differently with propositional connectives and quantifiers. Although propositional rules of inference admit direct proof search algorithms which successively reduce formulas to their principal subformulas, this is not the case of the rules of quantification. This is because the treatment of "for all x" and "there exist x" in both modus-ponens and sequent calculi actually imitate infinite model theoretical interpretations by viewing 'x' as a variable element in an arbitrary infinite structure. It is assumed that bound variables can every time be separated from the arbitrarily many given free variables, which implies that the set of all available (names of) variables should be infinite. That this assumption is not quite harmless shows standard completeness proofs either for cut free sequent calculi or resolution-type systems, which always provide infinite models consisting of (the names of) eigenvariables or suitable terms—although very often finite models would suffice. Note that strictly speaking Gentzen's rule of universal quantification does not preserve the subformula property, since the correlated premise-eigenvariable can (in fact, almost everywhere must) differ from all variables occurring in the conclusion.

By contrast, algebraic formalisms of predicate logic usually do not require any assumption of infinity—in fact, just finite models (algebras) are often more important there than infinite ones. These formalisms in turn are closely related to the modus-ponens predicate calculi with finitely many distinct variables. For example, the canonical formalism of relation algebras is equivalent to the modus-ponens logic (with equality) of four distinct variables. So what about proof theory of predicate logic with n distinct variables (abbreviated as n-VAR logic)? It turns out that sequent calculi proper are not suitable for n-VAR logic because they do really need infinitely many distinct eigenvariables. A possible improvement provides an idea of a rewriting system imitating Gentzen's rules (in the restricted variable domain) on arbitrary subformula levels simultaneously. To avoid obvious variable collisions one can strengthen "premise subformulas" either by "deleting" some "old" variables, or by adding the corresponding "new" universal quantifiers. Loosely speaking, these operations (reductions) produce new combinations of old subformulas.

This idea was implemented in [1] for n-VAR logic without function symbols and equality (engere Praedikatenlogik). The resulting rewriting systems (reduction calculi) RPCn prove all theorems of n-VAR logic, and they admit almost direct proof search algorithms, which is important to the automated theorem proving. Moreover, RPC4 and RPC5 share certain semi-completeness properties which make them very comprehensive. In the present lecture it will be shown how to extend this approach onto n-VAR logic with equality. There are strong connections between the resulting reduction calculi RPCEn and both relation and cylindric algebras. In particular, it will be shown that RPCEn enable us to solve some problems posed in [2] and [3].

[1] L. GORDEEV, ***Cut free formalization of logic with finitely many variables, Part I***, Lecture Notes in Computer Science, no. 933, Springer-Verlag, 1995, pp. 136–150.

[2] L. HENKIN, J. D. MONK, and A. TARSKI, ***Cylindric Algebras, Part I***, North-Holland, 1971.

[3] A. TARSKI and S. GIVANT, *A formalization of set theory without variables*, ***American Mathematical Society Colloquium Publications***, vol. 41 (1987).

► MARTIN HYLAND, *Proofs as process*.
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In this talk I aim to explain two related ideas.

(1) A game theoretic approach to the representation of proofs in minimal logic. This gives a fully compositional extension of dialogues first proposed by P. Lorenzen. The game theoretic semantics can be derived from one for intuitionistic linear logic, and has an interpretation in terms of essentially sequential processes.

(2) A new semantics for classical linear logic based on another notion of process derived from recent work of A. Joyal on free bicompletions. When combined with the ideas of Girard's LC and LU, this suggests a programme for analysing the 'constructive content' of classical proofs.

► STEFFEN LEMPP, *Decidability and undecidability in the enumerable Turing degrees*.
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Much of the work in classical computability theory over the last few decades has focused on the Turing degrees of the (recursively) enumerable sets. These degrees form one of the fundamental structures of mathematics and can be characterized in various ways, e.g., as the degrees of word problems of finitely presented groups, or as the degrees of solution sets of Diophantine equations.

I will summarize some of the research over the last decade, with particular attention given to questions of decidability and undecidability in its first-order theory. The initial algebraic investigations of this degree structure in the 1960s and 1970s led to the first proof of the undecidability of its first-order theory by Harrington and Shelah in 1982. This theory was then shown to be as complicated as first-order arithmetic by Harrington and Slaman, and to be non-$\aleph_0$-categorical by Lerman, Shore and Soare.

More recently, a sharper undecidability result, for the Π_3 fragment of the theory, was obtained by Lempp, Nies and Slaman, and a nontrivial automorphism of the degree structure was exhibited by Cooper. I will also talk about recent results on definability and on recent work in progress toward showing its Π_2-theory to be decidable, in particular about the characterization of the finite lattices embeddable into this structure, which is still open.

▶ ALI NESIN, *Omega stable groups of finite Morley rank*.
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Groups of finite Morley rank are groups on which a concept of dimension can be defined. These groups look like algebraic groups but are not necessarily so. However, simple groups of finite Morley rank are conjectured to be algebraic groups. This conjecture is known under the name of Cherlin Zil'ber Conjecture. I will give a survey of the present state of classification of groups of finite Morley rank and talk about the latest developments.

▶ GRAHAM PRIEST, *Inconsistent arithmetics and non-Euclidean geometries*.
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There are many pure geometries; and which one is to be applied (as the physical geometry of some region of space) is an a posterior matter. The paper argues that exactly the same is true of arithmetic. The existence of arithmetics other than standard arithmetic is illustrated with reference to inconsistent arithmetics based on a paraconsistent logic. In the process, what is known about the structure of these is briefly reported. A crucial fact about such arithmetics is that they allow solutions to equations such as: $n = n + 1$. To demonstrate that the status of arithmetic is a posteriori, a thought experiment is described to show that it is epistemologically possible that we might come to replace ordinary arithmetic, in one of its standard applications in physics, by such an inconsistent arithmetic.

▶ JAMES H. SCHMERL, *Arithmetically saturated models of Peano arithmetic*.
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Among all the models of Peano Arithmetic the recursively saturated ones stand out as particularly attractive and interesting ones. This has been known since the early 70s when Barwise and Schlipf characterized them in a surprising way. All the countable, recursively saturated models of PA tended to look alike; that is, proving a model-theoretic result for one of them was tantamount to proving it for all of them. However, the subclass of the arithmetically saturated ones shows this not to be the case.

This talk will be about arithmetically saturated models of PA; I will concentrate on $\mathrm{Aut}(M)$, which is the group of automorphisms of M, and on $\mathrm{Lt}(M)$, which is the lattice of elementary substructures of M, where M is a countable arithmetically saturated model of PA. Concerning $\mathrm{Aut}(M)$, I will discuss the following two theorems:

THEOREM 1. *If M, N are two countably arithmetically saturated models of PA such that M and N are elementarily equivalent and* $\mathrm{Aut}(M)$ *and* $\mathrm{Aut}(N)$ *are isomorphic, then M and N are isomorphic.*

THEOREM 2. *If M is a countable, recursively saturated model of PA, then the cofinality of* $\mathrm{Aut}(M)$ *is uncountable if and only if M is arithmetically saturated.*

Theorem 2 shows a difference between arithmetically and recursively saturated models. The reason that we cannot prove that $\mathrm{Aut}(M)$ has uncountable cofinality when M is not arithmetically saturated is simply that it is not true. On the other hand, the reason that we cannot extend Theorem 1 to models which are not arithmetically is that we just do not know how.

I will also discuss the next theorem, which is the analogue of Theorem 1 for substructure lattices.

THEOREM 3. *If M, N are elementarily equivalent, countable, arithmetically saturated models of PA such that* $\mathrm{Lt}(M)$ *and* $\mathrm{Lt}(N)$ *are isomorphic, then M and N are isomorphic.*

The extension of this result to recursively saturated models, like Theorem 1, is open.

[1] R. KOSSAK and J. H. SCHMERL, *The automorphism group of an arithmetically saturated model of Peano arithmetic*, ***Journal of the London Mathematical Society***, vol. 52 (1995), no. 2, pp. 235–244.

[2] ———, *Arithmetically saturated models of arithmetic*, ***Notre Dame Journal of Formal Logic***, vol. 36 (1995), pp. 531–546.

▶ SERGEI STARCHENKO, *Constructing groups and fields in o-minimal structures*.
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A linearly ordered structure $\mathcal{M} = \langle \mathcal{M}, <, \dots \rangle$ is *o-minimal* if every definable subset of M is a finite union of intervals whose endpoints lie in $M \cup \pm\infty$. (As usual by "definable" we mean "definable with parameters".)

A function $f : D \subseteq M^n \to M^m$ is definable if its graph is definable.

A group $\langle G, \cdot \rangle$ is definable in $\mathcal{M}$ if G is a definable subset of M^k and $\cdot$ is a definable operation.

In this talk we will consider groups definable in o-minimal structures. We will show that in many cases such groups are isomorphic to definable subgroups of algebraic groups.

Also, for 1-dimensional groups, we will discuss some surprising consequences of a growth dichotomy. In particular, we will show that in any o-minimal structure $\mathcal{M}$, up to definable isomorphism, there are at most 2 definable continuous groups with underlying set M.

▶ GAISI TAKEUTI, *Proof theory and problems of complexity theory*.
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The polynomial hierarchy PH is closely related to the systems S_2^i $(i = 1, 2, \dots)$ in Bounded Arithmetic introduced by S. Buss. Therefore the central problems in PH the separation problems are closely related to the separation problems of S_2^i $(i = 1, 2, \dots)$.

We introduce Gödel sentence φ^i and weak Gödel sentences $\varphi_1^i, \varphi_2^i, \dots$ and weak normal consistencies $\mathrm{Ncon}(S_2^i)$, $\mathrm{Ncon}^1(S_2^i)$, $\mathrm{Ncon}^2(S_2^i), \dots$ and discuss their relations to separation problems. Then we build Boolean valued models over nonstandard models of S_2 and discuss its relationship to $P \neq NP$ and $NP \neq co - NP$.

Abstracts of invited tutorials

▶ JACK H. LUTZ, *Tutorial: Resource-bounded measure*.
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In the 1990s, the measure-theoretic study of complexity classes has yielded a growing body of new, quantitative insights into various much-studied aspects of computational complexity. Benefits of this study to date include improved bounds on the densities of complete problems;

newly discovered relationships among circuit-size complexity, pseudorandom generators, and natural proofs; strong new hypotheses that may have sufficient explanatory power (in terms of provable, plausible consequences) to help unify our present plethora of unsolved fundamental problems; and a new generalization of the completeness phenomenon that dramatically enlarges the set of computational problems that are provably strongly intractable.

The principal method used in these investigations is *resource-bounded measure*, a complexity-theoretic generalization of Lebesgue measure developed by Lutz [3]. Using resource-bounded martingales (originally introduced by Schnorr [5] in his investigation of randomness), resource-bounded measure defines measurability and measure for subsets of many complexity classes. The measure 0 and measure 1 subsets of exponential time complexity classes are of particular interest, because these subsets interact informatively with many of the most extensively investigated properties in computational complexity. These properties include NP completeness, immunity, strong intractability, circuit-size and time-bounded Kolmogorov complexity, randomized complexity and pseudorandom generators, the structure of polynomial-time reductions, and natural proofs [2, 1, 4].

This tutorial will introduce the fundamental ideas and techniques of resource-bounded measure and its applications to the study of central problems in computational complexity. The objective will be to impart an intuitive understanding of the motivation and development of resource-bounded measure, a working knowledge of its most important techniques, a survey of its progress to date, and directions of current and future research.

This tutorial is intended for nonspecialists with a basic understanding of computability and the significance of polynomial-time computation.

[1] J. L. BALCÁZAR, J. DÍAZ, and J. GABARRÓ, ***Structural complexity II***, Springer-Verlag, Berlin, 1990.

[2] ———, ***Structural complexity I***, second ed., Springer-Verlag, Berlin, 1995.

[3] J. H. LUTZ, *Almost everywhere high nonuniform complexity*, ***Journal of Computer and System Sciences***, vol. 44 (1992), pp. 220–258.

[4] ———, ***The quantitative structure of exponential time***, ***Proceedings of the eighth annual structure in complexity theory conference***, 1993, pp. 158–175, updated version to appear in ***Complexity theory retrospective II*** (L. A. Hemaspaandra and A. L. Selman, editors), Springer-Verlag, 1996.

[5] C. P. SCHNORR, ***Zufälligkeit und Wahrscheinlichkeit***, Lecture Notes in Mathematics, no. 218, 1971.

▶ J.-J. CH. MEYER, *Logic for multi-agent systems*.
Intelligent Systems Group, Department of Computer Science, Utrecht University, P. O. Box 80.089, 3508-TB Utrecht, The Netherlands.
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(Joint work with W. van der Hoek (Utrecht), B. van Linder (Utrecht), F. Dignum (Eindhoven), R. J. Wieringa (V. U. Amsterdam), and R. Kuiper (Eindhoven).)

In this tutorial we present an overview of recent work on (modal) logics to give a formal framework for specifying systems of intelligent agents, where we concentrate on *informational* and *motivational* attitudes of agents. We start out from a basic framework, comprising a mixture of dynamic and epistemic logic to reason about the *knowledge* and *abilities* of agents and the possible *results* of actions that they have the *opportunity* to perform. In this basic set-up we can express notions of a *correct and feasible plan* for an agent to achieve a certain goal ([1]). Using this framework as a platform we treat various informational and motivational attitudes of agents, such as *observations*, *communication*, *default reasoning*, *belief revision*, having *intentions*, setting *goals*, making *commitments* and having *obligations*. For informative actions we use a general belief revision action which satisfies the AGM postulates; dependent on the credibility of the incoming information this revision action acts on one or more parts of the classified belief sets of the agents ([2]). As to *motivational attitudes* of rational agents we incorporate in our framework *wishes*, *goals*, *commitments* and *obligations* of agents ([3], [4]).

[1] W. van der Hoek, B. van Linder, and J.-J. Ch. Meyer, *A logic of capabilities*, ***Proceedings of the 3rd international symposium on the logical foundations of computer science*** (A. Nerode and Yu. V. Matiyasevich, editors), Lecture Notes in Computer Science, no. 813, Springer-Verlag, Berlin, 1994, pp. 366–378.

