THE BULLETIN OF SYMBOLIC LOGIC
Volume 3, Number 1, March 1997

1995 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '95

Haifa, Israel, August 9–17, 1995

The 1995 European Summer Meeting of the Association was held in Haifa, Israel, August 9–17, 1995. Hosts of the colloquium were the Technion—Israel Institute of Technology and the University of Haifa. This was the first time the Logic Colloquium was held in Israel. There were over 200 participants from 36 countries from five continents.

The topics of this conference were the four classical topics of Mathematical Logic—Set Theory, Model Theory, Proof Theory, and Recursion Theory—together with two topics which have evolved more recently under the influence of Computer Science: Finite Model Theory and Non-monotonic Logic. A tutorial on Interactive Proofs, Zero Knowledge, and Computationally Sound Proofs reported on recent developments in Theoretical Computer Science which shed new light on the nature of feasible computations.

Twenty-six leading experts gave plenary lectures or tutorials of 2–3 hours. They were: H. Becker (Set Theory, USA), Z. Chatzidakis (Model Theory, France), K. Compton (Tutorial, Finite Model Theory, USA), B. Cooper (Recursion Theory, Great Britain), A. Dawar (Finite Model Theory, Great Britain), R. Downey (Recursion Theory, New Zealand), I. Herzog (Model Theory, Germany), E. Hrushovski (Model Theory, Israel), J. Krajicek (Proof Theory and Complexity, Czech Republic), J. Lynch (Finite Model Theory, USA), P. Maddy (Set Theory: 100 years, USA), D. Marker (Tutorial, Model Theory, USA), M. Makkai (Categorical Model Theory, Canada), D. Martin (Set Theory: 100 Years, USA), S. Micali (Tutorial, Computer Science, USA), A. Nies (Recursion Theory, Germany), M. Pentus (Logic and Linguistics, Russia), Y. Peterzil (Model Theory, Israel), A. Pillay (Model Theory, USA), W. Pohlers (Proof Theory, Germany), M. Rathjen (Proof Theory, Germany), S. Shelah (Set Theory, Israel), T. Slaman (Tutorial, Recursion Theory, USA), L. Soukup (Set Theory, Hungary), A. Stolboushkin (Theoretical CS, USA), and S. Wainer (Proof Theory, Great Britain).

The remaining contributions were presented in parallel sessions. Special sessions of contributed papers were organized by M. Magidor (Israel) in Set Theory, E. Grädel (Germany) and J. Makowsky (Israel) in Finite Model Theory, R. Shore (USA) in Recursion Theory, G. Jäger (Switzerland) and S. Wainer (Great Britain) in Proof Theory, E. Bouscaren (France) and E. Hrushovski (Israel) in Model Theory, and W. Marek (USA) and D. Lehmann (Israel) in Non-monotonic Logic. There were more than 50 speakers in these special sessions and about an equal number of contributed papers.

A special banquet was held in honor of Professor S. Shelah from the Hebrew University in Jerusalem on the occasion of his 50th birthday and the completion of his 600th publication.

The organizing committee consisted of U. Abraham (Ben Gurion University), R. Aharoni (Technion), A. Avron (Tel Aviv University), N. Francez (Technion), V. Harnik (Haifa

1079-8986/97/0301-0005/$8.50

University), M. Kaminski (Technion), M. Koppel (Bar Ilan University), D. Lascar (Paris 7, France), A. Levy (Hebrew University), J. A. Makowsky (Chairman, Technion), and M. Rubin (Ben Gurion University). The organizing committee was greatly helped by the efficient and dedicated support of E. Ravve, M. Yelenevskaya, and Y. Sagi.

The program committee consisted of J. Barwise (USA), E. Bouscaren (France), E. Graedel (Germany), A. Hajnal (Hungary and USA), G. Jaeger (Switzerland), M. Magidor (Chairman, Israel), J. A. Makowsky (Coordinator, Israel), D. Mundici (Italy), R. Shore (USA), M. A. Taitslin (Russia), and M. Ziegler (Germany).

The conference gratefully acknowledges the services provided by the Department of External Studies and Continued Education of the Technion, especially those provided by Z. Keisari and N. Zonshine and their associates who were responsible for the daily logistics during the colloquium. In addition to financial support for student grants provided by the Association, contributions were received from Israeli universities and government agencies; the Kurt Gödel Society, Austria; the European Association for Logic, Language and Information, The Netherlands; and the North Holland Publishing Company, The Netherlands.

Proceedings containing most of the invited talks and tutorials will appear in the ASL series ***Lecture Notes of Logic*** (Springer). Selected papers of the special sessions will appear in special issues of various journals (Set Theory in ***Fundamenta Mathematicae***, Proof Theory and Finite Model Theory separately in the ***Archiv für Mathematische Logik und Grundlagen der Mathematik***, Recursion Theory in the ***Annals of Pure and Applied Logic*** and Model Theory in the ***Israel Journal of Mathematics***).

The abstracts of the invited plenary and special session talks and of the contributed papers (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
JOHANN A. MAKOWSKY

Abstracts of invited plenary lectures and tutorials

► HOWARD BECKER, *Path-connectedness, simple connectedness and descriptive set theory*.
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This talk is concerned with two classes of questions.

(1) Assume $\neg CH$. Does there exist a compact subset of $\mathbf{R}^n$ which has exactly $\aleph_1$ path-components? For $\mathbf{R}^3$ the answer is yes. For $\mathbf{R}^2$ the answer is no (assuming weak large cardinal axioms, which may or may not be necessary). The proof of both of these facts is descriptive set theoretic, and closely related to the proofs in (2), below.

(2) Let PC_n and SC_n denote the pointsets of path-connected and simply connected sets, respectively, in the space of compact subsets of $\mathbf{R}^n$. What is the complexity of PC_n and SC_n in terms of the projective hierarchy? For $n \geq 3$, PC_n is true $\mathbf{\Pi}^1_2$; SC_2 is true $\mathbf{\Pi}^1_1$; and for $n \geq 4, SC_n$ is true $\mathbf{\Pi}^1_2$. For PC_2 and SC_3, the problem is open, although some upper and lower bounds are known.

► ZOÉ CHATZIDAKIS, *Model theory of finite and pseudo-finite fields, a survey*.
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In this talk we give a survey of the model theory of finite fields, starting with the work of Ax, up to recent results using local stability theory.

In 1968, J. Ax gave an axiomatisation of the theory T_f of finite fields, and showed it is decidable. He also described the infinite models of the theory T_f, the so-called pseudo-finite fields, and gave simple invariants for their elementary theories. He isolated an important property shared by these fields: they are pseudo-algebraically closed (PAC). The study of PAC fields was later pursued by algebraists and model-theorists, and produced many interesting examples of well-behaved theories of fields.

Further developments in the model-theoretic study of finite fields arose in the 90's. Estimates of Lang-Weil type were obtained for definable subsets of finite fields: a definable subset of $\mathbf{F}_q$ has asymptotically size μq^d, for some positive rational μ and integer d; d can be interpreted as the dimension, and μ as a measure. These estimates have many applications, and show in particular that pseudo-finite fields, even though unstable, have a good behaviour: they don't have the finite cover property, nor the strict order property; the dimension of a definable set (which can be defined in purely algebraic terms) is a good notion of dimension.

These properties allow us to use local stability theory to study pseudo-finite fields, as well as related PAC fields; this yields various applications to algebraic groups defined over a finite or pseudo-finite field F: if G is such a group, then there are various bounds on the index of definable subgroups of $G(F)$, and definability of subgroups generated by Zariski irreducible definable subsets of $G(F)$. Moreover, going from pseudo-finite to finite, gives strong approximation results for dense subgroups of algebraic groups defined over $\mathbf{Z}$.

[1] J. Ax, *The elementary theory of finite fields*, ***Annals of Mathematics***, vol. 88 (1968), pp. 239–271.

[2] M. Fried and M. Jarden, ***Field Arithmetic, Ergebnisse 11***, Springer-Verlag, Berlin, Heidelberg, 1986.

[3] Z. Chatzidakis, L. van den Dries, and A. Macintyre, *Definable sets over finite fields*, ***Journal für die Reine und Angewandte Mathematik***, vol. 427 (1992), pp. 107–135.

[4] E. Hrushovski and A. Pillay, *Definable subgroups of algebraic groups over finite fields*, ***Journal für die Reine und Angewandte Mathematik***, vol. 462 (1995), pp. 69–91.

▶ KEVIN COMPTON, 0-1 *laws and finite model theory*.
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A class $\mathcal{C}$ of structures obeys a 0-1 law with respect to a logic $\mathcal{L}$ if, for every sentence $\mathcal{S}$ in $\mathcal{L}$, the probability that $\mathcal{S}$ holds in a random structure of size n in $\mathcal{C}$ approaches a limit of either 0 or 1 as n goes to infinity. The most famous result of this type is due to Glebskii et al. and Fagin. It says that the class of all structures over a relational vocabulary obeys a 0-1 law with respect to first-order logic. There have been many extensions and modifications of this theorem. Variations include the following.

- Change the class $\mathcal{C}$. Consider structures with functions, permutations, equivalence relations, etc. Allow underlying linear orders or other relations.
- Change the probability measure. Count structures with a fixed universe. Count isomorphism types. Assign edge probabilities in graphs. Generate partial order in an n-cube.
- Extend or change the logic $\mathcal{L}$. Consider monadic second-order logic. Consider fixpoint logic. Consider infinitary logic with a bounded number of variables. Consider Sigma-1-1 sentences with syntactic restrictions. Consider modal logic.
- If a 0-1 law does not hold, prove a weaker result. A limit law holds if the probability of every sentence approaches some limit. A slow variation law holds if the difference between consecutive probabilities approaches 0.

We will provide a framework to classify the many results now known, and give a general itemize of the techniques used to prove them.

▶ S. BARRY COOPER, *Beyond Gödel's theorem—the failure to capture information content*.
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The work of Gödel, Church, Turing and others from the thirties onwards presents us with by now familiar restrictions on mathematics. On the other hand, it has been possible to believe that science to some extent avoids such basic theoretical limitations. Recent work concerning Turing definability and automorphisms of Turing-related structures suggests that scientific observation may be inadequate for analysing physical reality. In science, as in mathematics, a 'theory of everything' may prove to be a theoretical impossibility.

▶ ANUJ DAWAR, *Model theoretic methods for complexity theory*.
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Research in the model theory of finite structures received a large impetus from the discovery that important computational complexity classes have natural characterizations as definability classes on finite models. Such results raised the hope that model theoretic methods could be brought to bear on notoriously outstanding problems in complexity theory.

This hope has been partly frustrated by the fact that the standard methods of model theory fail when only finite structures are considered. This failure is tied to the central role that first order logic has played in the classical model theory. First order logic does not occupy such a central position in finite model theory for two important reasons:

- First order logic is too strong. This is in the sense that any two elementarily equivalent finite models are isomorphic.
- First order logic is too weak. In the sense that any class of finite structures that is finitely axiomatizable is of very low complexity.

Both these weaknesses can be addressed by considering restrictions of first order logic to finitely many variables. This yields an interesting (from the model theoretic point of view) notion of elementary equivalence, that is also useful in studying extensions of first order logic. Moreover, many central problems of complexity theory can still be recast in this framework.

We discuss questions about the model theory of finite variable logics arising from such considerations.

▶ ROD DOWNEY, *On* low_2 *recursively enumerable degrees*.
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A set A is called low_n if its n-th Turing jump has the same degree as $\emptyset^{(n)}$. The low_n sets form a natural collection of degrees. The low_2 sets form a particularly interesting class since they seem to lie at the boundary of definability in, for instance, the recursively enumerable degrees. In this talk, I plan to describe some recent progress towards understanding and defining the low_2 (recursively enumerable) sets. I plan to speak on joint work with Richard Shore where we have proven that the low_2 (r.e.)(Δ_2) degrees are definable as are the low_2 r.e. sets under the ordering with two reducibilities $\{T, m\}$. I also plan to discuss a new technique developed with Shore which enables us to embed lattices below a given nonlow_2 degree. This is related to work with Shore and Cholak concerning nonembedding 1-3-1 into the r.e. degrees. Finally, I plan to briefly discuss recent work jointly with Steffen Lempp where we have proven that the low_2 class of contiguous degrees are definable in the uppersemilattice of recursively enumerable degrees, solving a 20 year old question of Ladner.

▶ IVO HERZOG, *The model theory of modules*.
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Let R be an associative ring with unit. The language $L(R)$ for (left) R-modules consists of the (nonlogical) symbols $L(R) = (+, 0, r)_{r \in R}$ necessary to express a linear equation

$$r_1 x_1 + \cdots r_n x_n \doteq 0$$

with (left) coefficients in R. A positive-primitive (pp-) formula of $L(R)$ is an existentially quantified system of linear equations. A brief history of the model theory of modules will clarify the central rôle pp-formulae play: Baur proved the elimination of quantifiers (modulo a complete theory of R-modules) up to pp-formulae and Baur, Garavaglia and Monk, independently gave a very nice classification of complete theories of R-modules in terms of pp-formulae.

If the ring R is a commutative field, then every pp-formula is equivalent to a system of linear equations. The model theoretic analysis of R-modules by use of the pp-formulae is thus a generalization of Linear Algebra. As with other such generalizations, a host of techniques from Abelian Category Theory is available. We shall describe the model theoretic significance of some standard homological notions.

Ziegler introduced the (left) spectrum ${}_R\mathrm{Zg}$ of a ring R as the topological space whose points are the pure-injective (pp-saturated) indecomposable R-modules with the support of a positive-primitive sort of $L(R)^{\mathrm{eq}}$ as a typical basic open subset. There is a Nullstellensatz for the Ziegler spectrum in the form of a bijective correspondence between the open subsets of ${}_R\mathrm{Zg}$ and the Serre subcategories of the pp-reduct of $L(R)^{\mathrm{eq}}$ (thought of as an Abelian category whose objects are the sorts and whose morphisms are the pp-definable functions between sorts). We shall survey the many consequences of this theorem.

Finally, we shall present the formal calculus of pp-formulae. Using the two quantifiers that preserve pp-formulae (existential and instantiation of 0) as face operations, the Goursat complex of the ring R arises. We shall calculate the Goursat homology of a field in the lower dimensions. Using the Nullstellensatz, we shall characterize the 0-dimensional integral Goursat cohomology of a ring R as the free group on the indecomposable R-modules M for which the Morley rank of the infinite sum $M^{(\omega)}$ is finite.

▶ EHUD HRUSHOVSKI, *Stability and its heirs*.
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From a decade's distance, we will view the culmination of classical stability theory in Shelah's demonstration of the main gap, and the less certain developments emanating naturally from there.

▶ JAN KRAJÍČEK, *Propositional proofs, proofs of membership in polynomial ideals, and their complexity*.
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Let $\mathbf{k}$ be a finite field. The *ideal membership problem* is the problem to decide, given polynomials $g_1, \dots, g_r, h \in \mathbf{k}[x_1, \dots, x_n]$, whether h is in the ideal $\langle \overline{g} \rangle$ generated by $\overline{g}$. A particular case of the ideal membership problem, relevant to propositional logic, is to decide whether the affine variety $V_{\mathbf{k}}(\langle \overline{g} \rangle)$ is empty. That is equivalent to the question whether the extended ideal $\langle \overline{g}, x_1^{|\mathbf{k}|} - x_1, \dots, x_n^{|\mathbf{k}|} - x_n \rangle$ contains 1.

A *proof system* for the ideal membership is a binary relation $P(\pi, \sigma)$ between finite words π over some fixed finite alphabet and words σ coding $\overline{g}, h$ as an $(r+1)$-tuple of terms (in the appropriate language with constants for elements of $\mathbf{k}$), and satisfying two conditions

(1) $h \in \langle \overline{g} \rangle$ iff $\exists \pi, P(\pi, \langle \overline{g}, h \rangle)$.

(2) The truth value of the relation $P(\pi, \sigma)$ is computable by a polynomial-time algorithm.

The efficiency of a proof system is measured by the length-of-proofs function

$$c_P(\langle \overline{g}, h \rangle) := \min\{|\pi| \mid h \in \langle \overline{g} \rangle \to P(\pi, \langle \overline{g}, h \rangle)\}.$$

The ultimate goal is to show, for every proof system P, a lower bound of the form $c_P(\langle \overline{g}, h \rangle) = m^{\omega(1)}$ (or even $\exp(m^{\Omega(1)})$) for some instances $\langle \overline{g}, h \rangle$ of length m related to propositional logic as earlier. That is equivalent to showing NP $\neq$ coNP.

We shall consider the question of proving such lower bounds for two particular proofs systems; one based on equational logic and one based on Hilbert's Nullstellensatz, and we outline a lower bound proof for the latter system.

Lower bounds for such proof systems imply lower bounds for some ordinary propositional proof systems and independence results for bounded arithmetic theories. We shall outline these connections as well.

[1] P. Beame, R. Impagliazzo, J. Krajíček, T. Pitassi and P. Pudlák, *Lower bounds on Hilbert's Nullstellensatz and propositional proofs*, ***Proceedings of the London Mathematical Society***, vol. 3 (1996), pp. 1–20, an extended abstract in ***Proceedings of the IEEE 35th Annual Symposium on Foundation of Computer Science***, 1994, pp. 794–806.

[2] J. Krajíček, ***Bounded arithmetic, propositional logic, and complexity theory***, Cambridge University Press, 1995.

► JAMES F. LYNCH, *Convergence laws for random graphs*.
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A convergence law in logic pertains to a language and class of finite structures on which a probability measure has been assigned to each set of structures of a given size. It states that, for every sentence in the language, the probability that it holds for a random structure approaches a limit as the size of the structure grows. A 0-1 law states that this limit must be 0 or 1. The original convergence law was a 0-1 law for first-order logic and relational structures with a uniform probability distribution. This expository article shows how it has been extended by numerous authors to more powerful logics and the class of random structures known as random graphs. In many cases, the 0-1 law no longer holds, but a convergence law can still be proven. Full proofs are not given, but a uniform framework is provided which emphasizes ideas common to all the proofs.

► PENELOPE MADDY, *Justifying the axioms* ($V = L$ *and maximize*).
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This talk is an attempt to describe, elaborate and defend a familiar line of argument against the axiom of constructibility. The full text will appear in the proceedings of this colloquium.

► MICHAEL MAKKAI, *Towards a categorical foundation of mathematics*.
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The talk will be based on three recent longer papers of mine, [8],[9] and [10]. Although the papers are purely mathematical in nature, in particular, they are placed within the usual set-theoretical foundational framework, they are motivated by the program of establishing what I call the Categorical Foundation of Mathematics (CFM), an alternative to Cantorian set-theory. The basis for CFM is a concept of set called abstract set, or set-as-type. An abstract set is a collection with an equality predicate, an arbitrary equivalence relation. No equality predicate is assumed with arguments ranging over possibly different sets. In particular, there is no unique "true equality" for all things. As a consequence, in CFM, there is no equality predicate for sets as arguments. As a result, sets do not form a set (in fact, a set is never an element of a set); sets form a category, a generalization of the notion of set. Objects of a category cannot be equal; they can be isomorphic. Categories themselves cannot be isomorphic; they can be equivalent. An infinite hierarchy of collective concepts, higher dimensional categories, is called for.

[8] introduces sketch-based syntax, a new general approach to formalized syntax. It is based on a natural generalization of the notion of sketch introduced by C. Ehresmann [4], and used extensively in category theory and theoretical computer science. The concept of elementary topos due to F. W. Lawvere and M. Tierney (see [7]) is an axiomatization of the concept of abstract set. It becomes a formal theory in a natural way when it is formalized in sketch-based syntax. Sketch-based syntax has its own semantics, and it is particularly suited to a variety of logics within categorical logic. It is intended as the basis for CFM as a formalized theory.

[9] engages in a revision of Category Theory itself; the concept of functor, which is seen implicitly to rely on the idea of equality of objects in a category, is replaced by that of anafunctor. The established higher dimensional concepts (bicategory [2], tricategory [6], and weak n-category [1], not yet explicitly defined in the literature) have resulting "ana"-versions; anabicategories are treated in [9]. [9] shows that using the "ana"- concepts, general category theory can be developed in a more canonical manner than usual, in particularly, largely without the use of the Axiom of Choice.

First-Order Logic with Dependent Sorts (FOLDS) introduced in [10] derives from P. Martin-Löf's type theory, and more directly from J. Cartmell's syntax [3]; it is intended as a metalanguage for the structures of CFM. The literature has certain concepts of "equivalence", one for each type of categorical structure (equivalence of categories, biequivalence of bicategories, etc.) which are recognized as the correct concepts of transformation under which all the properties ever to be considered should be invariant. It is shown that these notions of equivalence become a uniform model-theoretical concept within FOLDS, called FOLDS- equivalence, when one passes to the anaversions of the categorical structures. Properties formulated in FOLDS are precisely those first-order properties that are invariant under FOLDS-equivalence. Generalizing P. Freyd's result [5] on equivalence of categories, the main results of [10] are characterizations of first-order properties invariant under various categorical equivalences.

Although the work reported here does not directly belong to any established discipline in Mathematical Logic, it is quite close to Model Theory. [8] is written from the model-theoretic point of view as a framework in which the constructive content of representation theorems of categorical logic (completeness in algebraic form) can be directly displayed without recourse to translation into ordinary syntax. [10] consists in a large part of a general model theory of FOLDS.

[1] J. C. BAEZ and J. DOLAN, *Higher dimensional algebra and topological quantum field theory*, University of California, Riverside, preprint.

[2] J. BENABOU, *Introduction to bicategories*, ***Lecture Notes in Mathematics***, no. 47, Springer-Verlag, 1967, pp. 1–77.

[3] J. CARTMELL, *Generalized algebraic theories and conceptual categories*, ***Annals of Pure and Applied Logic***, vol. 32 (1986), pp. 209–243.

[4] C. EHRESMANN, *Esquisses et types de structures algebriques*, ***Bull. Instit. Polit., IASI***,

vol. XIV (1968), pp. 1–14.

[5] P. FREYD, *Properties invariant within equivalence types of categories*, ***Algebra, topology and category theory*** (A. Heller and M. Tierney, editor), Academic Press, 1976, pp. 55–61.

[6] R. GORDON, A. J. POWER and R. STREET, *Coherence for tricategories*, ***Memoirs of the American Mathematical Society***, vol. 117, no. 558, 1995.

[7] P. T. JOHNSTONE, ***Topos theory***, Academic Press, 1977.

[8] M. MAKKAI, *Generalized sketches as a framework for completeness theorems*, McGill University, preprint, to appear in three parts in ***Journal of Pure and Applied Algebra***.

[9] ———, *Avoiding the axiom of choice in general category theory*, ***Journal of Pure and Applied Algebra***, vol. 108 (1996), pp. 109–173.

[10] ———, *First-order logic with dependent sorts, with applications to category theory*, research monograph, accepted for publication in ***Lecture Notes in Logic***, Springer-Verlag.

► DAVID MARKER, *Strongly minimal sets and geometry*.
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The algebraic closure operator on a strongly minimal set gives rise to a combinatorial geometry. The nature of this geometry has global consequences about ambient structure. Zilber conjectured that these geometries have a natural simple classification. While this is false in general, it is true for a number of important special cases. In these lectures I will give an introduction to Zilber's proposed trichotomy and applications to algebraic and diophantine geometry.

► DONALD A. MARTIN, *Recent work on large cardinals and determinacy*.
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Both the subject of large cardinals and that of determinacy (the existence of winning strategies for infinite games) originated early in this century. During the last thirty years or so, these subjects have become intertwined, and considerable evidence has been amassed that they are not two subjects but one. We shall survey the known connections between large cardinals and determinacy, emphasizing work done in the past two or three years and discussing the main open problems.

► SILVIO MICALI, *Interactive proofs, zero knowledge and computationally sound proofs*.
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The Tutorial was based on three papers [1]–[3] and some new material on computationally sound proofs [4]. The first lecture gave a conceptual introduction and survey on interactive proofs and zero knowledge. The second lecture introduced a computationally-based notion of proof and explored its implications to computation at large. This work provides the beginnings of a theory of computational complexity that is based on *individual inputs* rather than languages.

[1] M. BLUM, A. DE SANTIS, S. MICALI and G. PERSIANO, *Noninteractive zero-knowledge*, ***SIAM Journal of Computing***, vol. 20 (1991), no. 6, pp. 1084–1118.

[2] O. GOLDREICH, S. MICALI and A. WIGDERSON, *Proofs that yield nothing but their validity or all languages in NP have zero-knowledge proof systems*, ***Journal of the Association for Computing Machinery***, vol. 38 (1991). no. 1, pp. 691–729.

[3] S. GOLDWASSER, S. MICALI and C. RACKOFF, *The knowledge complexity of interactive proof systems*, ***SIAM Journal of Computing***, vol. 18 (1989), no. 1, pp. 186–208.
[4] S. MICALI, *CS proofs*, ***Proceedings of the 35th Annual Symposium on Foundations of Computer Science***, IEEE Computer Society Press, 1994, pp. 436–453.

► ANDRÉ NIES, *Coding in distributive structures*.
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The application of coding methods to a structure A arising from computability theory has been a very successful tool for determining the degree of $\mathrm{Th}(A)$, obtaining limiting results on automorphisms and definability results for subsets of A. One uses a coding (also called an interpretation) of a structure B in A with a scheme of first-order formulas which possibly allows parameters from A. In particular, coding methods have been applied to structures based on enumerable sets, namely the distributive lattice $\mathcal{E}$ of enumerable sets under inclusion, and the uppersemilattices $\mathcal{R}_r$ of enumerable r-degrees, where $\leq_r$ is some reducibility. For semilattices, a notion of distributivity which generalizes distributivity for lattices has been introduced. This property is satisfied by the structures $\mathcal{R}_m$ of enumerable many-one degrees and $\mathcal{R}_{\mathrm{wtt}}$ of enumerable weak truth-table degrees (which is interesting since it behaves in some aspects similar to $\mathcal{R}_T$). We investigate coding in these distributive structures. Since distributivity is a property of being well-behaved, one can expect limitations to coding and definability. On one hand we present results in this negative direction. In joint work with W. Hodges, it is shown that no infinite linear order can be interpreted in any structure of the form $B \times B$. This implies that such a noninterpretability result holds for $\mathcal{E}$ and for certain initial segments of $\mathcal{R}_{\mathrm{wtt}}$. Moreover, for $\mathcal{E}$, we show that some nondefinable subset exists which satisfies the properties (necessary for definability) of being arithmetical and invariant under automorphisms.

On the positive side, we describe coding methods for each of the structures which allow one to obtain an interpretation of true arithmetic in the theory and, in the case of $\mathcal{R}_m$ and $\mathcal{R}_{\mathrm{wtt}}$, even give an interpretation without parameters of a standard model of arithmetic in the structure. The results are due to L. Harrington for $\mathcal{E}$ and to the author for $\mathcal{R}_m$ and $\mathcal{R}_{\mathrm{wtt}}$. In all proofs, distributivity is exploited in an algebraic way to make the coding schemes work. A key idea which was first applied for $\mathcal{R}_m$ and then carried over to $\mathcal{E}$ is to show uniform definability of a certain type of arithmetical subsets of the structure by induction on the arithmetical complexity. The proof for $\mathcal{E}$ was extended by the author to give an interpretation of true arithmetic in any interval of $\mathcal{E}$ which is not a boolean algebra.

► MATI PENTUS, *Lambek calculus and formal languages*.
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In 1958 J. Lambek introduced a calculus for deriving reduction laws of syntactic types. This Lambek calculus may now be regarded as a three-connective fragment of the non-commutative linear logic. The three connectives are product, left division, and right division (linear conjunction and two linear implications).

W. Buszkowski has established that the product-free fragment of the Lambek calculus is complete with respect to language models (free semigroup models), where formal types (i.e., linear logic formulas) are interpreted as languages in a fixed alphabet (i.e., subsets of a free semigroup). He has also proved that the full Lambek calculus is complete with respect to the class of all semigroup models.

In this paper we sketch a proof of a stronger result, namely that the full Lambek calculus is complete with respect to language models.

► YACOV PETERZIL, *Zilber's trichotomy and o-minimal structures*.
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Let $\mathcal{M}$ be a structure with a good notion of dimension for tuples and definable sets. $\mathcal{M}$ can be classified into one of three types according to the definable families of curves (1-dimensional sets) definable in it. In the early eighties B. Zilber has suggested a correspondence between this trichotomy and another one, depending on certain algebraic structures definable in $\mathcal{M}$. Roughly said, the latter trichotomy puts $\mathcal{M}$ into one and only one of the following:

(i) No algebraic structure.
(ii) A module-like structure.
(iii) A field-like structure.

The correspondence has recently shown up in the analysis of familiar mathematical objects such as Abelian varieties in algebraically closed fields and varieties in difference fields.

The class of linearly ordered, o-minimal structures has a good notion of dimension. After discussing the introductory notions I will present joint work with S. Starchenko in which we show the class of *o*-minimal structures can be classified according to the above trichotomy.

► ANAND PILLAY, *Superstable differential fields*.
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The talk is based around the problem of whether a differential field $(K, +, ., \delta)$ whose theory is superstable, must be differentially closed. Various partial results are obtained, generalising earlier work of the author and Sokolovic. The key result is that (for K superstable), K is differentially closed as far as the "generalised" strongly normal extensions recently introduced by the author are concerned. Together with results of Hrushovski and Sokolovic this yields:

THEOREM 1. *Let K be a superstable differential field and let L be its differential closure. Then (working in the structure L), for any $a \in L$, $\mathrm{tp}(a/K)$ is orthogonal to all nontrivial types of U-rank* 1.

Dave Marker and the author studied the "inverse Galois" problem for generalised strongly normal extensions. Together with above results this work yields the following:

THEOREM 2. *Let L be a differentially closed field, whose field of constants C has infinite transcendence degree. Let K be a differential subfield of L such that*

(i) *$C < K$, and*
(ii) *for any $a \in K$, $\mathrm{tp}(a/C)$ is analysable in C (in the sense of the structure L).*

Then K is not superstable.

► WOLFRAM POHLERS, *Subsystems of set theory vs. subsystems of second order number theory*.
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We describe the type of results which are obtained by a proof theoretical analysis of subsystems of set theory and show how these are connected to the results for subsystems of second order number theory. A full version of the talk appeared in this BULLETIN, vol. 2,

no. 2 (1996), pp. 159–188.

► MICHAEL RATHJEN, *The higher infinite in proof theory*.
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The first part of the talk will focus on "ordinally informative" proof theory and explain its current rationale and goals.

The second part of the talk will report on recent success in obtaining ordinal analyses for the subsystem of formal second order arithmetic with comprehension confined to Π^1_2-formulae and for theories that are reducible to iterated Π^1_2-comprehension, e.g., Δ^1_3 comprehension.

► SAHARON SHELAH, *An iterated forcing for the continuum*.
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We review preservation theorems for iterated forcing in "proper and improper forcing" (the new version of the book).

As examples, we will try to present three results and their relationship to the general theory:

(1) There may not be any ultrafilter (on **N**) for nowhere-denseness;
(2) No Gross space;
(3) The covering number of the reals by null sets may be $\aleph_\omega$.

► THEODORE A. SLAMAN, *Recent results in recursion theory*.
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Within the last year, Cooper has announced the existence of a nontrivial automorphism of the Turing degrees (D). In a sense to be made precise in these talks, with Cooper's announcement and the hoped for eventual understanding of its automorphism group $(\mathrm{AUT}(D))$, the global structure of D will completely characterized.

Although D is formed from the continuum at large, its global properties depend on the fine details of recursive enumerability and of the arithmetic sets. There is a beautiful interaction between the local behavior of relative computability on the arithmetically definable sets and the global structure of D.

Lecture 1. Introduction to the structure, analysis of its ideals, the Lerman-Shore decision procedure for its $\exists\forall$ theory, Slaman-Woodin first order coding of countable relations by parameters, characterization of the theory as recursively isomorphic to second order arithmetic.

Lecture 2. Shore's inhomogeneity theorem, Jockusch-Shore pseudo-jump operators, Cooper's definition of the Turing jump within $\langle D, \leq\rangle$, fixing automorphisms almost everywhere.

Lecture 3. Slaman-Woodin machinery: every element of $\mathrm{AUT}(D)$ is induced by an arithmetic function on reals, calculations of finite automorphism bases consisting of definable reals, characterizing the definable relations as those which are invariant under $\mathrm{AUT}(D)$ and definable in second order arithmetic.

► LAJOS SOUKUP, *Combinatorial principles from adding Cohen reals*.
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We prove combinatorial principles in models obtained by adding $\kappa > \omega_1$ many Cohen reals to models of ZFC + CH.

Using these principles we obtain new theorems and we also get new and simpler proofs of old consistency results concerning topological and algebraic problems. As an example we introduce principle $C(\kappa)$ and mention some of its consequences here.

DEFINITION 1. If $cf(\kappa) = \kappa > \omega$, then the principle $C(\kappa)$ is the following statement: If $T \subset \omega^{<\omega}$ and $\langle A(i,\alpha) : \alpha < \kappa, i < \omega \rangle \subset \mathcal{P}(\omega)$ then either (1) or (2) below holds:

(1) there is a stationary set $S \subset \kappa$ such that if $t \in T$ and $s \in S^{|t|}$, s is injective, then
$$\bigcap\{A(t(i), s(i)) : i < |t|\} \neq \emptyset,$$

(2) there are $t \in T$ and stationary subsets $D_0, D_1, \ldots, D_{|t|-1}$ of κ such that if $\delta(0) \in D_0, \ldots, \delta(|t|-1) \in D_{|t|-1}$ are pairwise different ordinals then we have
$$\bigcap\{A(t(i), \delta(i)) : i < |t|\} = \emptyset.$$

THEOREM 2. *If* $V \models$ GCH *and* $\kappa > \omega_1$ *is a regular cardinal then* $V^{\mathrm{Fn}(\kappa,2,\omega)} \models C(\kappa)$.

THEOREM 3. *If* $C(\kappa)$ *holds then*

(1) *every first countable, separable* T_2 *topological space* X *of size* κ *contains two disjoint open sets* U *and* V *of cardinality* κ.

(2) *there is no tower of size* κ,

(3) *there is no locally compact, thin scattered space of height* κ,

(4) *if* X *is a separable compact space of size* $> c^{<c}$ *then* X *contains a dyadic system of size* c, *consequently* X *maps onto* $[0,1]^c$ *and* $\beta\omega$ *can be embedded into* X.

► ALEXEI P. STOLBOUSHKIN, *Monotone properties in feasible models*.

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Many logical laws discontinue to hold when we try to restrict them to finite models only. We observe that this phenomenon is not at all specific for finite models. The situation is very similar for the class of all recursive models. In this latter class, the standard model of Arithmetic is finitely axiomatizable, and thus failures of compactness, axiomatizability, etc. in N translate into their failures for recursive models. The case we are trying to make is that the exact reason for those "logical" laws to hold is the existence of "weird" models. In applications related to computer science, only certain feasible models are of interest, where the notion of feasibility varies from one case to another, from "finite" to "recursive".

We closely consider Lyndon's Lemma—that every monotone formula is equivalent to a positive one—and establish its failure over recursive models. While the original proof of its failure for finite models is due to Ajtai and Gurevich, a recent short proof by the speaker [1] gives an additional insight into the nature of formulas monotone in feasible models.

While for a logician the main value of Lyndon's Lemma is in the implication that monotone and positive fixpoints are the same—which implication continues to hold both in finite models (Gurevich and Shelah) and in Arithmetic (Kleene)—for a computer scientist an effective syntax for monotone formulas is at least as important. For instance, monotone sentences provide a mechanism for decision-making under an incomplete information.

This naturally leads to the question on the status of Generalized Lyndon's Lemma—whether a recursive set of monotone formulas exists such that every monotone formula is equivalent to one in the class. This latter question is a representative of a wide variety of questions related to the existence of an effective syntax for a complex class. In a similar situation, the speaker together with M. A. Taitslin [2] recently showed that, for some feasible models, finite formulas do not have an effective syntax (the model-specific notion of finite formulas is relevant to database applications). The speaker hopes to get people interested in these problems.

[1] A. P. STOLBOUSHKIN, *Finite monotone properties*, ***Proceedings of the IEEE Symposium on Logic in Computer Science***, 1995, pp. 324–330.

[2] A. P. STOLBOUSHKIN and M. A. TAITSLIN, *Finite queries do not have effective syntax*, ***Proceedings of the ACM Symposium on Principles of Database Sytems***, May 1995, pp. 277–285.

► STAN WAINER, *The proof theoretic complexity of recursion*.
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A terminating recursive program over N defines a total recursive functional, taking 'given' total functions (oracles) g to the function f computed from them by the program. Thus $F(g) = f$. Termination means that the program has a well-founded 'computation-tree' with a recursive ordinal as its height. Kleene (1958) noted that, in contrast with the well-known 'collapsing phenomenon' for hierarchies of recursive functions, this hierarchy of total recursive functionals expands right the way through the recursive ordinals, and is notation-free (i.e., independent of how they are represented).

This talk surveyed the ways (some old, some new) in which the ordinal height of a program encodes its logical and computational complexity (though here we think of computational complexity 'in the large', not with prior bounds on resource; thus natural classes which arise are proof-theoretic in origin). By careful assignments of ordinal bounds to derivations in (i) Kleene's equation calculus, and (ii) arithmetic with the omega-rule, the standard Fast, Medium (Hardy), and Slow growing hierarchies all emerge as canonical complexity measures, allowing different forms of recursion to be classified and compared from the point of view of (i) computation, and (ii) proof.