[2] ———, *Seeing is believing (and so are hearing and jumping)*, ***Topics in artificial intelligence*** (M. Gori and G. Soda, editors), Lecture Notes in Computer Science (subseries LNAI), no. 992, Springer-Verlag, Berlin, 1995, pp. 402–423.

[3] ———, *Formalising motivational attitudes of agents: on preferences, goals and commitments*, ***Intelligent agents, Volume II, Agent Theories, Architectures, and Languages*** (M. Wooldridge, J. P. Müller, and M. Tambe, editors), Lecture Notes in Computer Science (subseries LNAI), no. 1037, Springer-Verlag, Berlin, 1996, pp. 17–32.

[4] F. Dignum, J.-J. Ch. Meyer, R. J. Wieringa, and R. Kuiper, *A modal approach to intentions, commitments and obligations: intention plus commitment yields obligation*, ***Deontic logic, agency and normative systems*** (M. A. Brown and J. Carmo, editors), Workshops in Computing, Springer-Verlag, Berlin, 1996, pp. 80–97.

Abstracts of contributed talks

► TAREK SAYED AHMED, HAJNAL ANDREKA, AND ISTVAN NEMETI, On neat reducts of algebras of logics

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We investigate generalisations of two results about neat reducts of cylindric algebras (CAs), to various classes of algebras that have the following in common with CAs:

(i) Can be considered as algebraisations of some versions of quantifier logics (e.g., of first order logic).

(ii) Can be conceived as expansions of boolean algebras from algebras whose elements are unary relations to algebras whose elements are relations of higher rank.

These algebras will include: Pinter's substitution algebras (RSCs), Sain-Thompson's quasi-polyadic (equality) algebras (QAs, QEAs) and Halmos polyadic (equality) algebras of infinite dimension (PAs, PEAs).

Previous results. Solving problem 2.11 formulated on p. 464 of [1], I. Nemeti showed in [3] that for ordinals $\alpha < \beta$:

(1) The class of all α-dimensional neat reducts of β-dimensional cylindric algebras is closed under the formation of subalgebras if and only if $\alpha \leq 1$, and is closed under the formation of homomorphic images.

As an easy consequence of (1) we obtain a confirmation of Tarski's conjecture (available in the form of an open problem in the introduction of [2]), which states that:

(2) For ordinals α and β, with $\omega \leq \alpha < \beta$, there exists a CA_α $\mathbf{A}$ which is a generating subreduct of a CA_β $\mathbf{B}$ and is distinct from Nr_α $\mathbf{B}$.

THE (MAIN) RESULTS. (1) (and (2)) generalise to RSCs, QAs and QEAs.

(2) does not generalise to PAs nor to PEAs. In particular for ordinals α, β with $\omega \leq \alpha \leq \beta$, the classes $\mathrm{Nr}_\alpha \mathrm{PA}_\beta$ and $\mathrm{Nr}_\alpha \mathrm{PEA}_\beta$ are varieties.

Other possible generalisations and the logical counterpart of the above are discussed.

[1] L. HENKIN, J. D. MONK, and A. TARSKI, ***Cylindric algebras, Part I***, North-Holland, Amsterdam, 1971.

[2] ———, ***Cylindric algebras, Part II***, North-Holland, Amsterdam, 1985.

[3] I. NEMETI, *The class of neat reducts of cylindric algebras is not a variety but is closed with respect to H and P*, ***Notre Dame Journal of Formal Logic***, vol. 24 (1983), no. 3, pp. 399–409.

► C. ANTHONY ANDERSON, *Apodictics*.

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In [3] John Myhill urged that the notion of *provability in an absolute sense* should be directly subjected to the logistic method. The project has begun to be carried out by Stewart Shapiro, Nicholas Goodman, and by Myhill himself ([5], [1], [4]). In those treatments a modal operator, or connective, is added to an underlying mathematical formalism, arithmetic or set theory, and axioms and rules of inference governing this new idea ('provability', symbolized by 'B', say) are proposed. Typically, they are the analogues of those of the modal logic S4:

(1) $Bp \supset p$,

(2) $B(p \supset q) \supset Bp \supset Bq$,

(3) $Bp \supset BBp$,

and 'From Φ, to infer $B\Phi$'. Leon Horsten, in [2], has usefully proposed that 'B' itself is to be seen as the juxtaposition of two connectives 'It is possible that', $\Diamond$ in ordinary modal logic, and 'It is proved that', say 'P', so that

$$B\Phi =_{\mathrm{df}} \Diamond P\Phi.$$

We here push this analysis one step further by developing a formalization of the usual informal identification of the *possibility of proving* with the (mathematical) *existence of an abstract proof*. A proof is here understood as a finite sequence of propositions (not sentences) related to one another in appropriate fashion. The task is seen to be non-trivial since the notion of a proof in this sense is not fixed by an antecedently given formal system.

We develop an intensional logic based on the primitive idea of (correct) inference and show that (1)–(3) are derivable therein on the basis of a natural definition of an absolute proof.

[1] NICHOLAS GOODMAN, *A genuinely intensional set theory*, ***Intensional mathematics*** (Stewart Shapiro, editor), North-Holland, Elsevier Science Publishers BV, Amsterdam, 1985.

[2] LEON HORSTEN, *Modal-epistemic variants of Shapiro's system of epistemic arithmetic*, ***Notre Dame Journal of Formal Logic***, vol. 35 (1994), pp. 284–291.

[3] JOHN MYHILL, *Some remarks on the notion of proof*, ***Journal of Philosophy***, vol. 57 (1960), pp. 461–471.

[4] ———, *Intensional set theory*, ***Intensional mathematics*** (Stewart Shapiro, editor), North-Holland, Elsevier Science Publishers BV, Amsterdam, 1985.

[5] STEWART SHAPIRO, *Epistemic and intuitionistic arithmetic*, ***Intensional mathematics*** (Stewart Shapiro, editor), North-Holland, Elsevier Science Publishers BV, Amsterdam, 1985.

▶ MOHAMMAD ARDESHIR, *A translation of intuitionistic predicate logic into basic predicate logic*.
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Basic Predicate Logic, BQC, is a proper subsystem of Intuitionistic Predicate Logic, IQC, in which *modus ponens* is weakened. (For motivations and history, see [1], [3], [5] and [4].)

We associate with every formula A in the language $\mathcal{L} = \{\vee, \wedge, \rightarrow, \perp, \top, \exists, \forall\}$, two sequences of formulas $\langle A_0, A_1, A_2, \dots \rangle$ and $\langle A^0, A^1, A^2, \dots \rangle$ such that the following hold:

(i) $\mathrm{BQC} \vdash A^n \Longrightarrow A^m$, $\mathrm{BQC} \vdash A_m \Longrightarrow A_n$, for all $m, n \in \mathbb{N}$ with $n \leq m$.

(ii) $\mathrm{IQC} \vdash A^n \Longrightarrow A^m$, $\mathrm{IQC} \vdash A_m \Longrightarrow A_n$, for all $m, n \in \mathbb{N}$ with $m \leq n$.

(iii) For every sequent $A \Longrightarrow B$, there is $n \in \mathbb{N}$ such that $\mathrm{IQC} \vdash A \Longrightarrow B$ if and only if $\mathrm{BQC} \vdash A_n \Longrightarrow B^n$.

Several applications of this translation are shown.

[1] M. ARDESHIR, *Aspects of basic logic*, ***Ph.D. thesis***, Marquette University, Wisconsin, 1995.

[2] M. ARDESHIR and W. RUITENBURG, *Basic propositional calculus I*, to appear.

[3] W. RUITENBURG, *Constructive logic and the paradoxes*, ***Modern Logic***, vol. 1 (1991), no. 4, pp. 271–301.

[4] ———, *Basic logic and Fregean set theory*, ***Dirk van Dalen Festschrift*** (H. Barendredt, M. Bezem, and J. W. Klop, editors), Department of Philosophy, Utretcht University, 1993, pp. 121–142.

[5] A. VISSER, *Aspects of diagonalization and provability*, ***Ph.D. thesis***, Rijsuniversiteit Utrecht, 1981.

▶ XABIER ARRAZOLA, *A set of rules of interaction in a cooperative multiagent system*.
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Following [1] we will present a set of rules that guide the interaction of agents in a cooperative multiagent context. The extension of the one agent context to a multiagent context permits, among other things, to consider actions not executable by an agent alone. This is, the accomplishment of the actions must be socialised given some disability of the agents. What theoretically one can carry out is not the same as what actually one can achieve. Given a system with communication, the standard approach in Artificial Intelligence [2]—and Philosophy [3]—presupposes a division of labour of the participating agents in the action. But to communicate is also to act and, in most cases, a precondition to act jointly.

According to the standard view, we consider an agent as a set of logics (belief, intention and goals), inserted in a multiagent context where the agents have the obligation to cooperate. The planning process starts when the agent is committed to an action, and this process offers the chain of subactions that must be accomplished in order to obtain the goal. Our proposal is applied to the process developed by an agent when committed to an action; this is, once the agent has a plan. The agent must achieve all of the actions of the plan to obtain the goal, and communicative actions are needed in the case that the agent could not carry out some subaction of the plan. We assume that communication is guaranteed. Rules are applied to the subsequent process of the planning, so they produce actions from the intentions and capacities of the agents.

[1] X. ARRAZOLA, *Acción colectiva: bases conceptuales y lógicas*, ***Ph.D. dissertation***, UPV-EHU, Donostia, 1996.

[2] M. E. POLLACK, *Plans as complex mental attitudes*, ***Intentions in communication*** (P. R. Cohen, J. Morgan and M. E. Pollack, editors), MIT Press, Cambridge, 1990, pp. 77–103.

[3] R. TUOMELA and K. MILLER, *We-intentions*, ***Philosophical Studies***, vol. 53 (1988), pp. 367–389.

▶ AGUSTíN ARRIETA URTIZBEREA, *Necessity and logical consequence in Aristotle's Prior Analytics*.

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Etchemendy, in *The concept of logical consequence*, after quoting Aristotle, tells us: "This modal characteristic, however dimly perceived and poorly understood, is clearly central to our intuitive understanding of the consequence relation". I will try to show that Aristotle has not a *dim perception* of that modal characteristic. Moreover, in Aristotle's *Prior analytics* there is a formal concept of logical consequence. In order to show that I make use of general ideas, though incorporating several new details, coming from Patzig's interpretation of Aristotle's work. Aristotle distinguishes between relative and absolute necessity, and it is the former that one related to logical consequence. In support of my claim, I show that Aristotle's concept of logical consequence has to be understood in light of his method of decision for inconcludent premise-pairs. In this way, I propose and formulate an Aristotelian definition for logical consequence in terms of substitution and universality, as it is done in Bolzano's proposal. Sebestik remarks that Bolzano considers Aristotle's "*éx anánkes*" as an antecedent of his own proposal.

Finally, I make several comments on Etchemendy's Tarski's fallacy, but concerning with Aristotle's account.

[1] ARISTOTLE, ***Prior analytics***, Hackett Publishing Company, Cambridge, 1989, translated by Robin Smith.

[2] J. CORCORAN, *Aristotle's natural deduction system*, ***Ancient logic and its modern interpretations*** (J. Corcoran, editor), Reidel, Boston, 1974.

[3] J. ETCHEMENDY, ***The concept of logical consequence***, Harvard University Press, London, 1990.

[4] G. PATZIG, ***Aristotle's theory of the syllogism***, Reidel, Boston, 1968.

[5] J. SEBESTIK, ***Logique et Mathématique chez Bernard Bolzano***, Vrin, París, 1992.

[6] L. VEGA, ***La Trama de la Demostración***, Alianza, Madrid, 1990.

▶ ANATOLY P. BELTIUKOV, *Hierarchy of small subrecursive classes based on bounded simultaneous recursion*.

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In this work the author gives a machine description of analogs of small function and predicate classes based on bounded recursion (by A. Grzegorczyk). These analogs are based on simultaneous bounded recursion of fixed dimension. Two kinds of classes are considered: classes of ordinary numerical functions and classes of word functions. It is proved that some classes of predicates from the above mentioned classes coincide despite different growth rates of their generating functions. It is proved that analogs of the unary functions class $\mathcal{E}_1^0$ have no finite bases with respect to superposition.

Let $\mathcal{E}^{n,k}$ denote the analog of the Grzegorczyk class $\mathcal{E}^n$ [1] for simultaneous k-dimensional bounded recursion. Let $\mathcal{E}_d^{n,k}$ denote the word analog of this class for d-letter alphabet. Let $\mathcal{E}^k(f)$ and $\mathcal{E}_d^k(f)$ denote the similar classes based on the function f as in [2]. Let A_* denote the class of all predicates (0-1-valued functions) of A.

Let us consider *k-dimensional stack register machines* (SRM-k) and *k-dimensional work stack register machines* (WSRM-k) that differ from the usual stack register machines [2] and word stack register machines [3] by having k components of *working* register and *usual stack* registers correspondingly. There are natural analogs of *space* and *word capacity* of computation for these machines. Let us denote the corresponding complexity classes as $\mathrm{SRM}^k(f)$ and $\mathrm{WSRM}^k(f)$ where f is a complexity bound, a function of maximum of input values.

THEOREM. *For all natural p, k and $d > 1$*

$$\begin{aligned}
\mathcal{E}_*^{0,k} &= \mathrm{SRM}^k(I), \quad \textit{where } I(x) = x \textit{ for any } x,\\
\mathcal{E}_{d\,*}^{0,k} &= \mathrm{WSRM}^k(\text{length}),\\
\mathcal{E}_*^{0,k} &= \mathcal{E}^k(\lambda x.x + x^{1-1/p})_*,\\
\mathcal{E}_{d\,*}^{0,k} &= \mathcal{E}_d^k(\lambda x.x \| a^{\langle \mathrm{length}(x)^{1-1/p}\rangle})_*,
\end{aligned}$$

where $\|$ is concatenation, $x^{\langle i\rangle} = x \ldots x$ (i times), a is a letter.

COROLLARY. *The classes of unary functions $\mathcal{E}_1^{0,k}$ and $\mathcal{E}_{d\,1}^{0,k}$ have no finite bases with respect to superposition.*

[1] A. GRZEGORCZYK, *Some classes of recursive functions*, ***Rozprawy Politechnika Poznańska***, vol. IV (1953), Warczawa.

[2] A. P. BELTIUKOV, *Machine description and hierarchy of initial Grzegorczyk classes*, ***Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniia Matematicheskogo Instituta***, vol. 88 (1979), pp. 30–46.

[3] ———, *Word stack register machines*, in ***Finite model theory and computer science***, Haifa, 1995, p. 31, proceedings of Logic colloquium '95.

► MARIA PAOLA BONACINA AND JIEH HSIANG, *On the notion of complexity of search in theorem proving*.

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We are concerned with the problem of defining notions of complexity that are suitable for the complexity of search in the infinite search spaces of theorem proving. In this work, our primary motivation is to capture the essence of *contraction* inferences, such as the subsumption and simplification, on the complexity of search. Such inference rules contract the search space and, from practical experience, usually produce simpler proofs. However, there has not been any formal analysis which justifies, mathematically, why contraction is effective in controlling the growth of the search space.

In this paper we try to answer this question by providing a general notion of complexity capable of describing and analyzing the behavior of the search space during a derivation. We begin by presenting a model for representing search. We point out that, unlike the conventional expansion inference rules such as resolution which merely visit the search space, contraction inferences visit and *modify* the search space at the same time. Our model captures this dynamic behaviour by introducing a notion of *marking* into the search graph, which allows us to describe the search plan of a strategy as well as its inference mechanism.

Based on the concept of marked search graph, we introduce a notion of complexity measure for the dynamic search spaces of strategies. Unlike the classical complexity measures which work with finite objects, our method deals with infinite search graphs and derivations which may not halt. Our notion of complexity analysis, based on well-founded orderings instead of natural numbers, captures both the present and the future of a derivation. We define notions of *ancestor-graph* and *dynamic distance* to replace the conventional notions of path and path-length. This allows us to "finitize" the future, the portion of the infinite search graph not yet discovered, into a sequence of finite search graphs within bounded distances.

We then apply the above framework to compare contraction-based strategies of different contraction power. We show how they affect the evolution of the respective search spaces during the derivation, and we analyze the variations of the complexity measure.

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Applying Rado's theorem we prove that partial well-orderings are absolutely characterizable in $L_{\infty,\omega}$, i.e., for each partial well-ordering P, there exists a sentence of $L_{\infty,\omega}$ such that it characterizes P except for isomorphisms. In order to prove that, we have to define a decomposition of the ordering by levels as in [4] and then we will extend the partial isomorphisms from finite sets of levels onto finite sets of levels, to a global isomorphism.

Secondly, we will prove by means of a counterexample that the property of not having any infinite antichains cannot be expressed in the language $L_{\omega_1,\omega}$. We see that the sentence $\forall X$ (X antichain $\rightarrow X$ is finite) is a Π^1_1 sentence over $L_{\omega_1,\omega}$. These formulas can be defined by disjunctive open game formulas as it is proved in [1]. We will define the Vaught's finite approximations of the sentence to prove the claim.

[1] J. Barwise, ***Admissible sets and structures***, Springer-Verlag, Berlin, 1975.

[2] R. Rado, *Axiomatic treatment of rank in infinite sets*, ***Journal of Mathematics***, vol. 1 (1949), pp. 337–343.

[3] R. Vaught, ***Descriptive set theory in*** $L_{\omega_1,\omega}$, Lecture Notes in Mathematics, no. 37, Springer-Verlag, Berlin, 1973, pp. 574–598.

[4] E. S. Wolk, *Partially well ordered sets and partial ordinals*, ***Fundamenta Mathematicae***, vol. 60 (1967), pp. 175–186.

► JOSÉ CARLOS CIFUENTES, *Cauchy ultrapowers and saturated model theory*.
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Given a structure $\mathcal{A} = \langle A, \dots \rangle \in \mathrm{St}^\tau$, where τ is a similarity type at most denumerable, and given $Y \subseteq A$ an at most denumerable subset of parameters of A, we define over A a zero-dimensional topology, and hence a uniformizable topology, which is given by the following basis of clopens: for all $\varphi(x) \in L^{\tau\cup Y}_{\omega\omega}$, $V_{\varphi(x)} = \{\, a \in A : \langle \mathcal{A}, y\rangle_{y\in Y} \models \varphi[a] \,\}$.

In this space, a sequence $\{a_n\}_{n\in\omega}$ is said to be a *Cauchy sequence* if and only if for all $\varphi(x) \in L^{\tau\cup Y}_{\omega\omega}$ there exists $n \in \omega$ such that for all $i, j \geq n$:

$$\langle \mathcal{A}, y\rangle_{y\in Y} \models \varphi[a_i] \iff \langle \mathcal{A}, y\rangle_{y\in Y} \models \varphi[a_j].$$

Let A^* ($\subseteq A^\omega$) be the collection of all the Cauchy sequences in A, and let D be a non-principal ultrafilter over ω. We define the *Cauchy ultrapower* of $\mathcal{A}$ by $\hat{\mathcal{A}} = \langle A^*/D, \dots \rangle$, where the quotient construction is similar to the construction of the ultrapower $\mathcal{A}^\omega/D$.

We prove that $\hat{\mathcal{A}} \succ \mathcal{A}$ and that $\hat{\mathcal{A}}$ realizes all the 1-types of $\mathcal{A}$. We also prove that we can define over $\hat{\mathcal{A}}$ a compact topology, which is induced by the topology of $\mathcal{A}$, in such a way so that $\mathcal{A}$ is a dense subspace of $\hat{\mathcal{A}}$, i.e., $\hat{\mathcal{A}}$ is a compactification, and hence a *Cauchy-completion*, of $\mathcal{A}$.

[1] J. C. CIFUENTES, *The problem of compactification in model theory*, ***Ph.D. thesis***, State University of Campinas, Brazil, 1993.

▶ FLÁVIO S. CORRÊA DA SILVA, *On reasoning with and reasoning about uncertainty in AI*.

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Uncertain reasoning is an active area of research in the field of Artificial Intelligence. Several different approaches to formalise this problem have been proposed [3, 6, 8, 10, 12, 16], each one better equipped to capture a different facet of the problem, and as a natural consequence there has been some debate on whether there could exist a "better", or a "more general" formalism to express uncertainty in formal reasoning.

The purpose of the present article is to argue against this possibility, by strengthening the view that different problems within the area of uncertain reasoning may require different formalisms, and that a pluralist approach to formalisation of uncertainty in reasoning may be the best alternative.

More specifically, in the present article we stress that, among the several systems proposed within this area, some have been developed to reason *with* uncertainty [2, 6, 7, 14, 15] whereas others have been developed to reason *about* uncertainty [1, 5, 4, 9, 10, 11, 13, 17].

We embody our view by developing a simple, yet—we believe—illustrative example: by adopting a single technique to measure uncertainty (namely, a simplified, lattice-based version of Necessity Measures [7, 8]) and a simple logical language (namely, classical propositional logic), we place the uncertainty measures differently on top of the language, producing first a language to reason *with* uncertainty, then another language to reason *about* uncertainty. After contrasting the interpretations of the measures in each case, we then show how they can be combined and still preserve their two different interpretations, thus producing languages to reason *with* uncertainty *about* uncertainty, to reason *about* reasoning *with* uncertainty, etc.

Since our purpose here is to build languages for artificial reasoning, we are careful during the development of each of these languages to preserve their computability properties, so that they can be implemented as systems for automated deduction.

[1] F. BACCHUS, ***Representing reasoning with probabilistic knowledge***, MIT Press, 1990.

[2] A. BUNDY, ***Incidence calculus***, Technical Report, no. 497, University of Edinburgh, Department of Artificial Intelligence, 1990.

[3] D. A. CLARK, *Numerical and symbolic approaches to uncertainty management in AI*, ***Artificial Intelligence Review***, vol. 4 (1990), pp. 109–146.

[4] P. R. COHEN, ***Heuristic reasoning about uncertainty: an artificial intelligence approach***, Pitman, London, 1985.

[5] F. S. CORREA DA SILVA, ***Automated reasoning about an uncertain domain***, Technical Report, University of Edinburgh, Department of Artificial Intelligence, 1991 (a preliminary version of this work was presented at the European Conference on Symbolic and Quantitative Aspects of Uncertainty under the same title, authored by F. S. Correa da Silva, D. Robertson, and P. Chung).

[6] F. S. CORREA DA SILVA, D. S. ROBERTSON, and J. HESKETH, *Automated reasoning with uncertainties*, ***Knowledge representation and uncertainty*** (M. Masuch and L. Pawlos, editors),

Lecture Notes in Artificial Intelligence, no. 808, Springer-Verlag, Berlin, 1994 (a preliminary version was presented at the Applied Logic Conference (Logic at Work) in 1992).

[7] D. Dubois, J. Lang, and H. Prade, *Automated reasoning using possibilistic logic: Semantics, belief revision and variable certainty weights*, in ***Proceedings of the 5th workshop on uncertainty in artificial intelligence***, 1989.

[8] D. Dubois and H. Prade, *An introduction to possibilistic and fuzzy logics*, ***Non-standard logics for automated reasoning*** (P. Smets, A. Mamdani, D. Dubois, and H. Prade, editors), Academic Press, 1988.

[9] R. Fagin, J. Y. Halpern, and N. Megiddo, *A logic for reasoning about probabilities*, ***Information and Computation***, vol. 87 (1990), pp. 78–128.

[10] J. Y. Halpern, *An analysis of first-order logics of probability*, ***Artificial Intelligence***, vol. 46 (1990), pp. 311–350.

[11] M. Kifer and V. S. Subrahmanian, *Theory of generalized annotated logic programs and its applications*, 1991, to appear in ***Journal of Logic Programming***.

[12] H. E. Kyburg Jr., ***Uncertainty logics***, Technical Report TR 337, University of Rochester, Department of Computer Science, 1990.

[13] N. J. Nilsson, *Probabilistic logic*, ***Artificial Intelligence***, vol. 28 (1986), pp. 71–87.

[14] Z. Pawlak, *Rough sets*, ***International Journal of Information and Computer Sciences***, vol. 11 (1982), pp. 145–172.

[15] J. Pearl, ***Probabilistic reasoning in intelligent systems: Networks of plausible inference***, Morgan Kaufmann, 1988.

[16] A. Saffiotti, *An AI view of the treatment of uncertainty*, ***The Knowledge Engineering Review***, vol. 2 (1987), no. 2, pp. 75–97.

[17] P. Smets, *The transferable belief model and other interpretations of Dempster-Shafer's model*, in ***6th conference on uncertainty in artificial intelligence***, 1990.

► INMA P. DE GUZMÁN AND MANUEL OJEDA-ACIEGO, *TAS methods in first-order logic*.

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Our research group GIMAC has presented TAS [2], a new general framework for generating ATPs as a part of a broader project. It has been a goal in our research to get a widely applicable, flexible and easily adaptable method to build ATPs with the property of generating countermodels in a natural manner.

The first TAS-based ATPs were TAS-D, for Classical Propositional Logic [1], and TAS-K_t, for Temporal Propositional Logic [3]. Currently, TAS has entered First-Order Logic, in [5] we introduce an efficient TAS method to transform a wff into Skolem normal form.

The problem of transforming a formula into a "good" normal form is an important problem in the field of Automated Theorem Proving. It is discouraging to see in specialised bibliography some examples of theorems whose clausal form is so complex that its treatment is difficult no matter the ATP we use, see for example [4]. Therefore, it is worth concentrating on investigating the existence of an efficient method to get a normal form.

The method to get a Skolem form, TAS-Sk, has all the features of the TAS methodology: the introduction of strategies which, by analysing the syntactic tree of the formula, detect tautologies, contradictions and subformulas which are equivalent to smaller-sized ones. These strategies avoid unnecessary distributions, allowing us to execute the non-avoidable distributions efficiently in a parallel manner. TAS-Sk has been designed to be not only an efficient method, but to obtain information about the (un)satisfiability of the wff. Furthermore, if satisfiability is detected, then the method gives a model of the wff, thus avoiding the ulterior application of an ATP. Indeed, some problems in Pelletier's paper [6] are solved during the translation to Skolem form.

The first idea in order to get a clausal form would be the following: given a wff A we first get a prenex form using the standard method, then apply TAS-NC, transformation to cnf, to the matrix and, finally, skolemize; this way, in the second process, we will have all the advantages of the TAS method. However, our aim has been more ambitious: to have these advantages "a priori," so we can simplify the matrix *and*, consequently, also the prefix. This goal is interesting because it simplifies the output of the method: it can even detect the (un)satisfiability of the wff, avoiding the application of an ATP.

The extension to First-Order has required a great effort of formalization but, in return, much has been given in terms of computational efficiency and new improvements in the propositional case.

[1] G. AGUILERA, I. P. DE GUZMÁN, and M. OJEDA, TAS-D^{++} ***syntactic trees transformations for automated theorem proving***, Lecture Notes in Artificial Intelligence, no. 838, Springer-Verlag, Berlin, 1994, pp. 198–216.

[2] ———, *A new general approach to ATPs*, in ***Proceedings of KI-95, workshop on computational propositional logic***, Bielefeld, 1995, pp. 77–78.

[3] I. P. DE GUZMÁN and M. ENCISO, *A new and complete theorem prover for temporal logic*, in ***Proceedings of the IJCAI workshop on executable temporal logics***, Montreal, 1995.

[4] J. D. MCCHAREN, R. A. OVERBEEK, and L. A. WOS, *Problems and experiments for and with automated theorem-proving programs*, ***IEEE Transactions on Computers***, vol. 25 (1976), no. 8, pp. 773–782.

[5] M. OJEDA, *Métodos formales para normalización en lógica de primer orden usando la metodología TAS*, ***Ph.D. thesis***, Universidad de Málaga, España, 1996.

[6] F. R. PELLETIER, *Seventy-five problems for testing automatic theorem provers*, ***Journal of Automated Reasoning***, vol. 2 (1986), pp. 191–216.