The close relationships between these hierarchies are given by ordinal transformations which correspond precisely to Cut Elimination Theorems of one form or another. These yield

(a) logical measures of transformational complexity (eg. recursive to tail-recursive) in terms of their corresponding ordinal trade-offs;

(b) comparisons between call-by-value derivations of recursions, and their computability by term-rewriting;

(c) the known classifications of provably recursive functions, and their connections with independence results via the Hardy hierarchy.

Abstracts of invited talks in the Special Session in Set Theory

► NORBERT BRUNNER, *Iterated permutation models*.
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Fraenkel-Mostowski permutation models are tools for independence proofs concerning the axiom of choice in ZFA set theory, a variant of ZF which admits atoms. The prototype of these models has been constructed by Abraham Fraenkel about 75 years ago. We show, that a permutation model which is built within a permutation model is again a permutation model. As an application it follows, that the modal logic **S5** has an interpretation as a modal logic of consistency by means of permutation models. Since permutation models are proper classes for which only local notions of truth are available, this result does not contradict Solovay's completeness theorem, where the use of a formalized notion of truth enforces the validity of **W** ($L(L\phi \to \phi) \to \phi$.)

[1] N. BRUNNER, *A modal logic of consistency*, ***Rendiconti del Seminario Matematico della Universitá di Padova***, vol. 93 (1995), in print.

[2] R. SOLOVAY, *Provability interpretations of modal logic*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 287–304.

► LEV BUKOVSKÝ, *Permitted sets for trigonometric thin sets*.
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We shall investigate permitted sets for the classical families of trigonometric thin subsets of [0, 1], namely for the families $p\mathscr{D}$, $\mathscr{N}_0$, $\mathscr{N}$, $\mathscr{A}$ and $w\mathscr{D}$ of pseudo Dirichlet sets, N_0-sets, N-sets, A-sets and weak Dirichlet sets, respectively.

Let $\mathscr{F}$ be a hereditary family of subsets of the interval [0, 1] not containing the whole interval. A set A is said to be $\mathscr{F}$-permitted if $A \cup B \in \mathscr{F}$ for each $B \in \mathscr{F}$. One can easily see that the family $\mathrm{Prm}(\mathscr{F})$ of all $\mathscr{F}$-permitted sets is an ideal. That was J. Arbault [1] who started the study of $\mathscr{N}_0$- and $\mathscr{N}$-permitted sets showing that every countable set is permitted.

Some results of [3] were used in the proof of the third assertion of

THEOREM (L. B., N. N. Kholshchevnikova and M. Repický).
(1) $\mathrm{non}(\mathrm{Prm}(p\mathscr{D})) \geq \min\{\mathfrak{s}, \mathfrak{b}\}$,
(2) $\mathrm{non}(\mathrm{Prm}(\mathscr{N}_0)) \geq \min\{\mathfrak{s}, \mathfrak{b}\}$,
(3) *every γ-set is $\mathscr{F}$-permitted for $\mathscr{F} = p\mathscr{D}, \mathscr{N}_0, \mathscr{N}, \mathscr{A}, w\mathscr{D}$.*

T. Bartoszyński and M. Scheepers [2] have shown that

$$\mathrm{non}(\mathrm{Prm}(\mathscr{N})) \geq \mathfrak{t}, \quad \text{and} \quad \mathrm{non}(\mathrm{Prm}(\mathscr{A})) \geq \mathfrak{t}.$$

More recently M. Repický has shown $\mathrm{non}(\mathrm{Prm}(\mathscr{F}))$ is greater than or equal to the additivity of Lebesgue measure.

[1] J. ARBAULT, *Sur l'ensemble de convergence absolue d'une série trigonométrique*, ***Bulletin de la Société Mathématique France***, vol. 80 (1952), pp. 253–317.

[2] T. BARTOSZYŃSKI and M. SCHEEPERS, *Remarks on sets related to trigonometric series*, ***Topology and its Applications***, vol. 64 (1995), pp. 133–140.

[3] L. BUKOVSKÝ, I. RECŁAW and M. REPICKÝ, *Spaces not distinguishing pointwise and quasinormal convergence of real functions*, ***Topology and its Applications***, vol. 41 (1991), pp. 25–40.

► JAMES CUMMINGS, *Reflection principles and their negations*.
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Stationary reflection principles involving different cofinalities are shown to be, to a certain extent, independent.

► MATTHEW FOREMAN, *On spectra of biliniear forms*.
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E-mail: `mforeman@math.uci.edu`.

We will consider the Γ-invariants for bilinear forms. Using a weakly compact cardinal we show the that it is consistent for the spectrum to be sparse and, extending work of Baumgartner, that it is consistent for the continuum to be arbitrarily large and the spectrum full.

[This is joint work with O. Spinas.]

► S. FRIEDMAN, *Fine structure and class forcing*.
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA, and UFR Mathématiques, Université de Paris 7, Paris, France.
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In this talk I will discuss aspects of my forthcoming book on the fine structure theory for L and the theory of class forcing. A modified Lévy hierarchy can be used to give a smoother treatment of fine structure theory and we use it to establish square, morass with square, fine scales and gap-2 morasses in L. Class forcings do not in general preserve ZF and we isolate the condition of Tameness required for this to be the case. The four basic examples of tame class forcings are: Easton, Long Easton, Reverse Easton and Amenable forcing.

We then explore the question of generic class existence. L is rigid in class forcing extensions of L and we introduce L[0#] as the least inner model in which L is not rigid. A class forcing is Relevant if it has a generic definable over L[0#]. Of the basic examples, Easton and Long Easton are relevant only if one avoids inaccessibles whereas Reverse Easton is relevant without this restriction.

Next comes the method of Jensen Coding which utilizes the fine structure theory introduced earlier. There are two proofs given, one assuming -0#, and the other more intricate but without the -0# hypothesis. Jensen Coding is of the Long Easton variety and is thereby shown to be relevant.

We use class forcing to answer three questions of Solovay: If a real does not construct 0# then must it be generic over L? Is there a Π^1_2-*Singleton* strictly between 0 and 0# in L-degree? Is there a real R such that the R-admissibles are exactly the recursively inaccessible ordinals? The first question is answered immediately by Jensen Coding if generic is taken to mean set-generic; otherwise hyperclass forcing is used. The second question requires a mixture of Reverse Easton and Jensen Coding techniques. The third question uses a refinement of Jensen Coding called Strong Coding.

Finally, there are other applications of class forcing to descriptive set theory and to other notions of set-theoretic definability.

► M. GOLDSTERN and S. SHELAH, *A partial order where all monotone maps are definable*.
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Using $\diamondsuit$ (or working in a Cohen extension), we construct a partial order $P = (\omega_1, \sqsubseteq)$ such that

(1) All monotone maps from P to P are first order definable from some countable subset of ω_1
(2) All countable subsets of ω_1 are first order definable in P

and hence all monotone maps are definable.

To achieve (1), we use an omitting types argument to ensure that all monotone maps f are bounded on the set $\{x : f(x) \neq x\}$. To achieve (2), we construct $\sqsubseteq$ from finite approximations in a quite "generic" way, such that many (simple) types are realized. We use and extend a technique from Shelah's paper *Constructions of many complicated uncountable structures and Boolean algebras*, ***Israel Journal of Mathematics***, 1993.

▶ T. JECH, B. BALCAR and J. ZAPLETAL, *Semi-Cohen Boolean algebras*.
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We investigate classes of Boolean algebras related to the notion of forcing that adds Cohen reals. A "Cohen algebra" is a Boolean algebra that is dense in the completion of a free Boolean algebra. We introduce and study generalizations of Cohen algebras: semi-Cohen algebras, pseudo-Cohen algebras and potentially Cohen algebras. These classes of Boolean algebras are closed under completion.

▶ I. JUHÁSZ, *When is an initially ω_1-compact separable T_3 space compact?*.
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A space X is called initially κ-compact if every open cover $\mathcal{U}$ of X with $|\mathcal{U}| \leq \omega_1$ has a finite subcover. Since CH implies that the weight of a separable T_3 space is $\leq \omega_1$, it is trivial that, under CH, an initially ω_1-compact such space is compact. We don't know if $\neg$CH implies the existence of a counterexample but it is well-known that if $t > \omega_1$ then there is one which is even T_4 and sequentially compact. On the other hand we have:

THEOREM 1 (Dow, Juhász, Soukup, Szentmiklóssy). *If we add any number of Cohen-reals to a model of* CH *then in the extension every initially ω_1-compact and sequentially compact separable T_3 space is compact.*

A space X is countably tight if $p \in \overline{A}$ implies $p \in \overline{B}$ for some countable $B \subset A$. It's easy to see that if an initially ω_1-compact and countably tight space X is not compact then there is a countable set $S \subset X$ such that $\overline{S}$ is not compact either. Hence under CH (and, by A. Dow, also under PFA) every initially ω_1-compact and countably tight T_3 space is compact.

M. Rabus has recently given a forcing construction, based on the Baumgartner-Shelah construction of a thin-very tall superatomic boolean algebra, that yields a counterexample to this. Replacing the boolean algebraic approach with that of a directly topological one L. Soukup and I have obtained the following result.

THEOREM 2 (Juhász, Soukup). *It is consistent with "practically" any cardinal arithemtic in which* CH *fails that there is a Frechet-Uryson, locally compact, countably tight, scattered T_4 space that is initially ω_1-compact but not compact.*

▶ VLADIMIR KANOVEI, *On nonwellfounded iterated Sacks extensions, with an application to the Glimm-Effros property*.
Moscow Transport Engineering Institute, Obraztsova 1S, Moscow 101475, Russia.
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We prove that if I is a p.o. set in a countable transitive model M of ZFC then M can be extended by a generic sequence of reals a_i, $i \in I$, such that $\aleph_1^M$ is preserved and every a_i is Sacks generic over $M[\langle a_j : j < i\rangle]$. I is not assumed to be wellfounded, linearly ordered, or countable in M.

This result generalizes a theorem in the recently published article of M. Groszek (*Journal of Symbolic Logic*, no. 3, 1994).

The structure of the degrees of M-constructibility of reals in the extension is investigated.

As an application, taking I, the "length" of iteration, to be $\omega_1^M \times Z$ (ω_1 copies of the integers, ordered lexicographically), we obtain a model in which the Σ^1_2 equivalence relation of "equiconstructibility" $x \mathrel{\mathrm{C}} y$ iff $\mathrm{L}[x] = \mathrm{L}[y]$ (x, y are reals) does not satisfy a reasonable form of the Glimm-Effros dichotomy theorem. More exactly, in the model the relation C:

(I) neither has a R-OD (real-ordinal definable) separating family (of any cardinality);

(II) nor admits a pairwise inequivalent uncountable R-OD set of reals (and then obviously does not embed E_0, the Vitali equivalence).

This shows that the dichotomy theorem proved by Harrington, Kechris, and Louveau (*Journal of the American Mathematical Society*, 1990, no. 4) for Borel relations and a similar theorem recently proved by the author for $\mathbf{\Sigma}^1_1$ relations (*Logic Eprints*, August 2, 1995) cannot be expanded to Σ^1_2 relations.

► PETER KOEPKE, *Gitik-Magidor-forcing from an elementary embedding*.
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A measurable cardinal can equally be characterized "combinatorially" using ultrafilters or "modeltheoretically" as critical point of an elementary embedding of the set theoretical universe V into a transitive subclass. This poses the question whether combinatorial constructions from measurable cardinals can be recast "modeltheoretically". We intend to demonstrate how this may be done for ordinary Prikry-forcing and its recent generalization by M. Gitik and M. Magidor where many Prikry-sequences are added simultaneously to violate the singular cardinals hypothesis in a generic extension. A sufficiently strong elementary embedding of V is directly used in the construction.

► PETER KOMJATH, *A graph decomposition property*.
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If CH holds and X is a graph on ω_1 with no independent subsets then X is the edge disjoint union of ω_1 such subgraphs. A complementary indepedence result also holds.

► S. SHELAH and LEE J. STANLEY, *Consistent negative partition relations for singular cardinals and the Erdös-Dushnik-Miller theorem*.
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We prove a number of consistency results about negative partition relations for certain singular cardinals. For example, consistently, $\aleph_{\omega_1} \not\to (\aleph_{\omega_1},\ \omega+1)^2$ and consistently, $2^{\aleph_0} \not\to (2^{\aleph_0}, \omega+1)^2$. This settles negatively a question of Erdös and Hajnal.

[The research of the first author was partially supported by the NSF and the Basic Research Fund, Israel Academy of Science. This is paper number 588 in the first author's list of publications. These theorems were proved in Spring 1995.]

► PETER VOJTÁŠ, *On the* (*Boolean*) *similarity of orderings of series, matrices and partitions*.
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For a divergent non-negative series $a \in c_0 \setminus \ell^1$ we denote

$$\mathcal{I}_a = \left\{ X \subseteq \omega : \sum_{n \in X} a_n < \infty \right\}.$$

$\mathcal{F}_a$ is the dual filter, considered as a (nowhere dense) closed subset of ω^*, the remainder of the Čech-Stone compactification of ω, and $\mathbf{F}_r = \left\{\mathcal{F}_a : a \in c_0 \setminus \ell^1\right\}$.

The separative factorisation of $(c_0 \setminus \ell^1, \leq^*)$ ordered by eventual dominance is isomorphic to $(\mathbf{F}_r, \subseteq)$. The boolean completion $RO(\mathbf{F}_r, \subseteq)$ is isomorphic to $RO(\omega^*)$ under $\mathbf{p} = cf(\mathbf{c})$, as we showed in our Ramat Gan 91 talk and at least under CH is $\mathcal{P}(\omega)/_{\mathcal{I}_a}$ isomorphic to $\mathcal{P}(\omega)/_{\text{fin}}$ (a late Erdös result). So there seems to be a lot of similarity between $(c_0 \setminus \ell^1, \leq^*)$ and $\mathcal{P}(\omega)/_{\text{fin}}$. We try to extend these witnesses. Moreover $\omega^* \setminus \bigcup \mathbf{F}_r$ are exactly rapid ultrafilters (semi Q-points).

The same pattern of similarities appears with an ordering of, and ideals connected to, Toepliz matrices (dual filters defining again rapid ultrafilters (joint result with W. Just)) and with an ordering of, and ideals connected to, partitions of ω into finite sets (dual filters defining Q-points). The P-point side behaves quite different.

[1] P. VOJTÁŠ, *Boolean isomorphism between partial orderings of convergent and divergent series and infinite subsets of* **N**, ***Proceedings of the American Mathematical Society***, vol. 117 (1993), pp. 235–242.

Abstracts of invited talks in the Special Session in Finite Model Theory

▶ JOHN T. BALDWIN, *On almost sure theories*.
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Let L^+ be a finite relational language. Let $(K_\alpha, \leq)$ be a class of finite L^+-structures with strong submodel determined by a dimension function depending on α (in the standard way). Consider random L^+ structures with "edge probability", $n^{-\alpha}$.

THEOREM. *T^α, the almost sure theory of random L^+-structures is the same as the theory T_α of the $\mathbf{K}_\alpha$-generic model. This theory is complete, stable, nearly model complete, and not finitely axiomatizable.*

▶ BRUNO COURCELLE, *Which sets of finite graphs have decidable monadic theories? Results and open problems*.
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Graphs can be represented by logical structures. These representations make it possible to express some of their properties by logical formulas. Monadic second-order logic is a language of particular interest in this respect because many graph properties (like k-colorability for fixed k, planarity, connectivity, Hamiltonicity) can be expressed with it, whereas several general decidability and complexity results hold for all graph properties that are expressible in this language.

A graph can be represented in several ways by a relational structure: as domain of the structure one can take either the set of vertices or the set consisting of vertices and edges, and in either cases, relations express adjacency and incidence. In the first case quantified variables can denote vertices or sets of vertices whereas in the second, they can also denote edges or sets of edges. Hence, we get two ways of expressing graph properties by monadic second-order formulas, depending on the chosen representation of graphs. We shall denote by MS2 the monadic second-order language using vertex and edge quantifications and by MS1 the less expressive one using only vertex quantifications. Some graph properties like the existence of a Hamiltonian cycle are expressible in MS2 but not in MS1.

QUESTION. Which classes of graphs have a decidable MS2-theory? Which classes of graphs have a decidable MS1-theory without having a decidable MS2-theory?

THEOREM 1 (Courcelle). *For each integer k, the* MS2-*theory of the class of finite graphs of tree-width at most k is decidable.*

The tree-width of a graph is an integer measuring how far it is from being a tree; trees have tree-width 1; tree-width is defined in terms of certain hierarchical decompositions of graphs called tree-decompositions introduced by Robertson and Seymour in their study of graph minors. The following result is some sort of converse.

THEOREM 2 (Seese). *If the* MS2-*theory of a class of finite graphs L is decidable, then the graphs in L have uniformly bounded tree-width.*

Is there an analogous result for MS1-decidable sets? Seese conjectured that if a set of graphs has a decidable MS1-theory then it is interpretable in a set of trees. We can prove:

THEOREM 3. *If a class of directed graphs is closed under arbitrary changes of edge directions and has a decidable* MS1-*theory, then its elements have uniformly bounded tree-width.*

We have a candidate "complexity measure" (analogous to tree-width and based on different tree-structurings of graphs) that is conjectured to characterize the classes of finite graphs having a decidable MS1-theory by a result similar to Theorem 2. We have proofs of special cases of this conjecture.The proofs of Theorems 2 and 3 use deep and difficult results on graph minors by Robertson and Seymour.

► ROD DOWNEY, *Fixed parameter tractability: Positive results*.
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A problem of the form

Input: $\sigma \in \Sigma^*$.
Parameter: k.
Question: Is $\langle \sigma, k \rangle \in L$?

is called fixed parameter tractable if there is a constant c independent of k and an algorithm A such that A decides if $\langle \sigma, k \rangle \in L$ in time $f(k)|\sigma|^c$. It turns out that while many problems are intractable if k is allowed to vary can be FPT. For instance, VETEX COVER is solvable in time linear in n for a fixed k yet is well known to be NP complete in practice. There are a number of algorithm design paradigms which have emerged which allow one to demonstrate parametric tractability. In this talk I hope to discuss some of these concentrating on some from logic. I may also discuss some parametric intractability results.

► ERICH GRÄDEL and ANDERS MALMSTRÖM, 0-1 *laws for recursive structures*.
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It is a well-known result due to Gaifman [1] that the set of all *extension axioms* of a finite relational vocabulary σ, has a unique countable model, called the *random σ-structure* $\mathcal{R}$, and that the isomorphism class of $\mathcal{R}$ has measure one (with respect to the Lebesgue measure) within the class of σ-structures with universe ω.