► GILLES DOWEK, *A type-free formalization of mathematics where proofs are objects*.
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This paper presents a formalization of mathematics where proofs are objects of the universe, like numbers, functions, sets, etc. This internalization of proofs, based on Heyting-Kolmogorov interpretation and not on Gödel numbering, respects proof structure: variable renaming, permutations of proof steps, elimination of cuts do not change the proofs. This theory is highly inspired by Martin-Löf's type theory and by the calculus of constructions of Coquand and Huet, but unlike these systems, it is an ordinary untyped first order theory. It is also inspired by Beeson's theory C, but it emphasizes the decidability of the provability relation.

The first step towards this theory is the internalization of propositions: the predicate symbol $\in$ is replaced by a function symbol, therefore the expression $a \in b$ is a term and not a proposition. The corresponding proposition is written $\varepsilon(a \in b)$, where ε is the single predicate symbol of the language. Connectors and quantifiers are also replicated at the level of terms and a typical axiom is

$$\varepsilon(A \wedge' B) \iff \varepsilon(A) \wedge \varepsilon(B).$$

A function symbol pr can then be introduced. The set $\mathrm{pr}(A)$ is the set of proofs of the proposition internalized by the term A. Typical axioms are

$$\mathrm{pr}(A \wedge' B) = \mathrm{pr}(A) \times \mathrm{pr}(B)$$
$$\mathrm{pr}(A \Longrightarrow' B) = \mathrm{pr}(A) \to \mathrm{pr}(B)$$

where $E \times F$ is the cartesian product of E and F and $E \to F$ the set of functions from E to F.

We can then state the axiom

$$(\exists x\ x \in \mathrm{pr}(P)) \iff P$$

without apparent contradiction. We discuss why Gödel-Tarski argument on the inconsistency of the axiom

$$\mathrm{Bew}(\text{‘}P\text{’}) \iff P$$

does not apply to this theory.

At last we design an algorithm that decides a small fragment of the theory and prove our main theorem: a proposition P is provable if and only if there is a term t such that the proposition $t \in \mathrm{pr}(P)$ is checked by the algorithm.

The consistency of this theory is left open.

[1] M. J. BEESON, ***Foundation of constructive mathematics***, Springer-Verlag, Berlin, 1980.

[2] T. COQUAND and G. HUET, *The calculus of constructions*, ***Information and Computation***, vol. 76 (1988), pp. 95–120.

[3] P. MARTIN-LÖF, *Intuitionistic type theory*, ***Bibliopolis***, Napoli, 1984.

▶ D. DZIERZGOWSKI, *Interpreting Heyting Arithmetic in an intuitionistic version of Russell's Simple Theory of Types*.

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Russell's Simple Theory of Types (TT) is a set theory whose language contains countably many variables x^i, y^i, ... of type i, for each $i \in \omega$. Its atomic formulae are of the form $x^i \in y^{i+1}$ and $x^i = y^i$ and its proper axioms are:

• extensionality: $(\forall x^{i+1}, y^{i+1})((\forall z^i)(z^i \in x^{i+1} \leftrightarrow z^i \in y^{i+1}) \to x^{i+1} = y^{i+1})$, for each $i \in \omega$;

• comprehension schema: $(\exists x^{i+1})(\forall z^i)(z^i \in x^{i+1} \leftrightarrow \varphi)$, for each $i \in \omega$ and each formula φ where x^{i+1} does not occur free.

In this paper we consider *Intuitionistic* TT, the theory whose axioms are given above and whose underlying logic is intuitionistic logic. It can be shown that the excluded middle cannot be derived in this theory. So, an intuitionistic version of TT is defined much more easily than an intuitionistic version of ZF.

We mainly prove that Heyting Arithmetic can be interpreted in Intuitionistic TT in a very natural way. Define a set E to be *N-inductive* if and only if

$$\emptyset \in E \wedge (\forall x)(x \in E \to (\forall z \notin x)(x \cup \{z\} \in E)) \tag{1}$$

(we omit type indices to improve legibility). Then we define the set of *N-finite sets* as

$$\mathrm{NFin} = \bigcap \{\, E : E \text{ is N-inductive} \,\},$$

the smallest N-inductive set, and the set of Frege *natural numbers* as N-finite cardinals:

$$\mathrm{Nn} = \{\, n : (\exists x \in \mathrm{NFin})(\forall z)(z \in n \leftrightarrow z \simeq x) \,\},$$

where $z \simeq x$ if and only if there exists a 1-1 function mapping z onto x. Addition, multiplication and order can be defined on Nn in the usual, natural way.

Defining the *Axiom of Infinity* as

$$\text{(AxInf)} \qquad (\forall n \in \mathrm{Nn})(\exists m \in \mathrm{Nn})(m > n),$$

we prove in Intuitionistic TT that Nn models Heyting Arithmetic if and only if (AxInf) is true. Proofs are lengthy and require careful computations.

Throughout the paper, we discuss precise formulations of definitions and properties. For example, we prove that (AxInf) is equivalent to

$$(\forall x \in \mathrm{NFin})(\exists x' \in \mathrm{NFin})(x \simeq x' \wedge (\exists z)(z \notin x')),$$

but is not equivalent to

$$(\forall x \in \mathrm{NFin})(\exists z)(z \notin x').$$

Also, (1) cannot be replaced with

$$\emptyset \in E \wedge (\forall x)(x \in E \rightarrow (\forall z)(x \cup \{z\} \in E))$$

nor with

$$\emptyset \in E \wedge (\forall x)(x \in E \rightarrow (\forall z)(\forall y)(y \subseteq \{z\} \rightarrow x \cup y \in E)).$$

Finally, we show that our interpretation of Heyting Arithmetic can be adapted to work when types are restricted to $\{0, 1, 2\}$ instead of ω, or when the axiom of extensionality is omitted. This interpretation also works in some fragments of an intuitionistic version of Quine's New Foundations which are known to be consistent.

[1] D. Dzierzgowski, *Models of intuitionistic TT and NF*, ***Journal of Symbolic Logic***, vol. 60 (1995), no. 2, pp. 640–653.

[2] ———, *Finite sets and natural numbers in intuitionistic TT*, to appear in ***Notre Dame Journal of Formal Logic***.

[3] ———, *Finite sets and natural numbers in intuitionistic TT without extensionality*, to appear in ***Studia Logica***.

▶ P. GENTILINI, P. FORCHERI, AND M. T. MOLFINO, *Conjectural provability logic based on logical information measures*.

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Starting from our works in the field ([4, 5, 6]), we present here an extension of classical Provability Logic ([1, 9]) through the formalization of a probabilistic notion of provability. In [3] and in [2] we have introduced an *Informational Logic* which defines probabilistic theorems (S, p) of a given system T, where p is the probability of S to be provable in T. Probability p is defined through a measure H of the *quantity of logical information* or *logical entropy* included in a syntactical logical structure, which is an extension to a logical context of the Shannon entropy measure ([7]) and which allows us to define the *logical entropy* $H(Q)$ of a proof Q. We establish p in (S, p) as the ratio $H(Q)/H(P)$ between the entropies of an estimated *virtual proof* Q of L in T, and of a *real proof* P of S in a powerful T-consistent extension Ω of T. Since the virtual proof Q is produced by proof-theoretical *estimate criteria* CR ([2, 8]) for proofs in T, we have a completely proof-theoretical definition of the notion of probability. We say that S is *probably T-demonstrable* with probability p in the *informational context* $\Gamma = (\Omega, T, H, CR)$.

In this paper, we define provability predicates which express *probable demonstrability*. We consider informational contexts Γ so that Ω is the sequent version of Arithmetic **PA** ([10]) with recursive functions, and T is a **PA**-subsystem. We introduce the arithmetization of the metatheory of Informational Logic, and the crucial step in this process is the arithmetization of the concept of estimate criterion CR for proofs in T. We formally establish CR as a 4-tuple (NF, Es-ax, Es-R, Es-l) where NF is a normal form theorem for proofs in T, and Es-ax, Es-R, Es-l are *effective procedures* NF-based for the estimate of the proper T-axiom occurrences, rule occurrences and length of a possible proof Q of a given sequent S in T. We so define the recursive predicate

$$\mathrm{cr}_T(x) = \text{“}x \text{ is an estimate criterion for proofs in } T\text{”}$$

and then the recursive predicate

$$\mathrm{virt}_T(Q(y), S) = \text{“}Q \text{ is a virtual } T\text{-proof of sequent } S \text{ based on the estimate criterion } y\text{”}.$$

Through these tools we can introduce our *conjectural provability predicates*, which formally express conjectures on the T-provability of a sentence. We define the recursive predicate

$$\begin{aligned}\text{Prob-dem}_T(P, Q(CR), CR, S, p) = \text{“}&S \text{ is probably demonstrable in } T\\ &\text{with probability } p \text{ starting from a real } \mathbf{PA}\text{-proof } P \text{ of } S,\\ &\text{the estimate criterion } CR \text{ and the } CR\text{-estimated } T\text{-proof } Q \text{ of } S\text{”}.\end{aligned}$$

Therefore, the *almost provability of S in T* is:

$$\text{Al-Pr}_T(S) \equiv \forall x \ \text{Prob-dem}_T(P, Q(x), x, S, (1 - 1/e))$$

where $e = \mathrm{card}\{\text{symbols occurring in } P\}$, and the *possible* or *virtual provability of S in T* is

$$\text{Virt-Pr}_T(S) \equiv \exists x \ \text{Prob-dem}_T(P, Q(x), x, S, (1 - 1/e)).$$

We wish to investigate a possible general link between almost-provability $\text{Al-Pr}_T()$ and standard provability $\mathrm{Pr}_T()$, stated by the following *reality condition for provability conjectures* (RC):

$$\text{Al-Pr}_T(S) \wedge \exists y\,(\mathrm{virt}_T(Q(y), S) \wedge H(Q(y))/\,\mathrm{card}\,AxQ > H(S)) \rightarrow \mathrm{Pr}_T(S),$$

where AxQ is the set of proper T-axioms in Q, that is: "if S is T-almost provable and a T-virtual proof Q of S exists so that its mean entropy is greater than the entropy of S, then S is T-provable". For the class of equational **PA**-subsystems RC is justified by a set of proof-theoretic results already obtained in [3, 2], which also provide effective examples of estimate criteria based on a Normal Form Theorem for proofs.

[1] G. Boolos, ***The logic of provability***, Cambridge University Press, 1993.

[2] P. Forcheri, P. Gentilini, and M. T. Molfino, *Research in automated deduction as a basis for a probabilistic proof-theory*, ***Logic and Algebra*** (P. Aglianò and A. Ursini, editors), Dekker, New York, 1995, pp. 491–538.

[3] P. Gentilini, *Informational proof-theory*, in ***International conference on logic methodology and philosophy of science***, Florence, 1995.

[4] ———, *Provability logic in the Gentzen formulation of arithmetic*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 38 (1992).

[5] ———, *Proof-theoretic modal PA-completeness*, to appear.

[6] ———, *Syntactical results on the arithmetical completeness of modal logic*, ***Studia Logica***, vol. 52, (1993), pp. 549–564.

[7] A. I. Khinchin, ***Mathematical foundations of information theory***, Dover Publications, New York, 1957.

[8] J. Krajicek, *On the number of steps in proofs*, ***Annals of Pure and Applied Logic***, vol. 41 (1989), pp. 153–178.

[9] C. Smorynski, ***Self-reference and modal logic***, Springer-Verlag, 1985.

[10] G. Takeuti, ***Proof theory***, North-Holland, 1987.

▶ A. J. GIL, A. TORRENS, AND V. VERDÚ, *On algebraizable Gentzen systems for the finite-valued Łukasiewicz propositional logics*.
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Let $\mathcal{L}$ be a propositional language and let m be a natural number. An m-sided $\mathcal{L}$-sequent is an expression of the form $\Gamma_0|\dots\Gamma_{m-1}$, where each Γ_i is a finite sequence of $\mathcal{L}$-formulas. An m-sided Gentzen system (see [4] and, for $m = 2$, [5]) is a pair $\mathcal{G} = \langle \mathcal{L}, \vdash_{\mathcal{G}} \rangle$, where $\vdash_{\mathcal{G}}$ is a finitary and structural consequence relation on the set of m-sided $\mathcal{L}$-sequents. Examples of m-sided Gentzen systems are the deductive systems and the k-dimensional deductive systems ([1, 2]), and the pairs $\mathcal{S}_K = \langle \mathcal{L}, \vDash_K \rangle$, where K is a class of algebras such that the equational consequence relation $\vDash_K$ is finitary. Other Gentzen systems can be obtained from a sequent calculus by using the rules of the calculus to deduce sequents from sets of sequents instead of deducing sequents from the axiom alone. In this way one can obtain a many-sided Gentzen system for every finite-valued logic by using the sequent calculi defined in [3].

We define the notion of equivalence between Gentzen systems, and we say that a Gentzen system is algebraizable if it is equivalent to a Gentzen system of the form $\mathcal{S}_K$. We obtain a characterization of the algebraizability of an m-sided Gentzen system in the line of the characterization for k-dimensional deductive systems ([1, 2]) and for 2-sided Gentzen systems ([4]).

We also prove that if $\mathbf{S}(m)$ is the finite linear MV-algebra of m elements, then the m-sided Gentzen system obtained from the sequent calculus defined in [3] is equivalent to the m-valued Łukasiewicz logic and to the system $\mathcal{S}_{\mathbf{S}(m)}$, so it is algebraizable.

[1] W. J. BLOK and D. PIGOZZI, *Algebraizable logics*, ***Memoirs of the American Mathematical Society***, vol. 396 (1989).

[2] ———, *Algebraic semantics for universal Horn logic without equality*, ***Universal algebra and quasigroups*** (A. Romanowska and J. D. H. Smith, editors), Heldermann Verlag, Berlin, 1992, pp. 1–56.

[3] M. BAAZ, C. G. FERMUELLER, and R. ZACH, *Elimination of cuts in first-order finite-valued logics*, ***Journal of Information Processing and Cybernetics***, vol. 29 (1994), no. 6, pp. 333–355.

[4] A. J. GIL, A. TORRENS, and V. VERDÚ, *On Gentzen systems associated with the finite linear MV-algebras*, Mathematics Preprint Series, no. 198, Universitat de Barcelona, 1996.

[5] J. REBAGLIATO and V. VERDÚ, *Algebraizable Gentzen systems and the deduction theorem for Gentzen systems*, Mathematics Preprint Series, no. 175, Universitat de Barcelona, 1995.

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The aim of this contributed paper is to develop a semantic account of the so-called particles that associate with the informational focus and, in particular, of the quantificational adverb *only*, using *Segmented Discourse Representation Theory* (SDRT). We will show that to get this semantics some pragmatic considerations are in order.

We will start reviewing briefly the *alternative semantics* ([5]) and the *structured meanings theory* ([4]) both of which have been used to formalise informational partition in order to determine, in a subsequent step, the scope of *only* and other focus particles. We will try to show that these proposals are problematic at various other points, the smallest of which it is not that they posit a semantic mechanism in order to account for a pragmatic phenomenon like the articulation of utterances in focus and topic. This view is also maintained in [3] and [6].

Next we will introduce some of SDRT's basic notions ([1]) and a discourse theory of informational partition ([2]) which distinguishes two pragmatic kinds of informational structures: focus dominant and topic dominant structures. We will show that SDRT is an adequate framework to build the semantics of *only* and other focus sensible particles because it is a framework where the interaction between semantics and pragmatics can be modeled.

[1] NICHOLAS ASHER, ***Reference to abstract objects in discourse***, Kluwer Academic Press, Dordrecht, 1993.

[2] NICHOLAS ASHER and ISABEL GÓMEZ TXURRUKA, ***Extending SDRT to integrate a theory of the informational partition***, Report no. ILCLI-95-LIC-2, UPV-EHU, 1995.

[3] KAI VON FINTEL, *Restrictions on quantifier domains*, ***Ph.D. thesis***, University of Massachusetts, 1994.

[4] MANFRED KRIFKA, *A compositional semantics for multiple focus constructions*, University of Texas, 1991, second draft.

[5] MATS ROOTH, *Association with focus*, ***Ph.D. thesis***, University of Massachusetts, 1985.

[6] ENRIC VALLDUVÍ, *The informational component*, ***Ph.D. thesis***, University of Pennsylvania, 1990.

▶ THEODORE HAILPERIN, *Ontologically neutral logic*.
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In sentential logic, considered as an independent branch of logic, questions of ontology do not arise. Sentences are simply true, or false, and whether the existence or non-existence of anything is implied is irrelevant to logical inference. Indeed, there is no need to consider the inner structure of any sentence apart from the truth-functional. Ontology does become relevant at a higher level of logic, namely when quantifiers ranging over individuals are introduced. The normal reading of '$\exists x\, \phi$', for example, is 'There is an x such that ϕ'. However, the notion of a quantifier to be introduced will be distinctly different. Here quantifications will be 'infinitely' extended conjunctions and (non-exclusive) disjunctions of sentences, formed independently of the content of the sentences. The formal language used will have atomic sentences that show no internal structure. Consequently there need not be—unless one wishes to have one—a notion of an individual with properties nor that of an 'existential' quantifier, i.e., one implying the existence of objects. Hence our descriptive epithet "ontologically neutral" for the associated logic.

Other considerations, of a semantic nature, also involve ontological presuppositions, though of a different kind. All proofs of (Gödel) completeness—justifying the adequacy of an axiomatic foundation for quantifier logic—require the assumption that there is an actual infinity of *symbols* of the logic. Making use of results of Herbrand we shall show that this strong assumption is not necessary, that one can get by on the assumption of only a potential infinity of symbols.

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In [1], Shapiro describes and investigates an epistemic formalization of arithmetic. This is a formalization of arithmetic in a language which contains a modal operator, which is interpreted as 'absolute provability', or as 'provability in principle'. In [2], I analyze Shapiro's absolute provability operator into (a) a modal operator ('it is possible that') and (b) a purely epistemic operator ('some mathematician has a proof that'). In this way, a modal-epistemic formalization of arithmetic is obtained.

The present paper extends this approach to the higher-order situation. Taking Flagg's *Epistemic Type Theory* (see [3]) as a paradigm for the formalization of higher-order epistemic theories, a higher-order modal-epistemic formalization of arithmetic is developed. In this theory an epistemic concept of randomness is formulated, on the basis of which I construct a classical model of Troelstra's theory of lawless sequences, as presented in [4].

[1] S. Shapiro, *Epistemic and intuitionistic arithmetic*, ***Intensional mathematics*** (S. Shapiro, editor), North-Holland, Amsterdam, 1985, pp. 11–46.

[2] L. Horsten, *Modal-epistemic variants of Shapiro's system of epistemic arithmetic*, ***Notre Dame Journal of Formal Logic***, vol. 35 (1994), no. 2, pp. 284–291.

[3] R. Flagg, *Integrating classical and epistemic type theory*, ***Annals of Pure and Applied Logic***, vol. 32 (1986), pp. 27–51.

[4] A. S. Troelstra and D. van Dalen, ***Constructivism in mathematics: An introduction***, volume II, North-Holland, Amsterdam, 1988.

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Let G be a domain in the complex plane, i.e., an open connected set. The analytic functions defined on G, together with pointwise addition and multiplication form a ring, which is denoted by $H(G)$. Let R be a subring of $H(G)$ satisfying the following conditions:

(i) R contains all polynomials.

(ii) If $f = gh$, where $f, g \in R \setminus \{0\}$ and $h \in H(G)$, then $h \in R$.

Then there is a first-order algebraic sentence φ, independent of R, such that $R \vDash \varphi$ if and only if $R = H(G)$. In particular, if R is elementarily equivalent to $H(G)$, then $R = H(G)$. This strengthens the main result of [5].

[1] L. Bers, *On rings of analytic functions*, ***Bulletin of the American Mathematical Society***, vol. 54 (1948), pp. 311–315.

[2] J. Becker, W. Henson, and L. A. Rubel, *First-order conformal invariants*, ***Annals of Mathematics***, vol. 112 (1980), pp. 123–178.

[3] T. Huuskonen, *The construction of a definable non-categorical domain*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 37 (1991), pp. 217–226.

[4] ———, *Constants are definable in rings of analytic functions*, ***Proceedings of the American Mathematical Society***, vol. 122 (1994), pp. 697–702.

[5] L. A. Rubel, *On the ring of differentially-algebraic entire functions*, ***Journal of Symbolic Logic***, vol. 57 (1992), pp. 449–451.

► ANDRZEJ INDRZEJCZAK, *Cut-free sequent calculus for S5*.
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It is well-known fact that S5, when formulated via Gentzen approach meets some difficulties. In standard approach of Ohnishi and Matsumoto (1957) cut is not eliminable. Since then many nonstandard cut-free formalizations were proposed; e.g., Kanger (1957), Mints (1970), Sato (1980), Belnap (1982), Wansing (1992). All of them depend on some modifications of standard sequent calculus. Present system is believed to be quite simple and not very deviant from standard apparatus. There is one major modification and two minor ones:

(a) Two types of sequents are applied: classical $\Gamma \Longrightarrow \Delta$ and modal $\Gamma \ \Box\!\!\Longrightarrow \Delta$. The rule for introducing $\Box$ into succedent is connected with elimination of modal sequent: $\Gamma \ \Box\!\!\Longrightarrow \Delta\varphi / \Gamma \Longrightarrow \Delta\Box\varphi$, on condition that Δ contains only modalised wffs or is empty (dually for introduction of possibility). There is also one rule for introducing modal sequent into the proof: $\Gamma \Longrightarrow \Delta / \Gamma \ \Box\!\!\Longrightarrow \Delta$, on condition that Γ (or Δ) is empty or contains only modalised wffs.

(b) All logical rules (including constants) are symmetric; they don't allow for shift of any wff from one side of sequent in premise(s) to the other in conclusion. It enables us to generalise standard Gentzen rules for connectives so that they can be applied also to modal sequents.

(c) There are special, structural rules for shifting wffs from antecedent to consequent and vice versa both for classical and modal sequents. Shifted wffs are temporally signed with '$-$'; this 'operator' is not iterated.

Consistency is proved by translation of sequents into standard modal language. No attempt is made in proof of cut-elimination. Instead, automatic, finite proof-search procedure is given, which for any finite sequent either produces its proof (not the most economical however) or enables us to devise a countermodel. Hence completeness and decidability follows.

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We prove that any speedable computably enumerable set may be split into a disjoint pair of speedable c.e. sets. This answers a longstanding question of J. B. Remmel concerning the behavior of c.e. sets in the machine independent complexity theory of Blum and Marques. Instead of using the standard weak-jump formulation of speedability we work directly with the enumerations in the original Blum-Marques definition given above. This allows us to exploit dynamic properties of the tree construction in giving implicit measurements and enforcements of speedability by embedding these into the substages of the tree construction. The usual approach of breaking up a higher level sentence into substrategies that are explicitly working on it would suffice to measure speedability, but not to endow a set with this complexity property—by the time we found them, the elements that are measured to witness speedability would no longer have the desired properties for the sets we are trying to build. In order to apply the usual Shelah-Mekkler-Slaman strategy we must make more subtle measurements that preserve these desired dynamic properties until we can utilize these witnesses in our construction.

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Frege structures which are introduced as a semantical concept in [1] can be syntactically considered as a truth theory over applicative theories (cf. [2]). Applicative theories have their roots in Feferman's theory T_0 [3, 4], and are based on the *Basic Theory of Operations and Numbers* BON (cf. [5, 2]). BON is a theory with a partial application operation and comprising combinatory logic, pairing, and explicitly defined natural numbers.

Because of strictness problems Frege structures are introduced only for the total version of BON (i.e., the application operation is total). They comprise besides a truth predicate T terms to represent formulae, and the axiomatization yields a truth definition as long as the formula is T-positive.

In our talk we present an approach to define Frege Structures also for the partial theory using a concept of pointers in applicative theories.

We show that this approach preserves the proof-theoretic strength of the theory in the presence of natural forms of induction. Moreover, it is adequate with respect to the total version since the axiom of totality added to the partial version yields a theory equivalent to the Frege Structures over total applicative theories.

[1] Peter Aczel, *Frege structures and the notion of proposition, truth and set*, ***The Kleene symposium*** (J. Barwise, H. Keisler, and K. Kunen, editors), North-Holland, Amsterdam, 1980, pp. 31–59.

[2] Michael Beeson, ***Foundations of constructive mathematics***, Springer-Verlag, Berlin, 1985.

[3] Solomon Feferman, *A language and axioms for explicit mathematics*, ***Algebra and Logic*** (J. Crossley, editor), Lecture Notes in Mathematics, no. 450, Springer-Verlag, 1975, pp. 87–139.

[4] ———, *Constructive theories of functions and classes*, ***Logic colloquium '78*** (M. Boffa, D. van Dalen, and K. McAloon, editors), North-Holland, Amsterdam, 1979, pp. 159–224.

[5] Solomon Feferman and Gerhard Jäger, *Systems of explicit mathematics with non-constructive μ-operator, Part I*, ***Annals of pure and applied logic***, vol. 65 (1993), no. 3, pp. 243–263.

► KEPA KORTA AND FERNANDO MIGURA, *Some constraints on the inference of conversational implicatures*.

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Grice ([5]) coins the term "implicature" for what a speaker means but does not say. Grice characterizes implicatures with respect to general features of discourse formulated in the celebrated Cooperative Principle (CP) and the more specific maxims of Quantity, Quality, Relation, and Manner in which it is developed. After supposing the observance of these maxims in our conversational exchanges, Grice gives an account of conversational particularized implicatures in terms of "exploitation" of them.

A previous question is to clarify the very notion of "conversational implicature" (CI), not only with respect to other phenomena related to pragmatic inference (indirect speech acts, presupposition, ...), but also with some cases that were considered CIs by Grice himself, but that are now put in question: we are talking about *generalized* (e.g., the scalar ones) or *standardized* or *conventionalized* CIs (as Bach and Harnish [2] call them). People like Sperber and Wilson [10], Carston [3] and Recanati [9] claim that some of these are not CIs but pragmatically determined elements of what is said. Therefore, if we want to have a clear notion of CI against "what is said", we need a coherent idea of the latter ([3]).

In this paper we will try to impose some constraints on the inference of CIs, interpreting it as an abductive process ([6, 4, 8]) of recognition of communicative intentions. There is an important problem in relation to an abductive model for recovering implicatures, namely, the indeterminacy associated to implicatures ([5]), since there may be various possible specific explanations for the same implicatum. Although the abductive model can face this problem giving as result of the abductive process the disjunction of all minimal explanations, it is not easy to see how could be built something like a procedure for generating and selecting the best explanation in an open conversational context using criteria beyond the general conditions of non-triviality and minimality imposed to explanations. Our constraints will be based on the notion of recognition of communicative intentions interpreted as shared beliefs ([1, 7]), exploiting the cancellability, calculability, and intentionality of CIs.

[1] G. AIRENTI, B. G. BARA, and M. COLOMBETTI, *Conversation and behavior games in the pragmatics of dialogue*, ***Cognitive Science***, vol. 17 (1993), pp. 197–256.

[2] K. BACH and M. HARNISH, ***Linguistic communication and speech acts***, MIT Press, Cambridge, 1979.

[3] R. CARSTON, *Implicature, explicature, and truth-theoretic semantics*, ***Mental representations: The interface between language and reality*** (R. Kempson, editor), Cambridge University Press, pp. 155–181.

[4] E. CHARNIAK and P. MCDERMOTT, ***Introduction to Artificial Intelligence***, Addison Wesley, Menlo Park, California, 1985.

[5] H. P. GRICE, *Logic and conversation*, ***Basic topics in the philosophy of language*** (R. M. Harnish, editor), Harvester Wheatsheaf, London, 1994, pp. 57–73.

[6] HOBBS et al., *Interpretation as abduction*, ***Artificial Intelligence***, vol. 63 (1993), pp. 69–142.

[7] K. KORTA, ***Mental states in conversation***, Report no. ILCLI-95-LIC-2, ILCLI, Donostia, 1995.

[8] F. MIGURA, ***Implicatures and abductive reasoning (A first approach)***, Report no. ILCLI-94-LIC-1, ILCLI, Donostia, 1994.

[9] F. RECANATI, ***Direct reference***, Blackwell, Oxford, 1993.

[10] D. SPERBER and D. WILSON, ***Relevance, communication and cognition***, Blackwell, Oxford, 1986.