A relational structure of finite vocabulary is called *recursive* if its universe is countable and each relation is recursive. Denote the class of all recursive σ-structures by $\texttt{recstr}(\sigma)$. To study 0-1 laws on recursive structures one has to define a reasonable measure for this class; since, obviously, the recursive structures have measure 0.

We use the *resource-bounded measure theory* introduced by Lutz [2]. A non-standard, but convenient way to define the Lebesgue measure on $\{0, 1\}^\omega$ is in terms of betting strategies or, equivalently, *martingales*. A player repeatedly bets part of his current capital on the successive bits of an unknown string $\chi \in \{0,1\}^\omega$. If he correctly predicts the next bit he wins twice the money he has bet, otherwise he gets nothing. The player *succeeds* on χ if he has a strategy to become infinitely rich. A class $\mathcal{C} \subseteq \{0,1\}^\omega$ has Lebesgue measure 0 iff there exists a strategy to succeed on every string of $\mathcal{C}$. By imposing restrictions on the martingales describing the strategies one can define *resource-bounded* measures on small subsets of $\{0,1\}^\omega$. For instance, by restricting the player to recursive martingales a measure on the set of recursive strings is obtained. More restrictive complexity bounds on the resources used by the player give measures for different classes.

Since recursive σ-structures can be identified with recursive strings in $\{0,1\}^\omega$ we obtain a measure on $\mathtt{recstr}(\sigma)$, called the rec-measure.

We show that the isomorphism class of the random σ-structure has measure one also on the class of recursive structures with respect to the rec-measure. We prove this by exhibiting a recursive betting strategy which is successful on all recursive structures on which at least one extension axiom does not hold. As a consequence, every logic has a 0-1 law on the class of recursive structures.

These results can even be extended to countable structures where the relations satisfy certain complexity restrictions.

An alternative approach has been proposed by Hirst and Harel [3] who study asymptotic probabilities of logical sentences on sequences of recursive structures.

[1] H. Gaifman, *Concerning measures in first-order calculi*, ***Israel Journal of Mathematics***, vol. 2 (1964), pp. 1–18.

[2] J. Lutz, *Almost everywhere high nonuniform complexity*, ***Journal of Computer and System Sciences***, vol. 44 (1992), pp. 220–258.

[3] T. Hirst and D. Harel, *More about recursive structures: Descriptive complexity and zero-one laws*, ***Proceedings of the 11th IEEE Symposium on Logic in Computer Science***, 1996, pp. 334–348.

► MANFRED KUDLEK, *Distributed catenation*.
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A new operation on words and sets of words is introduced, called *distributed catenation*. It is defined by partitioning the alphabet V into 2 disjoint sets Γ and Δ. Any word $w \in V^*$ consists of blocks over Γ and Δ. By distributed catenation the corresponding blocks are catenated yielding some kind of primitive shuffle where Δ has some priority.

This operation is associative on words and set of words, distributive with union, and has a neutral element λ or $\{\lambda\}$, respectively. Furthermore, the semiring over 2^{V^*} with operations distributed catenation and union, and corresponding neutral elements, is ω-complete. Defining also iteration closure of distributed catenation, algebraic and rational systems of equations with distributed catenation as operation have minimal solutions as in the case of pure catenation.

In the paper the closure properties of the classes of these rational and algebraic languages with respect to the classical operations are investigated, as well as the closure of classical language families under distributed catenation, and the relation between all these language families.

Rewriting with distributed catenation is introduced too, yielding corresponding type-3 and context-free language families which are precisely the rational and algebraic ones under distributed catenation.

Also considered are decidability problems, where among others the emptiness and infinity of intersections of rational languages is undecidable.

Finally, a pushdown automaton using distributed catenation on its pushdown is introduced. It is shown that any Turing machine can be simulated by such an automaton.

Finally, some generalization of catenation, distributed catenation and related operations is given.

▶ ZOÉ LACROIX, *Some results on expressive power of first order logic on constraint databases*.
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Until recently, databases were considered to be finite collections of data items. New applications such as those involving temporal and spatial data (e.g., geographical databases) lead very naturally to more general data models allowing infinite collections of items to be stored in the database. This is possible under the assumption that the databases admit a finite representation. In their seminal paper, Kanellakis, Kuper and Revesz [4], introduced *constraint query languages*. The basic idea is to generalize the relations of the relational model by defining generalized tuples as conjunctions of constraints. For instance, $x \leq y \wedge x \leq 0$ defines a binary generalized tuple. A generalized, or finitely representable, relation is then a finite set of such tuples.

We present some results on First-Order definability and data complexity of First-Order queries over constraint databases [4, 1, 2, 3, 5].

[1] S. GRUMBACH AND J. SU, *Finitely representable databases*, ***13th ACM Symposium on Principles of Database Systems***, Minneapolis, May, 1994, pp. 289–300.

[2] S. GRUMBACH, J. SU, and C. TOLLU, *Linear constraint databases*, ***Logic and computational complexity workshop*** (D. Leivant, editor), Springer-Verlag, Indianapolis, 1994, to appear in ***Lecture Notes in Computer Science***.

[3] ———, *First-order Definability over Constraint Databases*, ***Proceedings of the First International Conference on Principles and Practice of Constraint Programming, Cassis, France***, September, 1995.

[4] P. KANELLAKIS, G KUPER, and P. REVESZ, *Constraint query languages*, ***Proceedings of the 9th ACM Symposium on Principles of Database Systems***, Nashville, 1990, pp. 299–313.

[5] J. PAREDAENS, J. VAN DEN BUSSCHE, and D. VAN GUCHT, *First-order queries on finite structures over the reals*, ***Proceedings of the IEEE Symposium on Logic In Computer Science***, 1995.

▶ SHMUEL LIFSCHES and SAHARON SHELAH, *Random graphs in the monadic theory of order*.
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Monadic logic is a fragment of second order logic that allows quantification over unary relations.

We prove that it is consistent that the theory of Random Graphs cannot be interpreted in the monadic theory of linear orders.

It was proven by Gurevich, Magidor, and Shelah that “second order logic is interpreted in the monadic theory of ω_2” is consistent with ZFC. On the other hand, Gurevich and Shelah showed that it follows from ZFC that second order logic is interpreted in the monadic theory of the real line but this is done using a non-standard interpretation (into a Boolean valued model).

The question whether there exists a standard interpretation of non-trivial theories in the monadic theory of order in every model of ZFC is answered negatively by this result.

▶ BERND LOESCHER and AVY SHARELL, *Functions vs. relations on finite structures: A finer strict hierarchy in existential second order logic*.
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Let $\mathbf{\Sigma}_1^1$ be the existential fragment of second order logic, and for every $k \in \omega$ let $\mathbf{\Sigma}_{1,k}^1$ be its fragment with the maximal arity of the quantified relation variables restricted to k. We will use these symbols also for the classes of properties definable by the respective logical calculi over finite structures. In this sense, $\mathbf{NP} = \mathbf{\Sigma}_1^1$ ([4]).

Ajtai showed in [1] that $\mathbf{\Sigma}_{1,k}^1 \subset \mathbf{\Sigma}_{1,k+1}^1$ for all $k \in \omega$, so there is an infinite strict hierarchy inside $\mathbf{\Sigma}_1^1$. This gives rise to the problem to determine the expressive power of quantified function variables of a given arity.

For every $k \in \omega$ let $\mathbf{F\Sigma}_{1,k}^1$ be the fragment of existential second order logic where we quantify over function variables with arity at most k. Obviously, we have $\mathbf{\Sigma}_{1,k}^1 \subseteq \mathbf{F\Sigma}_{1,k}^1 \subseteq \mathbf{\Sigma}_{1,k+1}^1$ and by [1], for all $k \in \omega$ at least one of the two inclusions has to be strict.

We show both of them to be strict, as one would have expected.

Ajtai has shown that the parity of $k+1$-ary relation (i.e., the class of all $k+1$-ary relations over finite structures with an even number of $k+1$-tuples) is definable in $\mathbf{\Sigma}_{1,k+1}^1$, but not in $\mathbf{\Sigma}_{1,k}^1$. His proof can easily be strengthened to obtain the non-definability of this property even in $\mathbf{F\Sigma}_{1,k}^1$, so we have separated $\mathbf{F\Sigma}_{1,k}^1$ from $\mathbf{\Sigma}_{1,k+1}^1$.

To separate $\mathbf{\Sigma}_{1,k}^1$ from $\mathbf{F\Sigma}_{1,k}^1$, we consider the property $\mathbf{CONN}(2k)$ of a $2k$-ary relation P (essentially defined as connectivity of P seen as a graph over k-tuples).

$\mathbf{CONN}(2k)$ is in $\mathbf{F\Sigma}_{1,k}^1$ because one can define a spanning tree over the associated graph by using only existentially quantified k-ary function variables (this is a generalization of a result of Grandjean in [5]); and a graph is connected iff there is a spanning tree.

To prove that $\mathbf{CONN}(2k)$ is not in $\mathbf{\Sigma}_{1,k}^1$, we reduce graph connectivity with a built-in relation (called *k-representer relation*) that encodes elements of the universe as k-tuples to $\mathbf{CONN}(2k)$. Therefore it is sufficient to show that graph connectivity is not in $\mathbf{\Sigma}_{1,1}^1$ over the class of graphs with built-in k-representer relation. We accomplish this by playing the Ajtai-Fagin version ([2]) of generalized *Ehrenfeucht-Fraïssé* games ([3]) on this class.

Note that our k-representer relation is of maximal degree $n^{1-1/k}$, so the needed result is not provided by Schwentick's theorem ([6]) that graph connectivity is not in $\mathbf{\Sigma}_{1,1}^1$ in the presence of arbitrary built-in relations of degree at most $n^{o(1)}$.

[1] M. AJTAI, Σ_1^1*-formulae on finite structures*, ***Annals of Pure and Applied Logic***, vol. 24 (1983), pp. 1–48.

[2] M. AJTAI and R. FAGIN, *Reachability is harder for directed than for undirected finite graphs*, ***Journal of Symbolic Logic***, vol. 55 (1990), pp. 113–150.

[3] A. EHRENFEUCHT, *An application of games to the completeness problem of formalized theories*, ***Fundamenta Mathematicae***, vol. 49 (1961), pp. 129–141.

[4] R. FAGIN, *Generalized first-order spectra and polynomial-time recognizable sets*, ***Complexity of Computation*** (R. M. Karp, editor), SIAM-AMS Proceedings 7, 1974, pp. 43–73.

[5] E. GRANDJEAN, *First-order spectra with one variable*, ***Journal of Computer and System Sciences***, vol. 40 (1990), pp. 136–153.

[6] T. SCHWENTICK, *Graph connectivity and monadic* **NP**, ***Proceedings of the 35th IEEE Symposium on Foundations of Computer Science***, to appear.

▶ JUHA NURMONEN, *On winning strategies with unary quantifiers*.
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A combinatorial argument for two finite relational structures to agree on all sentences with bounded quantifier rank in first-order logic with unary generalized quantifiers $\mathrm{FO}(\mathcal{Q}_u)$, is given. The argument is based on the work of Fagin, Stockmeyer and Vardi ([1]), who studied d-equivalence of finite structures. The *d-type* of a point a is the isomorphism type of the neighborhood of radius d about a. Two structures are *d-equivalent*, if for each d-type there are as many points in both structures with that d-type. The combinatorial argument goes as follows.

Let r be a positive integer. There is a positive integer d, such that whenever $\mathbf{A}$ and $\mathbf{B}$ are d-equivalent structures, then $\mathbf{A} \equiv^r_{\mathrm{FO}(\mathcal{Q}_u)} \mathbf{B}$.

It is known that connectivity of finite graphs is neither in monadic existential second-order logic $\mathrm{Mon}\,\Sigma^1_1$ (even with built- in relations of moderate degree, [1]) nor in $\mathrm{FO}(\mathcal{Q}_u)$ ([2]). We consider connectivity in the logic $\mathrm{Mon}\,\Sigma^1_1(\mathcal{Q}_u)$, which is obtained, when the first-order part of $\mathrm{Mon}\,\Sigma^1_1$ is replaced by $\mathrm{FO}(\mathcal{Q}_u)$. The first-order part of the proof in [1] was based on counting d-types. Imitating that proof, with the argument above we prove that connectivity of finite structures in not even in $\mathrm{Mon}\,\Sigma^1_1(\mathcal{Q}_u)$ (even if built-in relations of moderate degree are allowed).

With the argument above, we give easy proofs for non-definability in $\mathrm{FO}(\mathcal{Q}_u)$ for the class of rigid graphs and the class of connected graphs. Classes defined by forbidden minors are also considered. The class of planar graphs is perhaps the best known example of such a class. By Kuratowski's theorem, a graph is planar if and only if it does not have $\mathbf{K}_5$ and $\mathbf{K}_{3,3}$ as a minor. We show that if the forbidden minors for a class are not in some sense trivial, that class is not definable in $\mathrm{FO}(\mathcal{Q}_u)$. Especially this holds for the class of planar graphs and the class of acyclic graphs.

[1] R. Fagin, L. Stockmeyer and M. Vardi, *On monadic NP vs. monadic co-NP*, ***Information and Computation***, vol. 120, no. 1 (1995), pp. 78–92.

[2] L. Hella and G. Sandu, *Partially ordered connectives and finite graphs*, ***Quantifiers: Logics, models and computation, volume II*** (M. Krynicki, M. Mostowski and L. Szczerba, editors), Kluwer Academic Publishers, Dordrecht, 1995, pp. 79–88.

▶ MARTIN OTTO, *Bounded variable logics*: 2, 3*, and more*.
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We consider infinitary logic with k variable symbols $L^k_{\infty\omega}$ and the richer fragments $C^k_{\infty\omega}$ that allow the use of all counting quantifiers $\exists^{\geq m}$ rather than just $\exists$. These logics provide an adequate logical framework for some important issues in finite model theory. In particular $L^\omega_{\infty\omega}$, the union of the $L^k_{\infty\omega}$, comprises the expressive power of the well known fixed-point logics FP and PFP. Similarly $C^\omega_{\infty\omega}$, the union of the $C^k_{\infty\omega}$, includes the extensions of these fixed-point logics that give recursive access to cardinalities of definable sets, $\mathrm{FP}+C$ and $\mathrm{PFP}+C$.

It is known from the work of Cai, Fürer and Immerman that even $C^\omega_{\infty\omega}$ fails to comprise all of Ptime in the general case of not necessarily ordered structures. Interestingly , the known examples for the separation of $\mathrm{FP}+C$ from Ptime are in fact separations even of $\mathrm{Ptime} \cap C^\omega_{\infty\omega}$ from Ptime. This is one motivation to study problems related to the question *whether there is a logical characterization of* Ptime in restriction to the non-uniform logical framework provided by the bounded variable fragments $L^k_{\infty\omega}$ and $C^k_{\infty\omega}$ of $L_{\infty\omega}$. These restrictions remain non-trivial since already $L^2_{\infty\omega}$ suffices to define arbitrarily complex and even non-recursive properties of finite graphs.

The problem of presenting Ptime on not necessarily ordered structures is due to the semantic condition on Ptime algorithms that their outcome be invariant with respect to isomorphism: An algorithm for instance that recognizes a graph property must not accept one representation of a finite graph and reject a representation of an isomorphic copy of that same graph. The restriction to the world of $L^k_{\infty\omega}$, say , corresponds to imposing the same invariance condition with respect to $L^k_{\infty\omega}$ -equivalence rather than isomorphism. This approach promises to be more tractable because the relations of $L^k_{\infty\omega}$- and $C^k_{\infty\omega}$ -equivalence can be analysed in Ehrenfeucht-Fraïssé style through related pebble games. Corresponding complete invariants that characterize finite structures up to $L^k_{\infty\omega}$- and $C^k_{\infty\omega}$ -equivalence can be abstracted from the games and yield complexity oriented characterizations of the expressive power of FP and PFP or FP $+C$ and PFP $+C$ in the general case—the fundamental concept of such invariants (in a computational framework) is due to Abiteboul and Vianu and has led to great success in their work on FP and PFP.

Recent results explore the possibilities to invert the passage from structures to their invariants in order to obtain PTIME computable *canonization procedures* up to $L^k_{\infty\omega}$- or $C^k_{\infty\omega}$-equivalence. Positive results here imply the recursive presentability together with a natural logical characterization for the corresponding fragments PTIME $\cap L^k_{\infty\omega}$ or PTIME $\cap C^k_{\infty\omega}$. For *two variables* this programme has been carried out successfully. Furthermore there is a reduction procedure that relates the essential requirements for the case of an arbitrary number of variables to the case of just *three variables*! For instance, if $C^3_{\infty\omega}$ admits canonization through PTIME inversion of the three-dimensional game invariant, then FP $+C$ captures PTIME $\cap C^\omega_{\infty\omega}$. The underlying reduction scheme is applicable to several other model theoretic questions as well.

▶ ERIC ROSEN, *Preservation theorems in finite model theory*.
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It is well known that most classical preservation theorems fail over the class of finite structures. We discuss the prospects for finding positive results in this direction by considering a generalized notion of preservation theorem applicable to various logics that have received attention in finite model theory. We focus our attention on classes of models that are closed under extensions, and the existential fragments of finite variable and fixed point logic. The following negative result is proved.

PROPOSITION 1. *There is a sentence $\phi \in L^\omega_{\infty\omega}$ such that* $\mathrm{Mod}(\phi)$ *is closed under extensions and for all $\theta \in L^\omega_{\infty\omega}(\exists)$, ϕ is not equivalent to θ, even over the class of finite structures.*

Certain partial positive results are also established, using modified Ehrenfeucht- Fraïssé games and related techniques.

▶ ALEXEI P. STOLBOUSHKIN AND MICHAEL A. TAITSLIN, *Normalizable linear orders*.
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A theory T_1 of a finite signature $\Omega_1 = \langle P^1, \dots, P^m\rangle$ is interpreted in a theory T_2 of a finite signature Ω_2 if the following properties hold:

(i) There are formulas $\Psi_1, \dots, \Psi_m, \Psi$ of the signature Ω_2 such that Ψ has the only free variable, and for any finite model A of T_2, the set $\{a \mid (A, a) \models \Psi\}$ together with the relations given by the formulas $\Psi_1, \dots, \Psi_m$ in A makes up a model of T_1 denoted by $\pi(A)$ and called the projection of A.

(ii) For any finite model B of T_1 there is a finite model A of T_2 such that B is isomorphic to the projection $\pi(A)$ of A.

(iii) For each finite model A of T_2, each isomorphism of the projection $\pi(A)$ of A can be extended to an isomorphism of A.

A theory of a finite signature is called a rigid theory if each of its finite model is rigid.

THEOREM 1. *A finite axiomatizable theory is rigid iff it can be interpreted in a finite axiomatizable theory with a first-order definable linear order.*

We are trying to capture cases when every imposed linear order can be *normalized* to the same resulting linear order.