► T. G. KUCERA AND PH. ROTHMALER, *Pure projective modules and positive constructibility*.

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One of the fundamental results of the model theory of modules is the fact, due to Baur, that every first-order definable subset of an R-module M (in the natural language for modules over the fixed ring R) is a Boolean combination of cosets of subgroups of M, definable by positive primitive formulas. As a result, most standard concepts of model theory have nice algebraic characterizations in theories of modules, and many interesting algebraic properties of modules generalize to a broader model-theoretic context.

The work being reported here is very much in this spirit. Natural concepts from algebra (projectivity) and from model-theory (atomicity, constructibility) come together by focusing on things defined in terms of positive primitive formulas, in particular on pure embeddings between modules. As a result, we study various concepts of pure-projective modules over different classes of modules, and we look at positive-atomic and positive-constructible modules in different classes of modules. The definitions of these concepts (which are too long to repeat here) are all direct analogs of the classical definitions from module theory or model theory, relativized to various classes of modules $\mathcal{C}$ and to various contexts for purity $\mathcal{P}$, so we are able to get some quite general results.

We investigate the relationship between constructibility and atomicity in this setting, proving results parallel to those for the standard concepts in model theory, and we show that every $\mathcal{C}$-constructible module is pure-projective for $\mathcal{C}$. We characterize pure-projectivity in pretorsion-free classes. This yields then the theorem that for any pretorsion-free class $\mathcal{F}$, the $\mathcal{F}$-constructible modules are precisely the pure-projectives for $\mathcal{F}$. Finally we prove that for countable rings R and any class of R-modules $\mathcal{C}$, any subset of a $\mathcal{C}$-constructible modules is $\mathcal{C}$-constructible. It follows that, for countable rings R and any pretorsion-free class of modules $\mathcal{F}$, every pure-submodule of a module pure-projective for $\mathcal{F}$ is itself pure-projective for $\mathcal{F}$.

► OLIVER KULLMANN, *Some remarks on extended resolution*.
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In [3] the concept of "Blocked Clauses" is introduced for the purpose of improved SAT decision (see also [4]): A clause C is called blocked with respect to a clause-set F if and only if there is a literal $l \in C$ such that every resolvent of C and $C' \in F$ with $\bar{l} \in C'$ is "not interesting".

By this concept parameterized generalizations of Extended Resolution ([5]) are obtained (which, on the other hand, can be simulated polynomially by Extended Resolution). For some restricted versions exponential lower bounds are given ([2]). Furthermore we discuss the connection to lower bounds for classes of SAT algorithms ([1]).

[1] O. KULLMANN, *An exponential lower bound for a general class of DPL-like algorithms for SAT-decision*, Frankfurt, 1995, preprint.

[2] ———, *A note on a generalization of extended resolution*, Frankfurt, 1995, preprint.

[3] ———, *A systematic approach to* 3-*SAT-decision, yielding* 3-*SAT-decision in less than* 1.5045^n *steps*, 1996, manuscript.

[4] O. KULLMANN and H. LUCKHARDT, *Various upper bounds on the complexity of algorithms for deciding propositional tautologies*, 1996, manuscript.

[5] G. S. TSEITIN, *On the complexity of derivation in propositional calculus* (A. O. Slisenko, editor), Seminars in Mathematics, no. 8, V. A. Steklov Mathematical Institute, Leningrad, 1968, English translation 1970.

► JUAN LIN AND EDUARDO MIZRAJI, *Logical decisions: a dynamical approach*.
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Mar and Grim have shown, using the infinite-valued logic of Lukasiewicz, that some classical semantic problems concerning the Liar paradox can generate dynamical models that reproduce acceptable cognitive evaluations. They have proved that the dynamics of self-reference, modeled as an iterated process, can exhibit deterministic chaos [1, 2]. Their work introduces an interesting way of dealing with paradoxes, one that also has the potential to clarify a complex and general question: what is the connection between meanings that belong to the cognitive domain and logical decisions?

It has been recently demonstrated that a vectorial formalism for the propositional calculus can be generalized very naturally to represent the quantifiers of first-order predicate calculus as well as the modal operators [3, 4]. These representations allow us to establish contact between neural network approaches to semantical problems and the theory of logical decisions applied to these problems. In this communication we use fuzzy matrix modalities and a probabilistic vector logic to generate interesting patterns of dynamic behavior out of a certain class of logical decisions.

We first analyze cognitive versions of the classical Lotka-Volterra competition and predator-prey models. In the competition model we find a tendency toward extinction of one of the logical options. In the predator-prey model we find a periodic dominance of one of the logical options, and possibly a chaotic or pseudo-random behavior within a narrow band of values for the modality parameter q, which defines the order of the universe of fuzzy logical decisions. We then go on to study two versions of the Liar paradox. For different values of the parameter q, it is found that these models display either regular behavior with different degrees of complexity or chaotic behavior.

[1] G. MAR AND P. GRIM, *Pattern and chaos: New images in the semantics of paradox*, ***Noûs***, vol. 25, 1991, pp. 659–693.

[2] P. GRIM, *Self-reference and chaos in fuzzy logic*, ***IEEE Transactions in Fuzzy Systems***, vol. 1, 1993 pp. 237–253.

[3] E. MIZRAJI, *Modalities in vector logic*, ***Notre Dame Journal of Formal Logic***, vol. 35, 1994, pp. 272–283.

[4] ———, *The operators of vector logic*, ***Mathematical Logic Quarterly***, vol. 42, 1996, pp. 27–40.

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Let $I_f(\mathfrak{M})$ denote the finite partial automorphism semigroup of the model $\mathfrak{M}$, and $I_r(\mathfrak{M})$ denote the partial recursive partial automorphism semigroup of the model $\mathfrak{M}$.

THEOREM 1. *Let* $\mathfrak{M}_0 = \langle|\mathfrak{M}_0|; P_1^{k_1}, \dots, P_n^{k_n}\rangle$, $\mathfrak{M}_1 = \langle|\mathfrak{M}_1|; R_1^{e_1}, \dots, R_m^{e_m}\rangle$ *be the models of the finite signatures. Then the following conditions are equivalent*:

(1) $I_f(\mathfrak{M}_0) \cong I_f(\mathfrak{M}_1)$.

(2) *There is the bijection* $\lambda\colon |\mathfrak{M}_0| \to |\mathfrak{M}_1|$ *such that for any* $P_i^{k_i}$ *there is the quantifier-free formula* φ_{P_i} *of the language of* $\mathfrak{M}_1$ *such that for any* $\bar{x} \in |\mathfrak{M}_0|^{k_i}$

$$\mathfrak{M}_0 \vDash P_i^{k_i}(\bar{x}) \iff \mathfrak{M}_1 \vDash \varphi_{P_i}(\lambda(\bar{x})).$$

And, conversely, for any $R_i^{e_i}$ *there is the quantifier-free formula* φ_{R_i} *of the language of* $\mathfrak{M}_0$ *such that for any* $\bar{x} \in |\mathfrak{M}_1|^{e_i}$

$$\mathfrak{M}_1 \vDash R_i^{e_i}(\bar{x}) \iff \mathfrak{M}_0 \vDash \varphi_{R_i}(\lambda^{-1}(\bar{x})).$$

The next proposition follows from Theorem 1.

PROPOSITION. *Let* $\mathfrak{N}_0$ *and* $\mathfrak{N}_1$ *be denumerable partial orderings and* $I_f(\mathfrak{N}_0) \cong I_f(\mathfrak{N}_1)$. *Then* $\mathfrak{N}_0$ *and* $\mathfrak{N}_1$ *are isomorphic or anti-isomorphic.*

THEOREM 2. *Let* $\mathcal{B}_0$ *and* $\mathcal{B}_1$ *be recursive Boolean algebras,* $\mathcal{B}_0$ *has an atomless element, and* $I_r(\mathcal{B}_0) \cong I_r(\mathcal{B}_1)$. *Then* $\mathcal{B}_0 \cong_r \mathcal{B}_1$.

[1] U. L. ERSHOV, *Decision problems and constructive models*, ***Nauka***, 1980.

[2] E. V. LIPACHEVA, *Partial automorphisms of denumerable models*, to appear in ***Izvestiya Vuzov Matematika***.

► ZHAOHUI LUO, *Logical truths in constructive type theory*.
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A constructive type theory such as Martin-Löf's type theory [4] has an internal logic based on the Curry-Howard principle of propositions-as-types. However, as compared with traditional logical systems such as the predicate logic, the internal logic of type theory and the associated notion of logical truth (as opposed to mathematical truth) are much less understood. This is partly because that, based on a rigid interpretation of the propositions-as-types principle as *identifying* propositions and types, as found in Martin-Löf's type theory, to understand the logic is to understand the whole type theory, including the types of mathematical structures.

We argue that, in type theory, there should be a clear distinction between the notion of a logical proposition and that of a type of mathematical structures, and on the basis of this distinction, a further distinction between the notion of (pure) logical truth and that of mathematical truth. Informally, this amounts to an argument for the independence of logic in the sense that the province of logic has a trait of being universal and hence independent of the existence of the entities to be reasoned about by means of the logic. A conceptual identification of logical propositions and types of mathematical structures could be misleading and may make it more difficult to understand the internal logic of type theory.

We consider an informal notion of logical truth in type theory and define a corresponding formal notion in the context of the type theory UTT [3], a natural combination of Martin-Löf's intensional type theory and Coquand-Huet's calculus of constructions [1], in which a distinction between logical propositions and data types is already present. We consider a compositional understanding based on the meaning theory of type theory as important in understanding the notion of logical truth, and study the notion of conservative extension as its formal counterpart. (This is related to Prawitz and Dummett's arguments for compositional understanding of languages (cf. [2]). But here, we consider the formal and mathematical language of type theory.) It is hoped that, through this investigation, we can obtain a better understanding of the use of the internal logic of type theory in formalisation of constructive mathematics and, for example, provide criteria as a basis to answer (informally) questions such as 'Is this formalisation adequate?'

[1] TH. COQUAND and G. HUET, *The calculus of constructions*, ***Information and Computation***, vol. 76 (1988), no. 2 and 3.

[2] M. DUMMETT, ***The logical basis of metaphysics***, Duckworth, 1991.

[3] Z. LUO, ***Computation and reasoning: A type theory for computer science***, Oxford University Press, 1994.

[4] B. NORDSTRÖM, K. PETERSSON, and J. SMITH, ***Programming in Martin-Löf's type theory: An introduction***, Oxford University Press, 1990.

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We consider superintuitionistic predicate logics with equality (s.i.e.l.). For simplicity, we assume that the language does not contain any functional symbols. The equality-sign is considered as a logical symbol. We say that A is an equality-formula if A does not contain any predicate symbol different from equality-sign. If $\mathbf{L}$ is a superintuitionistic predicate logic without equality (s.i.p.l.), denote by $\mathbf{LE}$ an extension of $\mathbf{L}$ by adding standard axioms for equality. The intuitionistic predicate logic with equality is denoted by $\mathbf{HQE}$. If $\mathbf{L}$ is a s.i.e.l., S any set of formulas, denote by $\mathbf{L} + S$ the least s.i.e.l. obtained from $\mathbf{L}$ by adding S as new axioms.

PROPOSITION 1.

(i) *Let* **L** *be a s.i.p.l. possessing Craig's interpolation property CIP (Lyndon's interpolation property LIP or Beth's property BP). Then* **LE** *also has CIP (respectively LIP or BP).*

(ii) *Let* **L** *be a s.i.e.l. possessing CIP (LIP or BP), S any set of equality formulas. Then* $\mathbf{L} + S$ *also has CIP (LIP or BP).*

Now we build an infinite set of independent formulas which are not equivalent to equality-formulas but preserve CIP, LIP and BP, too. Denote for any $n > 0$:

$$\begin{aligned} A_n &\iff \forall x_1 \dots \forall x_{n+1} \bigvee \{\, x_i = x_j \mid 1 \le i < j \le n+1 \,\}, \\ E_1 &\iff \exists x\, x = x, \\ E_{n+1} &\iff \exists x_1 \dots \exists x_{n+1} \bigwedge \{ \neg x_i = x_j \mid 1 \le i < j \le n+1 \,\}, \\ U_n &\iff A_n \wedge E_n. \end{aligned}$$

THEOREM 2. *Let* **L** *be a s.i.e.l. possessing CIP (LIP or BP), S any subset of a set*

$$\Gamma = \{\, (U_n \supset \phi \vee \neg\phi), (U_n \supset \neg\phi \vee \neg\neg\phi) \mid n \ge 1 \,\}$$

of axiom schemes. Then $\mathbf{L} + S$ *also has CIP (respectively LIP or BP).*

It is evident that this set Γ is not independent because a formula $(U_n \supset \neg\phi \vee \neg\neg\phi)$ is derivable from $(U_n \supset \phi \vee \neg\phi)$. But subsets $\{\, (U_n \supset \phi \vee \neg\phi) \mid n \ge 1 \,\}$ and $\{\, (U_n \supset \neg\phi \vee \neg\neg\phi) \mid n \ge 1 \,\}$ are both independent. Moreover, the following holds:

PROPOSITION 3. *A scheme $(U_n \supset \phi \vee \neg\phi)$ is independent in* **HQE** *on the rest schemes of Γ, and $(U_n \supset \neg\phi \vee \neg\neg\phi)$ is independent on*

$$\Gamma' = \{\, (U_m \supset \phi \vee \neg\phi), (U_m \supset \neg\phi \vee \neg\neg\phi) \mid m \ge 1, m \ne n \,\}.$$

Moreover, no scheme of Γ may be replaced by equality-formulas.

It is known that the intuitionistic predicate logic has LIP. Using Proposition 1 and Theorem 2, one can construct continua of s.i.e.l. possessing LIP and, therefore, CIP and BP.

▶ HUBERT MARRAUD, *Completeness on Restall's 4-valued Semantics*.
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In [1] Restall outlines three different approaches to the four valued semantics for relevant logics. Our concern here is with the third, of which he says: "[it] seems to capture a wide range of formal systems, but at the time of writing, lacks a completeness proof."

An interpretation is a 4-tuple $\langle g, W, R, I \rangle$, where $g \in W$, $R \subseteq W^3$, for any $x, y \in W$, $Rgxy$ if and only if $x = y$, $I : P \times W \to \mathrm{Sb}\{0,1\}$—'$P$' standing by the set of propositional variables of the given propositional language. Truth values at worlds are then assigned to formulae inductively.

$[\wedge, 1]$ $1 \in I(w, A \wedge B)$ if and only if $1 \in I(w, A)$ and $1 \in I(w, B)$,

$[\wedge, 0]$ $0 \in I(w, A \wedge B)$ if and only if $0 \in I(w, A)$ or $0 \in I(w, B)$,

$[\vee, 1]$ $1 \in I(w, A \vee B)$ if and only if $1 \in I(w, A)$ or $1 \in I(w, B)$,

$[\vee, 0]$ $0 \in I(w, A \wedge B)$ if and only if $0 \in I(w, A)$ and $0 \in I(w, B)$,

$[\neg, 1]$ $1 \in I(w, \neg A)$ if and only if $0 \in I(w, A)$,

$[\neg, 0]$ $0 \in I(w, \neg A)$ if and only if $1 \in I(w, A)$,

$[\to, 1]$ $1 \in I(w, A \to B)$ if and only if for all $x, y \in W$, if $Rwxy$ then (i) if $1 \in I(x, A)$, $1 \in I(y, B)$, and (ii) if $0 \in I(x, B)$ then $0 \in I(y, A)$,

$[\to, 0]$ $0 \in I(w, A \to B)$ if and only if for some $x, y \in W$, $Rwxy$ and either (i) $1 \in I(x, A)$

and $0 \in I(y, B)$, or (ii) $1 \in I(y, A)$ and $0 \in I(x, B)$.

I shall prove:

(1) Restall's suggested basic system,

$$\mathbf{DW} + A \wedge \neg B/\neg(A \to B) + \neg(A \to C) \wedge (A \to B) \to \neg(A \to \neg B),$$

is not sound for the above semantics; however it is both sound and complete with respect to the class of four-valued interpretations satisfying the additional constraint **DP** $Rabc \Longrightarrow Racb$.

(2) The weaker system

$$\mathbf{DW} + A \wedge \neg B/\neg(A \to B) + \neg(A \to \neg A) \wedge (A \to B) \to \neg(A \to \neg B)$$

is sound for this semantics, but its completeness is for the moment an open problem.

(3) A *bidimensional four-valued interpretation* is a 5-tuple $\langle g, W, R', R'', I\rangle$ with the same clauses as above, save

$[\to, 1]$ $1 \in I(w, A \to B)$ if and only if for all $x, y \in W$, if $R'wxy$ then (i) if $1 \in I(x, A)$, $1 \in I(y, B)$, and (ii) if $0 \in I(x, B)$ then $0 \in I(y, A)$,

$[\to, 0]$ $0 \in I(w, A \to B)$ if and only if for some $x, y \in W$, $R''wxy$ and either (i) $1 \in I(x, A)$ and $0 \in I(y, B)$, or (ii) $1 \in I(y, A)$ and $0 \in I(x, B)$.

(4) The latter system can be shown to be sound and complete with respect to the class of b. f. interpretations satisfying $R''abc \Longrightarrow (\exists x)((R''abx \vee R''acx) \wedge (R'abx \vee R'acx))$.

[1] Restall, *Four-valued semantics for relevant logics (and some of their rivals)*, ***Journal of Philosophical Logic***, vol. 24 (1995), no. 2, pp. 139–160.

▶ G. MINTS, *Cut-elimination for simple type theory with an axiom of choice*.
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Our proof is modeled after Takahashi's proof [3] of cut-elimination for simple type theory with extensionality. The same proof works when types are restricted, for example for second-order classical logic with axiom of choice. Some ideas of Yasuhara [4] inspired by the author's review [2] are used. Important feature is the use of a $(n+1)$-order notion $\{x^n\}A(x^n)$ (where A is a formula involving only types $\leq n$) in the definition of a relation "a set S *chooses* term $\varepsilon x^n A(x^n)$" of order n.

Derivable objects are *sequents* $\Gamma \to \Delta$. The rules of inference are those of simple type theory with extensionality plus the following rules for ε-symbol:

$$(\varepsilon) \qquad \frac{\Gamma \to \Delta, A(V) \quad A(\varepsilon x A(x)), \Gamma \to \Delta}{\Gamma \to \Delta}$$

where V is an abstract of a suitable type and ε-term $\varepsilon x A(x)$ occurs in the conclusion $\Gamma \to \Delta$.

(ext ε)

$$\frac{A(a), \Gamma \to \Delta, B(a) \quad B(a), \Gamma \to \Delta, A(a) \quad \forall z\, (\varepsilon x A(x)[z] \leftrightarrow \varepsilon y B(y)[z]), \Gamma \to \Delta}{\Gamma \to \Delta}$$

where both terms $\varepsilon x A(x)$, $\varepsilon y B(y)$ occur in the conclusion $\Gamma \to \Delta$ and a is a new variable of the same type as x.

Theorem. *Cut-elimination holds for simple type theory with extensionality and choice.*

[1] G. Mints, *A normal form theorem for second-order classical logic with an axiom of choice*, ***Matematika USSR Izvestiya***, vol. 32 (1989), no. 3, pp. 587–605.

[2] ———, *Review of a paper by L. Wessels*, ***Zentralblatt für Mathematik***, vol. 381 (1979), #03042.

[3] M. TAKAHASHI, *Simple type theory of Gentzen style with the inference of extensionality*, ***Proceedings of the Japanese Academy of Sciences***, vol. 44 (1968), pp. 43–45.

[4] M. YASUHARA, *Cut elimination in ε-calculi*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 28 (1977, 1982), pp. 311–316.

▶ TILL MOSSAKOWSKI, ANDRZEJ TARLECKI, AND WIESŁAW PAWŁOWSKI, *Combining and representing logical systems*.

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There is a wealth of various logical systems necessary for different purposes. As it is unlikely that there ever will be a single universal logical system serving all purposes equally well, the question arises how to handle the needed variety. One important idea is that logical systems should be built and used in a structured way. Two practically-motivated issues then are:

(1) Combining logical systems, which opens the possibility for introducing and adding new concepts in a step-by-step manner.

(2) Representing logical systems in order logical system(s) with well-known model theory and proof theory, with the possibility to re-use tools like theorem provers.

Several different meta-notions of logical system have been introduced. One of them, institutions [1], comes with two different notions of morphism:

(1) Institution morphisms, which capture how one logical system is built upon another and thus provide a rudimentary framework for combination of logics via category-theoretic limits;

(2) Institution representations, which encode one logical system in another and thus provide a basis for re-use of theorem provers (under an extra technical condition).

In [3] the role of these notions and the interplay between them have been studied. A suitable notion of map between institution representations, consisting of an institution morphism and an extra "fitting" component has been introduced. The main new theorem of [3] states that representations can be combined via category-theoretic limits, so that combined institutions can be represented provided that each component of the combination can be represented.

One deficiency of combinations of institutions via limits is that since sentences in institutions have no inner structure, they are simply united in the combination rather than being properly combined, and the features of the combined logics do not really interact in the result. The solution is to move to parchments, certain algebraic presentations of institutions providing an abstract syntax and evaluator-based semantics for sentences and therefore more useful for logic combination [2].

In the present work we deal with a slightly redefined notion of parchment, avoiding some foundational problems of the original parchment definition. We propose a notion of parchment representation and show that parchment representations interact with parchment morphisms quite similarly as institution representations interact with institution morphisms. In particular, we lift the theorem about combination of institution representations to the level of parchment representations. We give some examples illustrating intrinsic features of the combinations of parchment representations and the potential usefulness of the proposed framework.

[1] J. GOGUEN and R. BURSTALL, *Institutions: abstract model theory for computer science*,

Journal of the ACM, vol. 39 (1992), pp. 95–146.

[2] T. MOSSAKOWSKI, *Using limits of parchments to systematically construct institutions of partial algebras*, ***Recent trends in data type specifications: 11th workshop on specification of abstract data types*** (M. Haveraaen et al., editors), Lecture Notes in Computer Science, Springer-Verlag, vol. 1130 (1996), pp. 379-393.

[3] A TARLECKI, *Moving between logical systems*, ***Recent trends in data type specifications: 11th workshop on specification of abstract data types*** (M. Haveraaen et al., editors), Lecture Notes in Computer Science, Springer-Verlag, vol. 1130 (1996), pp. 478-502.

▶ MAREK NOWAK, *Two logics of strong implication*.
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In the paper a concept of strong implication introduced in [5] (cf. also [3, 4]) is formalized in the way as simple as possible: in the form of two propositional logics defined on the language $(L, \to)$ involving the only binary connective $\to$. The semantics for both logics makes use of the approach to Parry's analytic implication due to Fine [2] and may be placed in the general semantics framework of [1].

Let $\mathcal{M}$ be the class of all models $\mathfrak{M} = \langle (A, \vee), f, \mathrm{Val}\rangle$, where $(A, \vee)$ is a semilattice, f is a homomorphism of the algebras $(L, \to)$, $(A, \vee)$ and $\mathrm{Val} \subseteq \{0, 1\}^L$ is the class of valuations v fulfilling the condition:

$v(\alpha \to \beta)$ if and only if $(v(\alpha) = 0$ or $v(\beta) = 1) \wedge f(\beta) \vee f(\alpha) = f(\alpha)$,
any $\alpha, \beta \in L$.

Then $X \vDash \alpha$ if and only if $\forall \mathfrak{M} \in \mathcal{M}\, \forall v \in \mathrm{Val}\, (v(\alpha) = 1$ whenever $v(X) \subseteq \{1\})$, any $X \subseteq L$.

The following properties of the logic $\vDash$ are proved:

(1) $\vDash \alpha \to \beta$ if and only if $\vDash \alpha \Longrightarrow \beta \wedge V(\beta) \subseteq V(\alpha)$, where $\alpha \Longrightarrow \beta =_{\mathrm{df}} (((\alpha \to \beta) \to (\alpha \to \beta)) \to \alpha) \to (((\alpha \to \beta) \to (\alpha \to \beta)) \to \beta)$ (then $\forall \mathfrak{M} \in \mathcal{M}\, \forall v \in \mathrm{Val}$, $v(\alpha \Longrightarrow \beta) = 1$ if and only if $v(\alpha) = 0$ or $v(\beta) = 1$) and $V(\alpha)$ is the set of all variables occurring in α.

(2) $\vDash \alpha \to \beta$ implies $\vDash_{\mathrm{cl}} \alpha \to \beta$ but the conditions: $\vDash_{\mathrm{cl}} \alpha \to \beta$, $V(\beta \subseteq V(\alpha)$ do not imply that $\vDash \alpha \to \beta$, where $\vDash_{\mathrm{cl}}$ is purely implicational part of the classical logic.

The second logic $\vDash_1$ is defined by the class of all models $\mathfrak{M} = \langle (A, \vee), f, g\rangle$ where $(A, \vee)$ and f are such as above and $g\colon \mathrm{Val}_{\mathrm{cl}} \to \{0, 1\}^L$ is a function defined as follows: $\forall v \in \mathrm{Val}_{\mathrm{cl}}$, $g(v)(p) = v(p)$, p is a variable, and $g(v)(\alpha \to \beta) = 1$ if and only if $v(\alpha \to \beta) = 1 \wedge f(\beta) \vee f(\alpha) = f(\alpha)$, where $\mathrm{Val}_{\mathrm{cl}}$ is the class of all the functions $v\colon L \to \{0, 1\}$ fulfilling the condition: $v(\alpha \to \beta) = 1$ if and only if $v(\alpha) = 0$ or $v(\beta) = 1$.

$X \vDash_1 \alpha$ if and only if for any model $\mathfrak{M}$ and any $v \in \mathrm{Val}_{\mathrm{cl}}$, $g(v)(\alpha) = 1$ whenever $g(v)(X) \subseteq \{1\}$. For that logic the following holds true:

(3) $\vDash_1 \alpha \to \beta$ if and only if $\vDash_{\mathrm{cl}} \alpha \to \beta \wedge V(\beta) \subseteq V(\alpha)$.

For both logics an axiomatization with completeness proof is provided.

[1] R. EPSTEIN, ***The semantic foundations of logic***, M. Nijhoff, Hague, 1990.

[2] K. FINE, *Analytic implication*, ***Notre Dame Journal of Formal Logic***, vol. 27 (1986), pp. 169–179.

[3] M. NOWAK and D. VANDERVEKEN, *A complete minimal logic of the propositional contents of thought*, ***Journal of Symbolic Logic***, vol. 54 (1995), pp. 391–410.

[4] ———, *An algebraic analysis of the logical form of propositions*, ***Logique et Analyse***, vol. 141–142 (1993), pp. 135–148.

[5] D. VANDERVEKEN, *What is a proposition?*, ***Cahiers d'epistemologie***, Université du Quebec à Montreal, no. 9103 (1991).

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Constructive mathematics preserves the symmetry of Boolean logic's two truth values when it allows a proposition to be undecided: "could be either true or false". Intuitionistic logic destroys this symmetry by focusing on truth and offering only a nonconstructive "negation as failure", $\sim p = p \supset \bot$. Nelson's system N [1] of constructible falsity restores symmetry to intuitionistic logic, I, with an involutory *strong* negation $\neg p$ asserting the actual falsity of p. (Cf. Zaslavskiĭ [3] for another symmetrization.) Thomason [2] proposes a Kripke structure interpretation with three truth values for which he shows that N is sound and complete.