CONJECTURE 2. *In any finitely axiomatizable* (*in* FO) *class of rigid models linear order is* LFP-*normalizable.*

In the positive direction, we suggest a series of reductions of the conjuction to specialized classes of graphs.

[This work was carried out while the second author was visiting UCLA and supported in part by NSF Grant CCR 9403809.]

Abstracts of invited talks in the Special Session in Recursion Theory

▶ M. M. ARSLANOV, *Relative recursive enumerability and the difference hierarchy*.
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In this paper we study the following question (any degree we deal here is a degree below $\mathbf{0}'$): Suppose $\mathbf{a}$ r.e., $\mathbf{a} < \mathbf{b}$ and $\mathbf{b}$ is r.e. in $\mathbf{a}$ ($\mathbf{b}$ is REA($\mathbf{a}$)). What happens if $\mathbf{b}$ is α-r.e. for some $\alpha \leq \omega$?

First at all let us note that by a theorem of Jockusch and Shore (1984) there is a 2-REA degree $\mathbf{a}$ which is not ω- r.e.

The other result in the opposite direction is the following theorem.

THEOREM 1 (Arslanov, Lempp and Shore, 1995a). *There exists a r.e. degree* $\mathbf{a} < \mathbf{0}'$ *such that any* REA($\mathbf{a}$) *degree* $\mathbf{b} < \mathbf{0}'$ *is r.e.*

This means that there is a r.e. degree $\mathbf{a} < \mathbf{0}'$ such that

$$\mathrm{REA}(\mathbf{a}) = \Re(\geq \mathbf{a})(= \mathbf{D}_1(\geq \mathbf{a}).$$

Considering the question is there a r.e. degree $\mathbf{a} < \mathbf{0}'$ such that $\mathrm{REA}(\mathbf{a}) = \mathbf{D}_n(\geq \mathbf{a})$ for $n > 1$, we prove the following

THEOREM 2. *There exist r.e. degrees* $\mathbf{a} < \mathbf{0}'$ *and* $\mathbf{b} < \mathbf{0}'$ *such that any d-r.e. degree* $\mathbf{d} \geq \mathbf{a}$ *is r.e. in* $\mathbf{b}$.

Therefore, for some r.e. $\mathbf{a}, \mathbf{b} < \mathbf{0}'$

$$\mathbf{D}(\geq \mathbf{a}) \bigcap \mathrm{REA}(\mathbf{b}) \supseteq \mathbf{D}_2(\geq \mathbf{a}).$$

In the following theorem we proved that we cannot replace here $\mathbf{b}$ by $\mathbf{a}$:

THEOREM 3 (Arslanov, Lempp and Shore, 1995b). *Given r.e. degrees* $\mathbf{v} < \mathbf{u}$ *there is a d-r.e. degree* $\mathbf{d}$ *such that* $\mathbf{v} < \mathbf{d} < \mathbf{u}$ *and* $\mathbf{d}$ *is not r.e. in* $\mathbf{v}$.

But, despite the congestion of negative results the following proposal is true:

$$2\text{-}\,\mathrm{REA}\bigcap \mathbf{D}_{\omega} = \mathbf{D}_2.$$

The direction $\supseteq$ is obvious: as A. H. Lachlan proved any d-r.e. set is 2-REA. For the opposite direction we prove the following

THEOREM 4. *Let M and Q be sets such that M is ω-r.e., Q is r.e. in some r.e. set $C <_T Q$, and $Q \equiv_T M$. Then there is a r.e. in C d-r.e. set T such that $T \equiv_T M$.*

COROLLARY 1. *For any* 2-REA *degree* **a** *if* **a** *is ω-r.e. then* **a** *is d-r.e.*

► P. CHOLAK and R. DOWNEY, *Automorphic recursively enumerable sets*.
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Currently there is a lot of research being done to explore the relationship between definability, undefinability and automorphisms within the structure $\mathcal{E}$: the recursively enumerable sets with the subset relation. We will focus on automorphisms and show:

THEOREM 1. *There are two recursively enumerable sets which are Δ_4-automorphic but not Δ_3-automorphic.*

This is the first known example of such a pair of recursively enumerable sets. Hopefully, with some further exploration, this result will help answer the question of whether $\mathcal{E}$ is a prime model or the Slaman-Woodin Conjecture holds (the more likely of the two).

CONJECTURE 2 (Slaman-Woodin). *The set*

$$\{\langle e_0, e_1\rangle : W_{e_0} \textit{ automorphic to } W_{e_1} (\textit{ in } \mathcal{E})\}$$

is Π^1_1-complete.

[Research partially supported by a Mathematical Sciences Postdoctoral Research Fellowship (Cholak), a U.S.-New Zealand Binational Cooperative Research grant NSF INT 90-20558 (Cholak and Downey) and IGC of Victoria University, Wellington (Downey).]

► C. T. CHONG, *Degree of a Σ_n cut*.
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We report on the recent joint work with Y. Yang on models of fragments of Peano arithmetic. In any model of Σ_2 collection without Σ_2 induction, a Σ_2 definable cut I enjoys properties which are useful to the study of the degree theory of RE sets. Indeed the jump of a hyperregular RE set is recursive in $X = I \oplus \emptyset'$. This leads to the theorem that the existence of an incomplete high RE set, and the Sacks inversion Theorem, is each equivalent to Σ_2 induction over the base theory of Σ_2 collection. An analysis of the degree of X also yields results on the existence/nonexistence of low_2 RE sets. Obvious generalization to all $n \geq 2$ follow.

► PETER A. FEJER, *Decision procedures for distributive upper semi-lattices of degree structures*.
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We formulate general criteria that allow one to conclude that a distributive upper semilattice has a decidable two-quantifier theory. These criteria are applied to the weak truth-table degrees of the recursively enumerable sets and also to various substructures of the polynomial many-one (pm) degrees of the recursive sets. These applications to the pm degrees require no new complexity-theoretic results. The fact that the pm-degrees of the recursive sets have a decidable two-quantifier theory answers a question raised by Shore and Slaman. (This is joint work with Ambos-Spies, Lempp and Lerman.)

► S. S. GONCHAROV, *Morley's problem*.
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The theory of decidable models is connected with the study of algorithmic properties of models theoretical-model constructions. The basic problem is the existence of decidable and recursive models with different properties. One of the main results is Ershov-Vaught theorem about existence decidable models for decidable theories. M. Peretyatkin proved the decidability of prime models for decidable Ehrenfeucht theories. A. Lachlan , M. Peretyatkin and T. Millar constructed examples of decidable Ehrenfeucht theories with non-decidable models. S. Goncharov proved that any countable homogeneous model of decidable Ehrenfeucht theory with only recursive types is decidable.

M. Morley asked the question about decidability for countable models of decidable Ehrenfeucht theories with recursive types.

THEOREM 1. *There exists the decidable Ehrenfeucht theory with recursive types with non-decidable countable model.*

[1] M. MORLEY, *Decidable models*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 233–240.

► MARCIA GROSZEK, Π^0_1-*classes and minimal degrees*.
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We answer a question of Harvey Friedman and Stephen Simpson, by showing: There is an infinite binary recursive tree for which every infinite path computes a real of minimal Turing degree. This shows that the subsystem of second-order arithmetic WKL suffices to prove the existence of a minimal Turing degree.

[Joint work with Theodore Slaman.]

► VALENTINA HARIZANOV, *Frequency computable sets, their classes, and generalizations*.
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Unlike standard algorithms, whose answers must always be correct, approximating algorithms may give some erroneous answers. Frequency computations are approximating algorithms for which the maximum fraction of erroneous answers is known *a priori*. More precisely, a set $A \subseteq \omega$ is (m, n)-computable, $m, n \in \omega$ and $1 \leq m \leq n$, if there is an algorithm, called a frequency computation for A, which for given n distinct natural numbers outputs guesses for their membership to A and guarantees that at least m of these guesses are correct. We will present a variety of old and recent results on frequency computations, including their relationship to the cardinality theorem, recursively enumerable frequency computable sets, and joins of frequency computable sets.

The set of all (m, n)-computable sets forms a frequency class. Such a frequency class coincides with the class of all recursive sets exactly when $n - m < m$. All other frequency classes are pairwise distinct. We will present all known major results on the structure of frequency classes with respect to inclusion, including recent decision procedures for establishing whether a given frequency class is contained in another given frequency class. We will also show how the covering problem in coding theory can be used to obtain specific noninclusion relations for frequency classes.

Finally, we will discuss other classes of sets obtained by similar approximating algorithms, such as classes of weakly frequency computable sets, frequency classes with a single operator, classes of (strongly) verbose sets, classes of (weakly) selective sets, and effective search classes. We will discuss generalizing these classes and developing methods which can be used to generalize known results.

[1] V. Harizanov, M. Kummer and J. Owings, *Frequency computations and the cardinality theorem*, ***Journal of Symbolic Logic***, vol. 57 (1992), pp. 682–687.

[2] V. Harizanov, *Frequency computable sets and their classes*, preprint.

[3] M. Kummer and F. Stephen, *Some aspects of frequency computation*, ***Technical Report* 21/91**, Fakultät für Informatik, Universität Karlsruhe, 1991.

[4] ———, *The power of frequency computation*, preprint.

[5] T. McNicholl, *The inclusion problem for generalized frequency classes*, ***Ph.D. Dissertation***, The George Washington University, 1995.

► E. HERRMANN, *Boolean pairs formed by the Δ_n^0 sets*.
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We consider Boolean pairs of the form $(\Delta_m^0, \Delta_n^0)^\star$ for natural numbers $m, n \geq 1$ and $m > n$ which we get by the class Δ_m^0 together with the set operations and with a unary predicate satisfied exactly for all sets in Δ_m^0 which even are from Δ_n^0 and these all factorized by finite differences between the sets (symbolized above with the asterisk).

It will be shown that, nevertheless every Boolean algebra $\Delta_n^{0\star}$ obviously is the countable atomless algebra, hence a quite easy structure, all Boolean pairs $(\Delta_m^0, \Delta_n^0)^\star_{m>n\geq 1}$ are such complicated that their elementary theories are undecidable. This shows that the Δ_n^0 sets lay very inhomogeneously inside the class Δ_m^0 for $m > n$.

We get this by proving that in the pair $(\Delta_m^0, \Delta_n^0)^\star_{m>n\geq 1}$ the lattice $\mathcal{E}^{m+1}$ (i.e., the lattice of Σ_{m+1}^0 sets under inclusion) is elementary definable with two parameters. But the lattices $\mathcal{E}^m$, $m \geq 1$ have all a hereditarily undecidable theory and thus also the pairs $(\Delta_m^0, \Delta_n^0)^\star_{m>n\geq 1}$. This, in particular, gives that the elementary theory of every one of these Boolean pairs is undecidable.

Connected with this are the following two still open questions:

(1) Having in mind the recently shown fact by Harrington and Nies that all lattices $\mathcal{E}^n$, $n \geq 1$ have different elementary theories each to the other, nevertheless the proof of the undecidability is uniformly for all pairs, are the whole elementary theories of the pairs $(\Delta_m^0, \Delta_n^0)^\star_{m>n\geq 1}$ different for different numbers m and n?

(2) If we define Δ_0^0 to be equal to the class of all primitive recursive sets we get further Boolean pairs $(\Delta_m^0, \Delta_0^0)^\star_{m\geq 1}$. What can be said about the elementary theory of these pairs respectively to the point of view of undecidability ?

▶ JULIA F. KNIGHT, *The Ash-Nerode Theorem and the Friedberg-Muchnik theorem*.
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Let $\mathfrak{A}$ be a recursive structure, and let R be a recursive region on $\mathfrak{A}$. Ash and Nerode [3] gave syntactical conditions under which there is an isomorphism f from $\mathfrak{A}$ onto recursive $\mathfrak{B}$ such that $f(R)$ is not r.e. The Ash-Nerode result has led to a number of other results. Baker [4] gave conditions under which $f(R)$ can be made not Σ^0_α, where α is an arbitrary recursive ordinal. There are results of Ash-Knight [2] saying when $f(R)$ can be made not d-r.e., or not α-r.e. relative to some particular notation for α. There are results of Harizanov [5] saying when $f(R)$ can be given arbitrary r.e. degree. There is a result of Harizanov [5] and Ash-Cholak-Knight [1] saying when $f(R)$ can be given arbitrary Turing degree.

The following problem is related to Friedberg-Muchnik Theorem. Let $\mathfrak{A}$ be as above, and let R and S be recursive relations on $\mathfrak{A}$. Give conditions under which there is an isomorphism f onto a recursive $\mathfrak{B}$ such that $f(R)$ and $f(S)$ are r.e. and independent (neither is recursive in the other). We could vary the problem, asking for conditions under which $f(R)$ and $f(S)$ can be made Σ^0_α and independent over Δ^0_α. There are natural results, using the methods developed for the results above. We could vary the problem further, considering uniformly recursive families of relations $R_{\beta_{\beta<\alpha}}$ and $R_{\beta_{1<=\beta<\alpha}}$, and asking for conditions under which $f(R_\beta)$ and $f(S_\beta)$ can be made Σ^0_β (uniformly in β) and independent over Δ^0_β. Here, even for $\alpha = 2$, the old methods are insufficient. Michalski [4] has some results.

[1] C. J. ASH, P. CHOLAK and J. F. KNIGHT, *Permitting, forcing and copies of a given relation*, to appear in ***Annals of Pure and Applied Logic***.

[2] C. J. ASH and J. F. KNIGHT, *Recursive structures and Ershov's hierarchy*, ***Mathematical Logic Quarterly***, vol. 42 (1996), pp. 461–468.

[3] C. J. ASH and A. NERODE, *Intrinsically recursive relations*, ***Aspects of effective algebra*** (J. N. Croosley, editor), Upside Down A Book Co., Steel's Creek, Australia, 1981, pp. 26–41.

[4] E. BAKER, *Intrinsically Σ^0_α relations*, ***Annals of Pure and Applied Logic***, vol. 39 (1988), pp. 105–130.

[5] V. HARIZANOV, *Some effects of Ash-Nerode and other decidability conditions on degree spectra*, ***Annals of Pure and Applied Logic***, vol. 55 (1991), pp. 51–65.

[6] G. MICHALSKI, ***Ph.D. Dissertation***, University of Notre Dame, 1995.

▶ MARTIN KUMMER, *Kolmogorov complexity and instance complexity of recursively enumerable set*.
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We study in which way Kolmogorov complexity and instance complexity affect properties of r.e. sets. The well-known $2 \log n$ upper bound on the Kolmogorov complexity of initial segments of r.e. sets is shown to be optimal and the Turing degrees of r.e. sets which attain this bound are characterized. The main part is concerned with instance complexity introduced by Ko, Orponen, Schoening, and Watanabe (1986) as a measure of the complexity of individual instances of a decision problem. They conjectured that for every r.e. nonrecursive set the instance complexity is infinitely often at least as high as the Kolmogorov complexity. The conjecture is refuted by constructing an r.e. nonrecursive set with instance complexity logarithmic in the Kolmogorov complexity. This bound is optimal up to an additive constant. In the other extreme, the conjecture is established for many classes of complete sets, like wtt-complete sets and Q-complete sets. However, there is a Turing complete set for which it fails.

► MANUEL LERMAN, *An effective condition for embedding lattices without critical triples*.
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Ambos-Spies and Lerman have introduced both necessary conditions and sufficient conditions for embedding finite lattices into the recursively enumerable degrees. These conditions are not known to imply each other, nor has an example been found which satisfies neither condition. The necessary condition is an effective simple lattice-theoretic condition, but the sufficient condition is complicated and not effective as stated. We introduce a new effective condition, and show that every finite lattice which fails to embed critical triples but satisfies this condition can be embedded into the recursively enumerable degrees. This condition was obtained through the study of several examples, and it is hoped that this approach will lead to a necessary and sufficient condition.

Abstracts of invited talks in the Special Session in Proof Theory

► ARNON AVRON, *Gentzen-type systems, resolution and tableaux*.
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We show that both the tableaux and the resolution methods can be understood as attempts to exploit the power of cut-elimination theorems in Gentzen-type calculi. Another, related goal is to provide a purely syntactic basis for both methods (in contrast to the semantic proofs concerning resolution that can be found in the textbooks). This allows the use of a fruitful combination of the methods and might be helpful in generalizing them to other logics.

► L. BEKLEMISHEV, *On the expressive power of induction schemes*.
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A well-known result due, essentially, to D. Leivant states that, over basic Kalmar elementary arithmetic EA, the induction schema for Σ_n formulas is precisely equivalent to the uniform reflection principle for Σ_{n+1} formulas. We show that fragments of arithmetic axiomatized by various forms of induction *rules* admit a precise axiomatization in terms of reflection principles as well. Thus, the closure of EA under the induction rule for Σ_n (or Π_{n+1}) formulas is equivalent to ω times iterated Σ_n reflection principle. Moreover, for $k < \omega$, k times iterated Σ_n Reflection Principle precisely corresponds to the extension of EA by $\leq k$ nested applications of Σ_n Induction Rule.

The above relationship holds in greater generality than just stated. In fact, one can give general formulas characterizing in terms of iterated reflection principles the extension of a given theory (containing EA) by $\leq k$ nested applications of Σ_n or Π_n induction rules. For example, the closure of a theory T under just one application of Σ_1 induction rule is equivalent to T together with Σ_1-reflection principle for each finite Π_2 axiomatized subtheory of T.

These results have closely parallel ones in the theory of subrecursive hierarchies, which was popular in the 60's. The analogy can be roughly formulated as follows: counting applications of Σ_1 induction rule in a proof = counting operators of primitive recursion in a primitive recursive program.

For example, the above characterization of Σ_1 induction rule implies that, over a class of functions containing elementary ones, one application of *nested* recursion on ω can be reduced to one application of primitive recursion and composition. This statement is a slightly refined version of an old result of R. Peter.

▶ GIANLUIGI BELLIN, *The planar structure of proofs: Algorithmic and topological properties*.
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Girard's representation of proofs in linear logic by proof-nets [1986–1995] uses properties of abstract graphs as acyclicity, connectedness, etc. We show that if a proof-net is "planar", then the verification of its correctness is achieved in "linear" time; on the contrary, experience shows that in the case of abstract graphs the test of correctness cannot be done in general in less than quadratic time.

This algorithmic consideration is one motivation of the project of studying the planar structure of proofs, regarded as 2-dimensional CW-complexes; an explicit rule of Exchange is used to express the commutativity of the connectives. We present a new system of proof-nets for multiplicative linear logic with the rule of MIX. We compare it with existing characterizations, in particular, with Asperti's criterion [1993], yielding an instructive interpretation of logical deductions as concurrent computations.

[Joint work with Arnaud Fleury and Jacques van de Wiele.]

▶ V. DANOS, *Embeddings of classical logic into linear logic*.
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Joint work with J.-B. Joinet and H. Schellinx.