We give a Kripke model having only two truth values for which we show that N is sound and complete. Truth is defined on intervals (w_t, w_f) in the Kripke structure such that w_f is accessible from w_t. Truth is judged from one end w_t, falsity from the other w_f. A formula φ holds at an interval if and only if it holds at all non-trivial subintervals. We interpret strong negation as dualizing the Kripke structure with respect to both truth and accessibility. We formalize this as follows. We use a fact from [2] that every $\varphi \in N$ is equivalent to a formula where $\neg$ only scopes propositional atoms. For formulas of this form, let $A = \langle W, R, V : W \times P \to 2\rangle$ be a Kripke model for I, then for any $w_t \neq w_f \in W$ such that $w_t\ R\ w_f$,

$$\begin{aligned}
(w_t, w_f) \models_A p &= & V(w_t, p) = 1\\
(w_t, w_f) \models_A \varphi \wedge \psi &= & (w_t, w_f) \models_A \varphi \text{ and } (w_t, w_f) \models_A \psi\\
(w_t, w_f) \models_A \varphi \vee \psi &= & (w_t, w_f) \models_A \varphi \text{ or } (w_t, w_f) \models_A \psi\\
(w_t, w_f) \models_A \varphi \supset \psi &= \forall w', w''.\ w_t\ R\ w'\ R\ w''\ R\ w_f & (w', w'') \models_A \varphi \to (w', w'') \models_A \psi\\
(w_t, w_f) \models_A \neg p &= & V(w_f, p) = 0.
\end{aligned}$$

The atoms that Thomason assigns the third value are those propositional atoms which are assigned 0 at w_t and 1 at w_f.

Our completeness theorem below asserts that any underivable formula is refutable in a canonical model. In fact, the canonical refutation model K for positive intuitionistic logic I_p is the canonical refutation model for N. Any underivable formula in N can be extended to a complete underivable sequent $\Gamma \Longrightarrow \Delta$ that can be syntactically partitioned into $\Gamma_p, \Gamma_n, \Gamma_o \Longrightarrow \Delta_p, \Delta_n, \Delta_o$ where each φ in $\Gamma_p \cup \Delta_p$ is in I_p, and each φ in $\Gamma_n \cup \Delta_n$ is a formula of the form $\neg\psi$ where ψ is in I_p. Then $\Gamma_p \Longrightarrow \Delta_p$ is a complete sequent for I_p; hence $\Gamma_p \Longrightarrow \Delta_p$ is a world in K. The formula-wise contrapositive of $\Gamma_n \Longrightarrow \Delta_n$, denoted $\neg\Delta_n \Longrightarrow \neg\Gamma_n$, is also a world in K.

THEOREM. *If $N \nvdash \varphi$ then φ can be extended to a complete sequent, $\Gamma \Longrightarrow \Delta$, for N such that*

$$(\Gamma_p \Longrightarrow \Delta_p, \neg\Delta_n \Longrightarrow \neg\Gamma_n) \nvDash_K \varphi.$$

[1] DAVID NELSON, *Constructible falsity*, ***Journal of Symbolic Logic***, vol. 14 (1949), pp. 14–26.

[2] R. H. THOMASON, *A semantical study of constructible falsity*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 15 (1969), pp. 247–257.

[3] I. D. ZASLAVSKIĬ, *On predicate and arithmetic calculi of symmetric constructive logic*, ***Soviet Mathematics Doklady***, vol. 14 (1973), pp. 751–754.

▶ PAUL RUET, *Non-commutative linear logic with mobilities*.
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We introduce a new version of non-commutative linear logic. We consider two kinds of multiplicative connectives: the *non-commutatives* and the *commutatives*. The link between them is made by new modalities: the *mobilities*. We give a sequent calculus for this logic, and we prove cut-elimination.

The first motivation for this new approach was the semantics of concurrent processes, specifically that of concurrent constraint programs.

Syntax. We introduce a non-commutative version of Girard's multiplicative linear logic [2]. We consider two kinds of multiplicative connectives: the *non-commutatives* $\times$ and $+$, and the *commutatives* $\otimes$ and $\wp$; and two modalities, the *mobilities* $\star$ and $\bullet$. As in Abrusci's pure non-commutative linear logic [1], there are two negations: the left one ${}^{\perp}.$ and the right one $.^{\perp}$, such that ${}^{\perp}(A^{\perp}) = A$ and $({}^{\perp}A)^{\perp} = A$. They are defined by De Morgan laws, which express the duality between $\times$ and $+$, $\otimes$ and $\wp$, $\star$ and $\bullet$.

The link between the two kinds of multiplicatives is made by the mobilities $\star$ and $\bullet$, through the isomorphisms $\star A \times \star B \cong \star(A \otimes B)$, and dually $\bullet A + \bullet B \cong \bullet(A \wp B)$. This is to parallel with the fundamental isomorphisms $!\,A \otimes !\,B \cong !(A \,\&\, B)$ and $?\,A \wp\, ?\,B \cong ?(A \oplus B)$ of commutative linear logic. These formulas are made into a logic thanks to a sequent calculus.

The mobilities enjoy the usual rules of dereliction and promotion, expressing multi-functoriality and (co-)monadicity ($A \vdash \bullet A \dashv\vdash \bullet\bullet A$ and dually $\star\star A \dashv\vdash \star A \vdash A$).

These formulas are made into a logic thanks to a sequent calculus.

Sequent calculus. Formulas are first equipped with a polarity: *plain* A (just written A) and *parenthesized* A, written (A). The distinction between parenthesized formulas and not parenthesized ones is similar to the distinction between the classical and linear parts in sequents of Unified Logic [3]: (A) is a hypocrisy for $\bullet A$. This distinction is necessary in our non-commutative version, whereas it is not absolutely essential in commutative linear logic.

Sequents are contractible (partial) orders indexed by formulas (in the style of the ordered sequents of Retoré [4]). We give a sequent calculus for this logic, and we prove cut-elimination.

Pure non-commutative linear logic is a subsystem of ours, where the only authorized formulas are $\times$ and $+$. Constants can be easily accommodated.

Our first motivation for this new approach was a semantics of concurrent processes, specifically that of concurrent constraint programs [5], and the need for a 'sequential' connective (Retoré's *before* connective $<$ is not a solution, because $A \otimes (B < C) \vdash B < (A \otimes C)$).

[1] V.M. Abrusci, *Phase semantics and sequent calculus for pure noncommutative classical linear propositional logic*, ***Journal of Symbolic Logic***, vol. 56 (1991), no. 4, 1403–1451.

[2] J. Y. Girard, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987).

[3] ———, *On the unity of logic*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 201–217.

[4] Ch. Retoré, *Réseaux et séquents ordonnés*, ***Ph.D. thesis***, Université Paris 7, 1993.

[5] V. A. Saraswat, ***Concurrent constraint programming***, ACM Doctoral Dissertation Awards, MIT Press, 1993.

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We look at the Δ^0_2-Turing degrees, and in particular the computably enumerable (c.e.) degrees and the d.c.e. degrees. A result of Cooper and Yi [2] has shown the existence of so-called isolated d.c.e. degrees—those which have an immediate c.e. predecessor—and the c.e. degrees which isolate them. They also proved the existence of non-isolated properly d.c.e. degrees. Arslanov, Lempp and Shore [1] proved the existence of non-isolating c.e. degrees other than $\mathbf{0}$ and $\mathbf{0}'$. A further result of LaForte [4] (also proved independently by Ding and Qian [3]) shows that the isolating c.e. degrees (and isolated d.c.e. degrees) are dense in the upper-semi lattice of c.e. degrees. We show that such a density result cannot be obtained for the non-isolating c.e. degrees, and examine the best possible density result which can be proved.

[1] M. M. ARSLANOV, S. LEMPP, and R. A. SHORE, *On isolating r.e. and isolated d.r.e. degrees*, to appear, 1995.

[2] S. B. COOPER and X. YI, *Isolated d.r.e. degrees*, to appear, 1995.

[3] D. DING and L. QIAN, *Isolated d.r.e. degrees are dense in r.e. degree structure*, to appear.

[4] G. L. LAFORTE, *The isolated d.r.e degrees are dense in the r.e. degrees*, ***Mathematical Logic Quarterly***, vol. 42 (1996), no. 2.

[5] R. I. SOARE, ***Recursively enumerable sets and degrees***, Springer-Verlag, Berlin, 1987.

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In our thesis [3] (see as well [4]) we determined the proof theoretical strength of Martin Löf's type theory ($\mathrm{ML_1W}$) with W-type and one universe (there was parallel work on this by E. Griffor and M. Rathjen, see [1]) to be the collapse of the ωth admissible above the first recursive inaccessible. The intuition behind this result is that the universe is a fixed point under the formation of rules for it, and the definition depends on the inductive definition of the W-type. If we had that the universe is the least such fixed point, guaranteed by elimination rules as defined in [2] or [1], then, since the W-type corresponds to the step to the next admissible, the universe corresponds to one inductive definition closed under the step to the next admissible, which, since the inductive definition is strong enough, corresponds to one recursive inaccessible. However, in the definition of $\mathrm{ML_1W}$, we have no elimination rules and therefore only a fixed point. Note that axioms guaranteeing fixed points are usually far weaker than axioms for least fixed points. However elimination rules are not necessary, because in $\mathrm{ML_1W}$, we can define a least universe in the following sense:

THEOREM. *There exists a type $\phi(x)$ such that $\{x : \mathrm{U} \mid \phi(x)\}$ is the least universe closed under the formation rules for* U:

(a) $\{x : \mathrm{U} \mid \phi(x)\}$ *is closed under the formation rules*: *We can prove in* $\mathrm{ML_1W}$ $\phi(\mathbf{n}_0)$, $\forall a : \mathrm{U}.\forall b : (\mathrm{T}(a) \to \mathrm{U}).\phi(a) \to ((\forall x : \mathrm{T}(a).\phi(bx)) \to \phi(\sigma x : a.(bx)))$, *similarly for the other constructors.*

(b) $\{x : \mathrm{U} \mid \phi(x)\}$ *is the least such universe*: *If* $x : \mathrm{U} \Longrightarrow C(x)$ *type,* $C(\mathbf{n}_0)$, $\forall x : \mathrm{U}.\forall y : (\mathrm{T}(x) \to \mathrm{U}).C(x) \to ((\forall z : \mathrm{T}(x).C(yz)) \to C(\sigma z : x.yz))$, *similarly for the other constructors, then* $\forall x : \mathrm{U}.\phi(x) \to C(x)$.

This strengthens the intuition about why we achieve this strength, and is a step towards simplified well ordering proofs for $\mathrm{ML_1W}$.

For the proof of the theorem, we define $W_0 := Wx : \mathrm{U}.\mathrm{T}(x)$, for every $w : W_0$ the type $\mathrm{Subtree}(w)$ of subtrees of w (including a name root for w itself) and $\phi(v)$ iff there exists a tree $w : W_0$ and a function label from the subtrees of w to U such that $\mathrm{I}(\mathrm{U}, \mathrm{label}(\mathrm{root}), v)$ and for any $w' : \mathrm{Subtree}(w)$ we have one of the following conditions:

• $\mathrm{I}(\mathrm{U}, \mathrm{label}(w'), \mathbf{n}_0)$

• or $\mathrm{I}(\mathrm{U}, \mathrm{label}(w'), \sigma x : a.bx)$ for some $a : \mathrm{U}$, $b : \mathrm{T}(a) \to \mathrm{U}$ such that for some $w'', w'''(x) : \mathrm{Subtree}(w)$, which are immediate subtrees of w', we have $\mathrm{I}(\mathrm{U}, \mathrm{label}(w''), a)$

and $\forall x : \mathrm{T}(a).\mathrm{I}(\mathrm{U}, \mathrm{label}(w'''(x)), bx)$

- or an analogous condition for the other type constructions, under which U is closed.

[1] E. Griffor and M. Rathjen, *The strength of Martin-Löf type theories*, ***Archive for Mathematical Logic***, vol. 33 (1994), pp. 347–385.

[2] E. Palmgren, *Type-theoretic interpretation of iterated, strictly positive inductive definitions*, ***Archive for Mathematical Logic***, vol. 32 (1992), pp. 75–99.

[3] A. Setzer, *Proof theoretical strength of Martin-Löf type theory with W-type and one universe*, ***Ph.D. thesis***, University of Munich, 1993.

[4] ———, *Wellordering proofs for Martin-Löf's type theory*, submitted, 1997.

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Let intuitionistic propositional language be enriched by finite list $\bar{\varphi} = \{\varphi_1, \ldots, \varphi_n\}$ of additional propositional constants. A formula without occurrences of φ_i is said to be *pure*. By Int we denote the intuitionistic propositional logic (in pure language). A formula of the form $\varphi_i \leftrightarrow B$, where B does not contain φ_i is called *explicit expression* for φ_i.

By $\bar{\varphi}$*-logic* we mean any set of formulas of enriched language including Int and closed under modus ponens and substitution rules.

A $\bar{\varphi}$-logic L is said to be *conservative* over Int if for every pure formula B: $B \in L \Longrightarrow B \in$ Int.

Definition 1 (P. Novikov). A $\bar{\varphi}$-logic L is called *Novikov complete* if for every formula $A \notin L$ the $\bar{\varphi}$-logic $L + A$ is not conservative over Int. ($L + A$ means the least $\bar{\varphi}$-logic including $L \cup \{A\}$.)

Definition 2. Constants $\varphi_1, \ldots, \varphi_n$ are *independent* in $\bar{\varphi}$-logic L if, for every $i \leq n$, for every formula B without φ_i the $\bar{\varphi}$-logic $L + \varphi_i \leftrightarrow B$ is not conservative over Int (i.e., it is *impossible to enlarge* L by explicit expression). In the case $|\bar{\varphi}| = 1$ it is natural to say about *new* constant.

The *Novikov's problem* may be formulated as follows: to give explicit examples of Novikov complete $\bar{\varphi}$-logics with independent constants; to describe the family of all Novikov complete $\bar{\varphi}$-logics.

Main results:

- we present the "growth technique" for constructing examples of Novikov complete $\bar{\varphi}$-logics;
- the $\bar{\varphi}$-logic corresponding to given finite growth is finitely axiomatizable;
- for $|\bar{\varphi}| \geq 2$ the family of all Novikov complete $\bar{\varphi}$-logics is of the power of the continuum;
- the algorithmic problem "*whether the $\bar{\varphi}$-logic of the form Int + A is conservative over Int*" is undecidable;
- in contrast with preceding results, for $|\bar{\varphi}| = 1$ the family of all Novikov complete φ-logics is explicitly described as

$$L^1, L^2, L^3, L^5, \ldots, L^{2n+1}, \ldots, L^{\infty}.$$

Here L^1 contains $\neg\varphi$, L^2 contains φ, and each of the others determines new constant; all of L^α are decidable and, as a corollary, the conservativeness problem is decidable.