Extension to classical logic of Curry-Howard's isomorphism requires a suitable classical calculus, equipped with a deterministic noetherian normalization, a denotational semantics, and, as much as possible, robust w.r.t. linear boolean equivalences.

Several solutions were proposed: Barbanera-Berardi, Girard, Parigot, etc., which we classify by means of embeddings in linear logic. New suitable calculi are found as well, but whether they correspond to interesting programming languages remains to be seen.

▶ W. DEKKERS, M. BUNDER and H. BARENDREGT, *Completeness of two systems of illative combinatory logic for propositional and predicate calculus*.
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The theory of combinators and the lambda calculus are theories that succesfully analyze the notion of effective computability. However, the original founders of these subjects, Curry and Church, also had as aim to provide a basis for logic. Formal systems intended to achieve this were developed by Church and Curry in the 1930s. It was shown by Kleene and Rosser that these systems are inconsistent. Curry and his school then started a program of defining several systems of illative combinatory logic, ICL's, of varying strength. The goal was to 'find stronger and stronger systems which are consistent and weaker and weaker systems which are inconsistent but strong enough to interpret logic, hoping to end up with a consistent system in which logic can be interpreted'.

In [1] we defined four systems of illative combinatory logic IP, Iξ, IF and IG. We interpreted first-order propositional and predicate calculus into illative systems in two ways: following the propositions-as-types paradigm, in which derivations become combinators, and in a more direct way, in which derivations are not translated. All translations turned out to be sound. The two direct translations from PROP into IP and from PRED into Iξ turned out to be complete. We conjectured completeness for the two indirect interpretations from PROP into IF and from PRED into IG. This conjecture will be proved in the present paper.

[1] H. BARENDREGT, M. BUNDER and W. DEKKERS, *Systems of illative combinatory logic complete for first-order propositional and predicate calculus*, ***Journal of Symbolic Logic***, vol. 58 (1993), pp. 769–788.

► ULRICH KOHLENBACH, *Real growth in standard parts of analysis*.
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To measure the growth of uniform bounds which are extractable from proofs in various parts of analysis we introduce a hierarchy $(\mathrm{G}_n\mathrm{A}^\omega)_{n\in\mathbf{N}}$ of subsystems of arithmetic in all finite types whose definable functions correspond to the Grzegorczyk hierarchy. Many basic notions of the analysis of continuous functions can be introduced already in $\mathrm{G}_2\mathrm{A}^\omega$ while some (as e.g., the unrestricted exponential function exp) are available only in $\mathrm{G}_3\mathrm{A}^\omega$. Hence we consider the impact of higher analytical principles on the growth of bounds relatively to $\mathrm{G}_2\mathrm{A}^\omega$ and $\mathrm{G}_3\mathrm{A}^\omega$. For a certain set Γ of analytical axioms which cover the main parts of classical analysis of continuous functions (including e.g., Brouwer's fixed point theorem, the Cauchy-Peano theorem, Dini's theorem and many others) we obtain the following result: Let $A_0(u^1, k^0, v^\tau, w^0)$ denote a quantifier–free formula of $\mathrm{G}_n\mathrm{A}^\omega$ containing only u, k, v, w as free variables, where τ is an arbitrary type, t is a closed term of $\mathrm{G}_n\mathrm{A}^\omega$ and $\leq_\tau$ is defined pointwise. AC–qf denotes the axiom schema of quantifier-free choice. Then for $n \geq 2$ the following rule holds:

$$\begin{cases} \text{From a proof } \mathrm{G}_n\mathrm{A}^\omega\text{+AC–qf } + \Gamma \vdash \forall u^1, k^0 \forall v \leq_\tau tuk \exists w^0 A_0 \\ \text{one can extract a closed term } \Phi \text{ of } \mathrm{G}_n\mathrm{A}^\omega \text{ such that} \\ \forall u^1, k^0 \forall v \leq_\tau tuk \exists w \leq_0 \Phi uk\, A_0 \\ \text{is true in the full set-theoretic type structure.} \end{cases}$$

For $n = 2$ (resp. $n = 3$) Φuk is a polynomial (an elementary recursive function) in $u^M := \lambda x^0.\max(u0, \dots, ux)$ and k, where tf^1x^0 is called a polynomial (an elementary recursive function) in f, x if $\forall f, x(tfx =_0 \hat{t}[f, x])$ for some term $\hat{t}[f, x]$ which is built up from $0, x^0, f^1, S^1, +, \cdot$ (resp. $0, x^0, f^1, S^1, +, \cdot, \lambda x, y.x^y$) only. (Note that the bound Φuk does not depend on v.)

Furthermore additional use of **single sequences of instances** (which may depend on u, k, v) of the convergence of bounded monotone sequences in $\mathbf{R}$, the existence of a greatest lower bound for sequences in $\mathbf{R}_+$, the Bolzano-Weierstraß property for bounded sequences in $\mathbf{R}^d$ and the Arzelà-Ascoli lemma contributes to the bound Φ at most by a certain primitive recursive functional (in the sense of Kleene) so that also Φ is primitive recursive in u, k (in the sense of Kleene). We discuss important special cases where the growth is much lower and even polynomial.

Finally we show that the growth caused by the use of the existence of lim sup for single bounded sequences in $\mathbf{R}$ (which may depend on u, v, k) can be captured using the Gödel recursor R_1 for type-1-recursion (which however suffices to define the Ackermann function).

► G. MINTS, *Natural deduction in the intuitionistic linear logic*.
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A natural deduction system described here admits normalization and has subformula property. It has standard axioms $A \Rightarrow A$, $\Rightarrow 1$, standard introduction and elimination rules for $\&, \rightarrow$ (linear implication), $\oplus$ and quantifiers. The rules for $\otimes$ are now standard too. Structural rules are (implicit) permutation, contraction for !-formulas and weakening for 1 and the formulas beginning with ! (cf. below). The rules for ! use an idea of D. Prawitz. By a

!-formula we mean 1, any formula beginning with !, and any expression $\langle\Gamma\rangle$, where Γ is a list of formulas and !-formulas. Then weakening, contraction and the rule for ! are as follows:

$$\langle\rangle I\,\frac{\Gamma \Rightarrow !A}{\langle\Gamma\rangle \Rightarrow !A} \quad !I\,\frac{\mathbf{\Gamma} \Rightarrow A}{\mathbf{\Gamma} \Rightarrow !A} \quad \langle\rangle E\,\frac{\langle\Gamma\rangle \Rightarrow A}{\Gamma \Rightarrow A} \quad \text{weak}\,\frac{\Gamma \Rightarrow \mathbf{A} \quad \Sigma \Rightarrow B}{\Gamma, \Sigma \Rightarrow B} \quad c\,\frac{\mathbf{B}, \mathbf{B}, \Gamma \Rightarrow D}{\mathbf{B}, \Gamma \Rightarrow D}$$

provided $\mathbf{\Gamma}$ consists of !-formulas, $\mathbf{B}$ is a !-formula and $\mathbf{A}$ is 1 or begins with !. Normal deduction is one without *maximal segments*.

THEOREM. *Every derivable sequent has a normal deduction.*

PROOF. Apply Prawitz translation to a cut-free sequent derivation.

[1] J.-Y. GIRARD, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–102.
[2] G. M. KELLY and S. MACLANE, *Coherence in closed categories*, ***Journal of Pure and Applied Algebra***, vol. 1 (1971), pp. 97–140.
[3] P. LINCOLN and J. MITCHELL, *Operational aspects of linear lambda calculus*, ***Proceedings of the IEEE Symposium on Logic in Computer Science***, 1992, pp. 235–247.
[4] G. MINTS, *Lewis' systems and system T*, in [7], Russian original 1974.
[5] ———, *Closed categories and the theory of proofs*, in [7], pp. 183–212, Russian original 1977.
[6] ———, *Normal forms of sequent derivations*, ***CSLI Technical Report No. CSLI-94-193***, Stanford, 1994.
[7] ———, ***Selected papers in proof theory***, North-Holland-Bibliopolis, 1992.
[8] D. PRAWITZ, ***Natural deduction***, Almquist and Wiksell, 1965.

► WOLFRAM POHLERS, *Extending the predicative universe*.
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We discuss an approach to extend the predicative universe by adjoining large ordinals as points.

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Theories with self-application provide an elementary framework for many activities in (the foundations of) mathematics and computer science. They were first introduced by Feferman [3, 4] as a basis for his systems of explicit mathematics, e.g., the theory T_0; these theories are broadly discussed in the literature from a proof-theoretic and model-theoretic point of view.

It is the aim of the present work to propose a first order theory PTO of operations and binary words, which allows full self-application and whose provably total functions on $\{0, 1\}^*$ are exactly the polynomial time computable functions. In spite of its proof-theoretic weakness, PTO has an enormous expressive power due to the presence of full (partial) combinatory logic, i.e., there are terms for every partial recursive function.

The formulation of PTO is very much akin to well-known theories of operations and numbers, namely PTO can be viewed as the polynomial time analogue of the theory BON plus set induction of Feferman and Jäger [5]. The choice of a unary predicate W for binary words instead of a predicate N for natural numbers is not mandatory, but more natural in the context of polynomial time computability.

The proof of the fact that PTO captures exactly polynomial time is very much in the spirit of reductive proof theory. More precisely, we show that PTO contains Ferreira's system of polynomial time computable arithmetic PTCA (cf. [7, 8]) via a natural embedding. Furthermore, PTO is reducible to the theory $\mathrm{PTCA}^+ + (\mathrm{BCP})$, where PTCA^+ denotes the extension of PTCA by NP induction and (BCP) is the collection principle for bounded formulas. $\mathrm{PTCA}^+ + (\mathrm{BCP})$ is known to be a Π^0_2 conservative extension of PTCA by the work of Buss [1], Cantini [2], or Ferreira [6].

Finally, let us mention that our approach can easily be extended in order to provide applicative theories which capture the nth level of the Grzegorczyk hierarchy.

[1] SAMUEL R. BUSS, *A conservation result concerning bounded theories and the collection axiom*, ***Proceedings of the American Mathematical Society***, vol. 100, no. 4 (1987), pp. 709–715.

[2] ANDREA CANTINI, *Asymmetric interpretation for bounded theories*, ***Mathematical Logic Quarterly***, vol. 42 (1996), pp. 270–288.

[3] SOLOMON FEFERMAN, *A language and axioms for explicit mathematics*, ***Algebra and Logic*** (J. N. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer-Verlag, Berlin, 1975, pp. 87–139.

[4] ———, *Constructive theories of functions and classes*, ***Logic Colloquium '78*** (M. Boffa, D. van Dalen, and K. McAloon, editors), North Holland, Amsterdam, 1979, pp. 159–224.

[5] SOLOMON FEFERMAN AND GERHARD JÄGER, *Systems of explicit mathematics with non-constructive μ-operator. Part I*, ***Annals of Pure and Applied Logic***, vol. 65, no. 3 (1993), pp. 243–263.

[6] FERNANDO FERREIRA, *A note on a result of Buss concerning bounded theories and the collection scheme*, ***Portugaliae Mathematica***, vol. 52 (1995), pp. 331–336.

[7] ———, *Polynomial time computable arithmetic and conservative extensions*, ***Ph.D. Thesis***, Pennsylvania State University, 1988.

[8] ———, *Polynomial time computable arithmetic*, ***Contemporary Mathematics***, vol. 106, 1990, pp. 137–156.

► ANDREAS WEIERMANN, *Term rewriting applications in proof and recursion theory*.
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We present a general framework, which has been developed together with E. A. Cichon, for relating term rewriting techniques to (sub-) recursive function theory. We give illuminating and simple proofs of classical results of R. Peter concerning nontrivial closure properties of the primitive recursive functions [joint work with E. A. Cichon]. We give a solution to Cichon's problem on the relationship of the hydra battle rewrite system and the Howard Bachmann ordinal. We prove a nontrivial bound on derivation lengths for term rewriting systems for which termination has been proved with ordinal notation based termination orderings like the multiset or lexicographic path ordering, etc. We give a term rewriting classification of the polytime functions [joint work with A. Beckmann].

Abstracts of invited talks in the Special Session in Model Theory

► OLEG V. BELEGRADEK, *On mutual interpretability of a ring R and the group $\mathbf{UT}_n(R)$*.
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All the notions concerning interpretations we use below can be found in [3, 5.3]. Let R be an associative ring with unit and $n \geq 3$. We denote by $\mathbf{UT}_n(R)$ the group of all upper unitriangular $n \times n$ matrices over R. There is an obvious interpretation of $\mathbf{UT}_n(R)$ in R without parameters and an interpretation of R in $\mathbf{UT}_n(R)$ with parameters; see [4, 2]. Since there is a ring R with undecidable theory such that the theory of $\mathbf{UT}(R)$ is decidable [1] we have

Proposition 1. *There exists a ring R which is not interpretable in the group $\mathbf{UT}_n(R)$ with definable parameters.*

Propositions 2 and 3 below are based on results of [B2]:

Proposition 2. *If R is commutative then R is interpretable in $\mathbf{UT}_n(R)$ with definable parameters.*

Proposition 3. *Let R have no zero divisors. Then the group $\mathbf{UT}_n(R)$ and the ring $R \times R^{\mathrm{op}}$ are interpretable with definable parameters in each other. In particular, R is interpretable in $\mathbf{UT}_n(R)$ with definable parameters iff R is interpretable in $R \times R^{\mathrm{op}}$ with definable parameters.*

Remark. For a ring R without zero divisors, if $R \simeq R^{\mathrm{op}}$ or $R \not\equiv R^{\mathrm{op}}$ then R is interpretable in $R \times R^{\mathrm{op}}$ with definable parameters. Note that all the rings R with $R \not\simeq R^{\mathrm{op}}$ and $R \equiv R^{\mathrm{op}}$ known to the author have zero divisors.

Proposition 4. *Let R be an algebraically closed field. Then the ring R and the group $\mathbf{UT}_n(R)$ are not bi-interpretable with parameters.*

A proof of Proposition 4 is based on the fact that an algebraically closed field is strongly minimal but no group of the form $\mathbf{UT}_n(R)$ is almost strongly minimal. Note that, in contrast with the latter fact, a ring R is uncountably categorical iff the group $\mathbf{UT}_n(R)$ is [2].

Conjecture. *For any infinite R, the group $\mathbf{UT}_n(R)$ and the ring R are not bi-interpretable with parameters.*

[1] O. V. Belegradek, *Unitriangular groups and the undecidability*, ***Izvestia Vysshikh Uchebnykh Zavedeniĭ, Matematicheskaya***, vol. 370, no. 3 (1993), pp. 19–22.

[2] ———, *The model theory of unitriangular groups*, ***Annals of Pure and Applied Logic***, vol. 68 (1994), pp. 225–261.

[3] W. Hodges, ***Model theory***, Cambridge University Press, 1993.

[4] A. I. Mal'tsev, *On a certain correspondence between rings and groups*, ***Matematicheskiĭ Sbornik***, vol. 50 (1960), pp. 267–266.

▶ BERNHARD HERWIG, *Extending partial isomorphisms and its applications*.
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We present extension theorems of the following type for different classes K of finite structures: "For every structure A in K there exists a structure B in K such that every partial isomorphism of A extends to an automorphism of B."

There are two applications of these theorems:

(1) One can use them to prove that certain subsets of a freely finitely generated group are closed (e.g., finite products of finitely generated subgroups, which is a theorem of Ribes and Zalesskii).

(2) One can use them to prove the Small Index Property for certain countable homogeneous omega-categorical structures.

► ANATOLE KHELIF, *Existentially closed models via constructible sets*; *There are continuously many pairwise non elementarily equivalent existentially closed ordered groups*.
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We prove that there are continuously many pairwise non elementarily equivalent existentially closed ordered groups, which solves the main open problem in this area.

A simple direct proof is given of the weaker fact that the theory of ordered groups has no model companion; the case of the ordered division rings over a field k is also investigated.

Our main result uses constructible sets and can be put in an abstract general framework. Comparison with the standard methods which use forcing is sketched.

► LUDOMIR NEWELSKI, *$\mathcal{M}$-gap conjecture and m-normal theories*.
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Assume T is small superstable. The feature that distinguishes such theories from ω-stable ones is the presence of types with infinite multiplicity. Hence investigating multiplicities seems a necessary step towards a proof of Vaught's conjecture for T. For $p \in S(A)$, A finite, $\mathrm{Tr}_A(p)$ denotes the set of stationarizations of p over A. I introduced an ordinal valued rank $\mathcal{M}$ measuring the topological shape of $\mathrm{Tr}_A(p)$ and proved that when T has few countable models, then $\mathcal{M}(p)$ is finite.

$\mathcal{M}$-rank leads to notions of meager forking and meager type. Any meager type is locally modular. This generalizes many previous results on existence of locally modular types and provides us potentially with many such types. I found a way to produce meager types using $\mathcal{M}$-rank, similar to U-rank yielding regular types. Unfortunately, in order for this way to be productive we need a type p with $\mathcal{M}(p)$ infinite (to start with). There are examples of small weakly minimal types with $\mathcal{M}(p) = \infty$ and with $\mathcal{M}(p)$ arbitrarily large finite.

$\mathcal{M}$-GAP CONJECTURE. *If T is small then there is no p with $\omega \leq \mathcal{M}(p) < \infty$.*

All the known examples of small superstable theories have a common feature, which I call m-normality, since it is parallel to weak normality (1-basedness). I proved the $\mathcal{M}$-gap conjecture for m-normal theories.

Developing the theory of multiplicities parallel to the theory of forking I introduced yet m-independence (refining forking independence) and *-finite tuples, so that ($\mathcal{M}$, m-independence, *-finite tuples) correspond to (U, forking, imaginaries). *-finite tuples may be useful in many-models arguments.

The above is covered by the following papers [1–6]. In the talk I will focus on [6].

[1] L. NEWELSKI, *A proof of Saffe's conjecture*, ***Fundamenta Mathematicae***, vol. 134 (1990), pp. 143–155.

[2] ———, *A model and its subset*, ***Journal of Symbolic Logic***, vol. 57 (1992), pp. 644–658.

[3] ———, *Meager forking*, ***Annals of Pure and Logic***, vol. 70 (1994), pp. 141–175.

[4] ———, *$\mathcal{M}$-rank and meager types*, ***Fundamenta Mathematicae***, vol. 146 (1995), pp. 121–139.

[5] ———, *$\mathcal{M}$-rank and meager groups*, ***Fundamenta Mathematicae***, vol. 150 (1996), pp. 149–171.

[6] ———, *$\mathcal{M}$-gap conjecture and m-normal theories*, preprint, 1995.

► E. RABINOVICH, *On definability in algebraically closed fields*.
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A rich incidence system is a structure whose universum is a union of two subsets called set of points and set of lines, and the incidence relation between them satisfies the following: any two different lines intersect in a finite number of points, the set of points of any line is infinite, the set of lines through any point is infinite.

A structure is called rich if a rich incidence system is interpretable in it.

In 1980 Zilber conjectured: For every rich structure M interpretable in an algebraically closed field, the field is interpretable in M.

I will present the proof of the conjecture for the rich structures with an algebraic curve as the universum.

► ZELJKO SOKOLOVIC, *Smoothly approximable structures and small index property*.
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Cherlin and Hrushovski characterized up to biinterpretability every smoothly approximable structure as a structure coordinatized by (roughly speaking) classical geometries (projective, unitary, symplectic or orthogonal) over a finite field. Using this characterization and the criterion by Hodges, Hodkinson, Lascar and Shelah we proved that from the abstract automorphism group G of such a structure M one can recover the topological structure of G by showing the small index property for M.

[Joint work with A. Chowdhury and B. Hart.]

► FRANK O. WAGNER, *Local superstability*.
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Let P be a $\emptyset$-invariant family of stationary types in a stable theory. We define a type $\mathrm{tp}(a/B)$ to be a P-forking extension of its restriction to some subset $A \subset B$ iff there is an element $b \in B$ such that $\mathrm{tp}(b/A) \in P$ and $a \not\downarrow_A b$ We can then study the structure of P-forking. If we define U_P-rank analogously to the usual definition of Lascar rank, but taking into account merely P-forking extensions, it turns out that this rank satisfies many of the properties of Lascar rank. In particular the Lascar inequalities hold, and the sum of the coefficients in the Cantor normal form of U_P-rank gives a bound for the P-weight of a type p, i.e., for the maximal number of independent realizations of types in P some realization of (a nonforking extension of) p can fork with at the same time. Clearly, Lascar rank equals U_P-rank if P is the family of all stationary types. In a superstable theory, Lascar rank also equals U_P-rank if P is the family of all stationary regular types, or even the family of all minimal types.

We can also define Shelah rank with respect to the family P, which we denote by D_P. It can then be shown that a formula has ordinal D_P-rank iff it only contains types of ordinal U_P-rank. Furthermore, if $A \subset B$, then $U_P(a/B) < U_P(a/A)$ iff the canonical base of $\mathrm{tp}(a/B)$ has nonzero U_P-rank over A. The question whether this holds iff $D_P(a/B) < D_P(a/A)$ is as yet open.

These notions generalize both ordinary Lascar rank and P-weight, where P is either a single regular type or a regular family of regular types. They restrict attention to forking with a family P (which need neither be regular, nor consist of regular types), but contrary to weight, they also take into account the behaviour of forking extensions of the original type. They allow the extension of Lascar rank techniques into the non-superstable domain (if a suitable family P can be found such that U_P-rank is ordinal). In that case, they give rise in the usual manner to regular types, namely those of monomial U_P-rank.

Abstracts of invited talks in the Special Session in Non-monotonic Logic

▶ GIANNI AMATI, CARLUCCI AIELLO, DOV GABBAY, and FIORA PIRRI, *Provability logic for default reasoning*.
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The goal of our research is to highlight the connection between modal provability theory and default logic. Our work continues the research tradition in which definitions of extensions in default logic are related to modal systems. The main innovation is to try to get the most out of reflection results due to Gödel and developed by Boolos, Smorynski, Bernardi, and Solovay.

Default logic deals with a dynamic of reasoning that accounts for notions like provability and consistency. This dynamic consists in accepting multiple explaining theories to a given problem. As the deductive closure operator of a logic is monotonic, it seems hardly possible to model the deduction of multiple, even conflictual, situations from a given theory. Fixed point constructions are, usually, adopted in the non monotonic literature to deal with this problem; they can be seen as forms of self-reference as, in fact, solutions are established according to particular instantiations of a parametric theory (a context). The above mentioned fixed point constructions are based on an equation exploiting the deductive closure under some family of modal logics. Modal logics, in fact, have been largely used to model linguistic notions or meta linguistic notions such as necessity, beliefs, knowledge, provability in Peano Arithmetic, etc. Therefore these logics have been rightly considered good formal tools to express metalanguage constructions (in the non monotonic community, e.g., by McDermott & Doyle and Marek & Truszczynski).

Following this tradition, we have shown how is it possible to use a modal logic, namely *KD*4Z (*KD*4 plus the discreteness axiom Z), to solve the problem of representing self-reference as an internal construction. A logical problem which admits multiple solutions, such as the definition of extensions for default logic, can be dealt with via self-referential sentences in *KD*4Z.

We propose *KD*4Z as *the* modal provability logic that can express default reasoning. In other words, *KD*4Z, is the *single* modal logic that "hosts" the meta logical constructions proper of non monotonic reasoning. Our results answer the question of "which modal logics can host self-reference, hence internal fixed point operators adequate for non monotonicity".

There is a well established area of pure logic where self reference has been studied and used to interpret the notion of "provable" in $\mathcal{PA}$, the first order Peano Arithmetic. Following Gödel's procedure for numbering theorems of $\mathcal{PA}$, a first order "provability" predicate can be constructed. Once it is interpreted as $\Box$, this predicate satisfies some conditions which resemble Axioms K, 4 and $W = \Box(\Box\alpha \rightarrow \alpha) \rightarrow \Box\alpha$ (the so called Löb axiom) and the necessitation rule in modal logic. Solovay has indeed proved that this connection is perfectly well defined. The normal modal logic $G = K4W = KW$ corresponds to $\mathcal{PA}$. A consequence of the Gödel's second incompleteness theorem is that no sentence in $\mathcal{PA}$ can imply its own consistency: correspondingly, for no formula ϕ, $\vdash_G \phi \rightarrow \Diamond\phi$ holds. G is an interesting modal logic because it provides a machinery for defining modal predicates. A modal predicate $E(p)$ is said to be *explicitly definable* in the logic Λ when there is a formula T such that $\vdash_\Lambda T \leftrightarrow E(T)$. The notion of definability we give in *KD*4Z is weaker than that in G, where the uniqueness of fixed points is a consequence of Löb axiom. Although the existence of a solution for fixed point equations is always unique and guaranteed in G, a necessary condition for its existence is required in $KD4Z$. Moreover, $KD4Z$ has fixed point equations with multiple solutions, hence $KD4Z$ is more expressive than G and it is perfectly suited to represent the multiple extensions of default reasoning.

In fact, G cannot serve for the purpose of formalizing forms of "negation as failure to prove" paradigm. In fact, if this construction were definable in G through some predicate $E(p)$, then "negative introspection" (i.e., $\{\Diamond\alpha : \neg\alpha \notin T\}$, which contains $\Diamond T = \{\Diamond\alpha : \alpha \in T\}$) would require the formula T (if T is finitely axiomatizable) to logically imply its own consistency, that is $\Diamond T$. On the other hand, if we restrict default logic to decidable fragments of first order logic, then a theorem on the definability of modal predicates can be proved in $KD4Z$ analogously to G. In contrast to G, the provability operator in $KD4Z$ is supposed to rely on the seriality axiom $\Diamond\top$.

We prove that a class of modal predicates (those implying the provability of their own consistency) are definable in $KD4Z$. We apply these results to non monotonic logic and show that a default extension E (w.r.t. Reiter's default logic) of a given default theory $\langle W, D\rangle$ corresponds to the modal sentences $\mathcal{T}$ such that the self-referential sentence $\mathrm{tr}(\mathcal{T}) \leftrightarrow \mathcal{T}$ is a theorem of $KD4Z$, where $\mathrm{tr}(\_)$ is a systematic translation of $\langle W, D\rangle$. Let $\mathscr{L}$ be propositional classical logic, the correspondence is established by proving $E \vdash_{\mathcal{L}} \phi$ if and only if $\mathcal{T} \vdash_{KD4Z} \Box\Box\phi$ holds. In other words, $\mathcal{T}$ is a solution of the fixed point equation $\mathrm{tr}(p) \leftrightarrow p$, where p is a propositional variable (parameter). The systematic translation $\mathrm{tr}(\_)$ of $\langle W, D\rangle$ contains an adequate translation of defaults that faithfully represents their intended meaning:

> "If α has been *proved* from a set of beliefs $\mathcal{T}$ and the *consistency* of β with $\mathcal{T}$ is *provable*, then γ is *provable* from $\mathcal{T}$."

Indeed, this is expressed by the following modal sentence:

$$\Box(\mathcal{T} \to \alpha) \wedge \Box\Diamond(\mathcal{T} \wedge \beta) \to \Box(\mathcal{T} \to \gamma).$$

More details can be found in [1].

[1] G. Amati, L. Carlucci Aiello, D. Gabbay and F. Pirri, *Provability logic for default reasoning*, ***Technical Report 29.94***, Università di Roma "La Sapienza".

▶ SHAI BEN-DAVID and RACHEL BEN-ELIYAHU, *A modal logic for subjective default reasoning*.

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There have been many attempts in the Artificial Intelligence community to provide a formal framework for reasoning by default. Such a framework is essential for the development of systems that reason with incomplete information.

Various features were suggested as appropriate for characterizing implication by default. In this work we offer a very general semantic framework in which logics having such features can be embedded, analyzed and compared to each other.

More specifically, we introduce a logic endowed with a two-place modal connective that has the intended meaning of "If α, then normally β". Our logic allows nesting of the default operator. The semantics is a "neighborhood modal semantics", and it allows for *subjective* defaults, that is, defaults may vary within different worlds that belong to the same model. The semantics has an appealing intuitive interpretation and may be viewed as a set-theoretic generalization of the probabilistic interpretations of default reasoning. We prove soundness and completeness theorems for several proof systems that are known in default reasoning.

We show that our semantics is most general in the sense that any modal semantics that is sound for some basic axioms for default reasoning is a special case of our semantics. Such a generality result may serve to provide a semantical analysis of the relative strength of different proof systems and to show the nonexistence of semantics with certain properties.

▶ MICHAEL GELFOND, *Representing knowledge in declarative logic programming*.
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It was long realized that sophisticated software systems should contain substantial build-in knowledge about their environment. This realization led to development of logic programming paradigm which strives to reduce a substantial part of a programming process to the description of objects comprising the domain of interest and relations between these objects. After such description is produced by a programmer it can be queried to establish truth or falsity of statements about the domain or asked to find objects satisfying various properties. Initially, classical logic was used to describe the domains as well as to formulate queries. In the seventies, advances in automatic theorem proving lead to the development of the programming language, Prolog (Programming in Logic), which allowed applications of logic to natural language processing, databases, and even to the writing of compilers and operating systems. To further transfer the ideas and methods of mathematical logic to non-mathematical domains, the researchers often had to introduce new, non-standard logical connectives suitable for expressing knowledge in a given class of domain, to understand the meaning of these connectives and to find and mathematically model reasoning patterns common for the domains of this class. Many such patterns used in everyday, common sense reasoning appear to be defensible, i.e., allow conclusions which have to be withdrawn when new information about the domain becomes available. These observations lead to the development of new logical systems called "non-monotonic logics".

I will talk about one such system, called Declarative Logic Programming (DLP), its semantics and applications to formalization of common sense, "non-monotonic" reasoning. The main non-monotonic operator of DLP is a generalization of negation-as-finite-failure of Prolog. There are also close connections with Autoepistemic Logic of Moore, Default Logic of Reiter, and Nelson's constructive logic with strong negation.

▶ GEORG GOTTLOB, *Expressive power of non-monotonic formalisms*.
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In this talk we give an overview of recent results concerning the expressive power of various non-monotonic formalisms such as Circumscription, Disjunctive Logic Programming, and different versions of Default Logic. We deal with both the expressive power in the propositional case and the expressive power in the first order case in the sense of finite model theory. We will also announce a number of new results.

▶ HEINRICH HERRE, *Model theory of non-monotonic inference operations*.
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In the present talk an approach to non-monotonic reasoning is discussed which is inspired by the paradigm of model theory. The aim is to analyse non-monotonic reasoning within the framework of inference operations based on model operators. This is done by using the concept of a deductive frame and its semantical counterpart, a semantical frame. This framework is then extended to belief set operations to formalize and to analyse multiple belief sets in an abstract setting. Within this extended framework all known non-monotonic systems can be classified. The following topics are studied in detail:

(1) *Representation theorems for deductive frames and belief set frames.* Starting from an inferential system (L, C), which is determined by an operation $C : \mathrm{Pow}(L) \to \mathrm{Pow}(L)$, one constructs a deductive frame (L, C_L, C) satisfying certain desirable properties e.g. distributivity or maximality. Then the system (L, C_L, C) is represented by a semantical frame $(L, M, \models \Phi)$, where $\Phi : \mathrm{Pow}(L) \to \mathrm{Pow}(M)$. The same idea can be carried out for belief set systems (L, B), where $B : \mathrm{Pow}(L) \to \mathrm{Pow}(\mathrm{Pow}(L))$.

(2) *Applications to minimal reasoning in partial logics.* The basic logic $L(\sigma)$ is built on a predicative signature σ and the following logical connectives $-, \sim, \wedge, \vee, \forall, \exists$; $-$ is called weak negation, and $\sim$ strong negation. $L_0(\sigma)$ is the set of all quantifier free formulas in $L(\sigma)$, and $L_0^+(\sigma)$ the set of formulas in $L_0(\sigma)$ not containing weak negation $-$. The semantics for $L(\sigma)$ is given by partial models (four-valued, three-valued coherent, and two-valued models). Several versions of minimality are introduced and their compactness properties are studied.

(3) *Applications to extended logic programs.* A general program rule is an expression of the form $F_1, \ldots, F_m \Rightarrow G_1, \ldots, G_n$, where F_i are formulas from $L_0(\sigma)$, and G_j formulas from $L_0^+(\sigma)$. The declarative semantics of a rule is defined by $\mathcal{I} \models \bigwedge_i F_i \supset \bigvee_j G_j$, where $\mathcal{I}$ is a partial interpretation and $F \supset G =_{\mathrm{df}} -F \vee G$. Several generalizations of stable model semantics are introduced and discussed.

(4) *Classification of specification levels.* There are at least three levels of abstraction at which non-monotonic reasoning can be specified. The system can be specified by a global set of possible (intended) worlds and the beliefs that hold in them, it can be specified by the belief sets abstracting from the specific reasoning patterns generating them, and it can be specified by a set of intended reasoning patterns that lead to the intended possible belief sets. It is shown that there exist connections between these levels, and that they are necessary to get a full picture about non-monotonic reasoning.

[1] J. Engelfriet, H. Herre and J. Treur, *Non-monotonic belief set frames and reasoning frames*, ***Proceedings of ECSQARU 95***, Lecture Notes in Artificial Intelligence, vol. 946, Springer-Verlag, 1995, pp. 189–196.

[2] H. Herre, *Generalized compactness of non-monotonic inference operations*, ***Journal of Applied Non-classical Logic***, vol. 5 (1995), pp. 121–135.

► V. WIKTOR MAREK and MIROSLAW TRUSZCZYNSKI, *Current state of modal non-monotonic logics.*

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Modal Non-monotonic Logics have been introduced by D. McDermott in his 1982 paper [3], extending his earlier work with J. Doyle [4]. McDermott assigns to each modal logic $\mathcal{S}$ a nondeterministic partial operator. This operator assigns to modal theories I their S-extensions, that is, theories T satisfying the following fixpoint equation:

$$T = \mathrm{Cn}_{\mathcal{S}}(I \cup \neg L\bar{T}),$$

where $\neg L\bar{T} = \{\neg L\varphi : \varphi \notin T\}$. Here L is the necessity operator of the modal language, and the operator $Cn_{\mathcal{S}}$ is the provability operator in $\mathcal{S}$ allowing for applying necessitation rule to all formulas in the proof.

It turns out that McDermott construction is very powerful and allows to formalize a number of important non-monotonic formalisms. These include: default logic of Reiter [6], autoepistemic logic of Moore [5], the MBNF logic of Lifschitz [2], stable semantics of logic programs (Gelfond and Lifschitz [1]) and other constructions.

We present an outline of properties of modal non-monotonic logics, including recent results of Gottlob, Schwarz, Gabbay and others.

[1] M. GELFOND and V. LIFSCHITZ, *The stable semantics for logic programs*, ***Proceedings of the 5th international symposium on logic programming*** (R. Kowalski and K. Bowen, editors), MIT Press, Cambridge, Massachusetts, 1988. pp. 1070–1080.

[2] V. LIFSCHITZ, *Minimal belief and negation as failure*, ***Artificial Intelligence***, 1994, to appear.

[3] D. MCDERMOTT, *Non-monotonic logic II: non-monotonic modal theories*, ***Journal of the Association for Computing Machinery***, vol. 29 (1982), pp. 33–57.

[4] D. MCDERMOTT and J. DOYLE, *Non-monotonic logic I*, ***Artificial Intelligence***, vol. 13 (1980), pp. 41–72.

[5] R. C. MOORE, *Semantical considerations on non-monotonic logic*, ***Artificial Intelligence***, vol. 25 (1985), pp. 75–94.

[6] R. REITER, *A logic for default reasoning*, ***Artificial Intelligence***, vol. 13 (1980), pp. 81–132.

► VLADIMIR V. RYBAKOV, *Consequence relations and admissibility*.

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A very general conception originated from German and Polish logical schools is the idea of logical consequence relation $\vdash$, which is a binary relation on sets of formulas and individual formulas with some natural properties (cf., for instance, Wojcicki [4]). The present research is concerned with the enriching of consequence relations $\vdash$ by all admissible for $\vdash$ inference rules, such enriching of $\vdash$ we call the *admissible closure*. Before the conception of admissible inference rules has been rather developed for some individual non-standard logics (cf. [2,3]). We study here general properties of admissible closures without any limitations on $\vdash$, and in an important particular case, - when $\vdash$ is an algebraizable relation (cf. for definitions and denotations Blok, Pigozzi [1]).

We consider arbitrary consequence relation $\vdash$. By Los-Suzko theorem $\vdash$ coincides with a derivability relation $\vdash_{\mathcal{AS}}$ of individual formulas from sets of hypothesis in some axiomatic system $\mathcal{AS}$, i.e., $\vdash=\vdash_{\mathcal{AS}}$. In this event we call $\mathcal{AS}$ the *base* for $\vdash$. Let Γ be a set of formulas, $\mathrm{Th}_{\vdash}(\Gamma) := \{\alpha \mid \Gamma \vdash \alpha\}$. A structural rule r is said to be *admissible for* $\vdash$ iff $\mathrm{Th}_{\vdash}(\emptyset)$ is closed with respect to rule r. A rule $r := \Gamma/\alpha$ is called *derivable* in $\vdash$ if $\Gamma \vdash \alpha$. Relation $\vdash$ is *structurally complete* if any admissible for $\vdash$ rule is derivable in $\vdash$.

If $\mathcal{AS}$ is an axiomatic system then the *admissible closure* of $\mathcal{AS}$ is the system $\mathcal{AS}^+$, obtained from $\mathcal{AS}$ by adding of all admissible for $\vdash_{\mathcal{AS}}$ inference rules.

PROPOSITION 1. *If a logical consequence relation* $\vdash$ *has two different representations:* $\vdash=\vdash_{\mathcal{AS}_1}=\vdash_{\mathcal{AS}_2}$ *then* $\mathcal{AS}_1^+ = \mathcal{AS}_2^+$.

For every consequence relation $\vdash$, the *admissible closure* of $\vdash$ is the relation $\vdash^{\mathrm{Ad}}:=\vdash_{\mathcal{AS}^+}$, where $\vdash=\vdash_{\mathcal{AS}}$. Because the notion of admissible closure is stable with respect to the choice of axiomatic systems for $\vdash$, this definition is correct.

PROPOSITION 2. (i) *For any* $\vdash$, $\vdash\subseteq\vdash^{\mathrm{Ad}}$; (ii) $\vdash_1\subseteq\vdash_2 \Rightarrow \vdash_1^{\mathrm{Ad}}\subseteq\vdash_2^{\mathrm{Ad}}$ (*so the taking of admissible closure is absorbing and monotonic*).

PROPOSITION 3. *Given a consequence relation* $\vdash$, (i) $\vdash^{\mathrm{Ad}}$ *is structurally complete;* (ii) $\vdash$ *is structurally complete* $\Rightarrow \vdash=\vdash^{\mathrm{Ad}}$; (iii) $\vdash_{\mathrm{Ad}}$ *is a minimal structurally complete consequence relation containing* $\vdash$.

THEOREM 4. (i) *For any logical consequence relation* $\vdash$, $\vdash^{\mathrm{Ad}}$ *has an adequate matrix semantics consisting of the quasi-variety generated by single Lindenbaum matrix* $\mathcal{M}(\emptyset,\vdash)$ *for* $\vdash$ *over* *the empty set*; (ii) *If* $\vdash$ *is a logical algebraizable consequence relation* (*with equivalent to* $\vdash$ *semantics consisting of a quasi-variety of algebras* Q) *then so is* $\vdash^{\mathrm{Ad}}$. *In this event the free algebra of countable rank* F_ω *from* Q *forms an equivalent to* $\vdash^{\mathrm{Ad}}$ *algebraic semantics with the same defining equations and equivalence formulas.*

For any quasi-variety Q, $\mathrm{Th}_{pq}(Q)$ denotes the *pure quasi-equational theory* of Q i.e., the set of all quasi-identities without occurrences of $=$ (which we call *pure quasi-identities*) which are valid on Q. A pure quasi-equational base for Q is a set S of pure quasi-identities from $\mathrm{Th}_{pq}(Q)$ such that for any pure quasi-identity q from $\mathrm{Th}_{pq}(Q)$, q is a consequence of S (which means that for any algebraic system $\mathcal{A}$, if $(\forall q_1 \in S)\mathcal{A} \models q_1$ then $\mathcal{A} \models q$).

THEOREM 5. *Let* $\mathcal{AS}$ *be an axiomatic system with axioms* $\mathcal{A}x$ *and inference rules* $\mathcal{R}$. $\mathcal{AS}$ *is a base for the admissible closure* $\vdash^{\mathrm{Ad}}$ *of a logical consequence relation* $\vdash$ *iff the set of quasi-identities* $\mathcal{A}x^e \cup \mathcal{R}^q$ *where* $\mathcal{A}x^e := \{D(\alpha) \mid \alpha \in \mathcal{A}x\}$, $\mathcal{R}^q := \{(\bigwedge_{\beta\in\Gamma} D(\beta))\Rightarrow D(\alpha) \mid (\Gamma/\alpha) \in \mathcal{R}\}$ *is a pure quasi-equational base for the quasi-variety* Q *generated by single Lindenbaum matrix* $\mathcal{M}(\emptyset,\vdash)$ *for relation* $\vdash$ *over the empty set of formulas.*

Let $\vdash$ be an algebraizable relation, $\delta_i = \gamma_i$ is a corresponding system of defining equations, t are equivalence formulas. Then the quasi-variety Q equivalent to $\vdash$ has the free algebra F_ω of countable rank. Let D be the set of inference rules:

$[x \,/\, t(\delta(x), \gamma(x))]$, $[t(\delta(x), \gamma(x)) \,/\, x]$, $[z \,/\, t(x, x)]$, $[t(x, y) \,/\, t(y, x)]$,
$[t(x, y), t(y, z) \,/\, t(x, z)]$, $[x, t(x, y) \,/\, y]$, $(\forall x \in \mathrm{Var}(\alpha))\,[t(y, z) \,/\, t(\alpha(\tbinom{x}{y}), \alpha(\tbinom{x}{z}))]$,
where α is any term of view $\langle$logical connective on variables$\rangle$.

THEOREM 6. *An axiomatic system* $\mathcal{AS}$ *consisting of axioms* $\mathcal{A}x$ *and inference rules* $\mathcal{R}$*, where* $D \subseteq \mathcal{R}$*, forms a base for the admissible closure* $\vdash^{\mathrm{Ad}}$ *of some algebraizable relation* $\vdash$ *iff the set of quasi-identities* $\mathcal{A}x^e \cup \mathcal{R}^q \cup K^A$*, were*

$$\mathcal{A}x^e := \{\delta_i(\alpha) = \gamma_i(\alpha) \mid i, \alpha \in \mathcal{A}x\}$$
$$\mathcal{R}^q := \{(\bigwedge_{\beta\in\Gamma, i} \delta_i(\beta) = \gamma_i(\beta)) \Rightarrow \delta_j(\alpha) = \gamma_j(\alpha) \mid (\Gamma/\alpha) \in R,\ j\}$$
$$K^A := \{x = y \Rightarrow \delta_i(t_j(x, y)) = \gamma_i(t_j(x, y))\} \cup \{(\bigwedge_{i,j} \delta_i(t_j(x, y)) = \gamma_i(t_j(x, y))) \Rightarrow (x = y)\},$$

is a base for the quasi-variety generated by F_ω.

[1] W. J. BLOK and D. PIGOZZI, *Algebraizable logics*, ***Memoirs of the American Mathematical Society***, no. 396 (1989), pp. 1–78.
[2] V. V. RYBAKOV, *Rules of inference with parameters for intuitionistic logic*, ***Journal of Symbolic Logic***, vol. 57, no. 3 (1992), pp. 912- -923.
[3] ———, *Problems of substitution and admissibility in modal logic and intuitionistic propositional calculus*, ***Annals of Pure and Applied Logic***, vol. 49 (1990), p. 1–34.
[4] R. WOJCICKI, ***Lectures on Propositional Calculi***, The Polish Academy of Science, Institute of Philosophy and Sociology, Warsaw, 1984.

► KARL SCHLECHTA, *Semantics for non-monotonic* (*and other*) *logics*.
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We shall present the key arguments for various completeness results for non-monotonic and other logics. Preferential models—Kripke style structures—provide a natural semantics for non-monotonic and deontic logics: They consist of a set of classical models, equipped with a binary relation $<$, which expresses preference. In non-monotonic logic, preference expresses "more normal", in deontic logic, it expresses "more moral".

A preferential structure determines a choice function, by (in slight simplification)

$$f(X) := \{x \in X : x \text{ is } < \text{-minimal in } X\}.$$

These choice functions have a very simple characterization: In the most general case, by

(1) $f(X) \subseteq X$,

(2) $X \subseteq Y \rightarrow f(Y) \cap X \subseteq f(X)$.

"Smooth" structures are characterized by the additional property,

(3) $f(X) \subseteq Y \subseteq X \rightarrow f(X) = f(Y)$—provided certain closure conditions hold.

Completeness proofs based on these algebraic characterizations are almost totally independent of logic, their main arguments can be used in different situations. Thus, they carry over quite easily to the Lewis/Stalnaker semantics for counterfactual conditionals, which are also based on an (indexed) preference relation, or to the first order case. But they can also be used to obtain representation results in still other contexts, e.g. for a very poor sequential calculus, and for descriptions of dynamic situations.

Abstracts of contributed talks in the Special Session in Set Theory

► DORELLA BELLÈ and FRANCO PARLAMENTO, *The decidability of the $\forall^*\exists$ class and the axiom of foundation.*

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Let NW be the theory in the first order language with equality $=$ and the membership relation $\in$, whose axioms in skolemized form are (N) $\forall y(y \notin x)$ and (W) $\forall x \forall y \forall z(x \in \mathbf{w}(y,z) \leftrightarrow x \in y \vee x = z)$. Let E be the Extensionality Axiom; R the Regularity Axiom; AFA_1 (every graph has at least one decoration) and AFA_2 (every graph has at most one decoration) the two parts of the Antifoundation Axiom AFA as in [1]. We show that:

(a) the decision problem for $\forall^*\exists$ sentences with respect to NW, NWE, $\mathrm{NW} + \mathrm{AFA}_2$ is undecidable;

(b) the decision problem for $\forall^*\exists$ sentences with respect to NWR and $\mathrm{NW} + \mathrm{AFA}_1$ is decidable;

From result in [2] it follows that (b) holds also for the theories NWER and $\mathrm{NW} + \mathrm{AFA}$ which are actually complete with respect to $\forall^*\exists$ sentences.

Thus, for the membership theories under consideration, the Axiom of Foundation as well as AFA_1, play a crucial role in determining the decidability of the $\forall^*\exists$ class while extensionality is irrelevant in that respect.

[This work has been supported by funds MURST 60% and 40% of Italy.]

[1] P. ACZEL ***Non-well-founded sets***, CSLI Lecture Notes, 1988.

[2] E. OMODEO, F. PARLAMENTO and A. POLICRITI, *Decidability of $\exists^*\forall$-sentences in membership theories*, in ***Mathematical Logic Quarterly***, vol. 42 (1996), pp. 41–58.

▶ M. DZAMONJA, *On variants of* ♣.
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We shall discuss certain variants of ♣. Unlike for $\diamondsuit$, it turns out that changing the definition of ♣ slightly, gives a different principle. Consider, for example the requirement of the existence of a sequence $\langle A_\delta : \delta < \omega_1 \rangle$ such that for each limit δ, the set A_δ is a cofinal subset of δ of order type ω, and for each unbounded subset A of ω_1, there are stationarily many δ such that $A_\delta \setminus A$ is finite. It is consistent that this principle is different than ♣ (which is the same statement, except that we require that $A_\delta \subseteq A$). The results presented are from joint work with Saharon Shelah, and answer a question of Sakae Fuchino.

▶ HEIKE MILDENBERGER, *Non-constructive Galois-Turkey connections*.
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Vojtáš [2] introduced a framework in which cardinal characteristics of the continuum can be regarded as norms of corresponding relations $\mathbf{A} = (A_-, A_+, A)$ with $A_-,A_+ \subseteq 2^\omega$, $A \subseteq A_- \times A_+$, and the norm $||\mathbf{A}|| = \min\{|\mathcal{Z}| : \forall x \in A_- \exists z \in \mathcal{Z} A(x,z)\}$. A Galois-Tukey connection from a relation $\mathbf{A}$ to a relation $\mathbf{B}$, which we call simply *morphism*, is a pair of functions (α, β) such that

$$\begin{array}{ll} \alpha\colon B_- \to A_-, & \beta\colon A_+ \to B_+, \\ \forall b \in B_- \forall a \in A_+ & (A(\alpha(b), a) \to B(b, \beta(a))). \end{array}$$

If there is a morphism from $\mathbf{A}$ to $\mathbf{B}$, then $||\mathbf{B}|| \leq ||\mathbf{A}||$, and indeed the proofs of the inequalities usually exhibit morphisms between the corresponding relations. Some inequalities are proved by Borel morphisms between products or more complicated compositions of the given relations. We deal with the unsplitting relations: For $n \geq 1$, we have

$$\begin{aligned} \mathbf{R}_n &= (n^\omega, \mathcal{P}_\infty(\omega), \{(f, Y) : f \text{ is almost constant on } Y\}), \\ \mathbf{R}_n^\sharp &= (n^\omega, \mathcal{P}_\infty(\omega), \{(f, Y) : f \text{ is constant on } Y\}), \\ \mathbf{r}_n &= ||\mathbf{R}_n||, \\ \mathbf{r}_n^\sharp &= ||\mathbf{R}_n^\sharp||. \end{aligned}$$

$\mathbf{r}_2$ is the usual unsplitting number. Blass [1] gives a notion of sequential composition of two copies of $\mathbf{R}_2$, called $\mathbf{R}_2;\mathbf{R}_2$, and a Borel morphism from $\mathbf{R}_2;\mathbf{R}_2$ to $\mathbf{R}_3$, which proves $\mathbf{r}_2 \geq \mathbf{r}_3$. He conjectures that there are no Borel morphisms between the simple relations. We prove this conjecture in some cases and show that a sharper conjecture may be true.

THEOREM 1. *For $n > m$, there is no morphism (α, β) with a continuous α (and arbitrary β) from $\mathbf{R}_m^\sharp$ to $\mathbf{R}_n^\sharp$.*

THEOREM 2. *For $m \geq 1, n \geq 2$, there is no morphism (α, β) with a Borel α from $\mathbf{R}_m$ to $\mathbf{R}_n^\sharp$.*

Similarly we prove that that the notion of category-product and the older notion of component-product are weaker than the sequential composition.

[1] A. BLASS, *Reductions between cardinal characteristics of the continuum*, to appear in ***Contemporary Mathematics***, 1995.

[2] P. VOJTÁŠ, *Generalized Galois-Tukey connections between explicit relations on classical objects of real analysis*, ***Set theory of the reals*** (H. Judah, editor), Israel Mathematical Conference Proceedings 6, American Mathematical Society, 1993, pp. 619–649.

▶ ALEKSANDER RUTKOWSKI, *On η-extensions of ordered sets*.
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We try to answer the following J. Łoś's question:

> *When a partial order can be extended (with the preservation of its underlying sets) to a linear order of type η (i.e., type of the set of rationals)?.*

The class of all ordered sets with an η-extension is quite rich. The most trivial examples are: set of all rationals itself and a countable antichain (set of pairwise incomparable elements). On the other side it is pretty easy to prove that this class is closed w.r.t. lexicographic sums over any finite or countable index set and to products. We present here one necessary and few sufficient conditions for the existence of such an extension.

THEOREM 1. *Let P be an ordered set with an η-extension. Then P contains an infinite antichain or a nontrivial dense saturated chain.*

(X is a saturated chain if, for each $p \notin X$, if $X \cup \{p\}$ is a chain then p is either lower or upper bound of X). Now, for a chain X and $p \in P$, let $X(p)$ be the set of all $q \in P$ such that $X \cup \{p\}$ and $X \cup \{q\}$ are order-isomorphic.

THEOREM 2. *Let P be a countable ordered set. If P contains a maximal chain X of the type η such that for each $p \in X$, $X(p)$ is either $\{p\}$ or it has an η-extension then P has an η-extension.*

Denote by $X^{\wedge}$ (resp. $X^{\vee}$) an order ideal (resp. filter) generated in P by its subset X and by ($\star$) the following property:
Let X and Y be finite antichains in P such that

$$(\forall x \in X)(\forall y \in Y) y \not\leq_P x.$$

Then there exists $p \in P$ such that $p \notin X^{\wedge}$ and $p \notin Y^{\vee}$.

Note that if P is linearly ordered then ($\star$) means just its density. The proof of the following theorem is based on a Cantor's "back-and-forth" technique.

THEOREM 3. *If an ordered set P satisfies ($\star$) then it has an η- extension. Moreover, $\leq_P$ is the intersection of all its η-extensions.*

COROLLARY 1. *Let P be a countable ordered set with the property*:
There is an infinite antichain $A \subseteq P$ such that, for each $p \in P$, A_p, the set of all elements of A which are comparable with p, is finite.
Then P has an η-extension.

COROLLARY 2. *Let P be a countable ordered set with no infinite chains. If each element of P is comparable with only finitely many elements of P, then P has an η-extension.*

COROLLARY 3. *Let P, Q be countable ordered sets. If P contains a chain with no upper bound and Q contains a chain with no lower bound, then $P \times Q$ has an η-extension.*

▶ Z. SIKIC, *On unfounded universe with individuals and proper classes*.
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Our aim is to show that the restriction of ZFC-FA, which admits individuals and proper classes, is the appropriate framework for the introduction of unfounded classes (i.e., unfounded sets and unfounded proper classes). Namely, in this framework it is quite easy and natural to prove the equivalence of Aczel's antifoundation axiom and the solution lemma for unfounded sets.

To prove the equivalence we have to modify Aczel's antifoundation axiom so as to admit individuals and proper classes. Furthermore, a system of equations in a set of individual indeterminates X (where X contains no individuals) is to be defined as a function $c : X \to V[\mathrm{Ind} + X] + \mathrm{Ind}$ where $V[A]$ denotes the universe of sets based on the set of individuals A). A solution to this system of equations is a function $s : X \to V[\mathrm{Ind} + X] + \mathrm{Ind}$ such that $s = s'c$ where s' is the unique substitution function generated by s. The solution lemma asserts that for each system of equations c there exists a unique solution s. The proof of the equivalence uses only the restricted version of the solution lemma. This version asserts the existence of a unique solution only for the systems of equations of the suitably defined first order (it follows that any system of equations can be reduced to a system of the first order).

In conclusion we add that the solution lemma gives the most transparent explication of the unfounded universe. Hence the lemma should be the axiom, and Aczel's axiom should be the lemma.

[1] P. Aczel, *Non-wellfounded sets*, ***CSLI Lecture Notes***, no. 14, 1988.

[2] Z. Sikic, *On the equivalence of the solution lemma and Aczel's antifoundation axiom*, ***Grazer Mathematische Berichte***, vol. 323 (1994), pp. 69–78.

► JURIS STEPRĀNS, *Decomposition with smooth sets*.
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Let $\partial_{m,n}$ denote the least number of sets into which m-dimensional Euclidean space can be decomposed so that each member of the partition has an n-dimensional tangent plane at each of its points. It will be shown to be consistent that $\partial_{m,n}$ is different from $\partial_{m,n+1}$. These cardinals will be shown closely related to the invariants associated with the problem of decomposing continuous functions into differentiable ones.

► FRANKLIN D. TALL, *Preserving topological properties by forcing*.
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The question of which topological properties of which spaces are preserved by which kinds of forcing over which models of set theory is interesting in itself as well as for its applications, especially to reflection theorems. We shall mention some of the work of the author and his students in this area, and highlight several problems of particular interest.

Abstracts of contributed talks in the Special Session in Recursion Theory

► VALERIY BULITKO, *Segment complexity of reduction and jump inversion*.
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We want to show that introduction of complexity concepts into the degree theory allows to make thinner the degree hierarchy and obtain some new specifications of well known results such as the jump inversion theorems [1, 2].

Let $\mathbf{T}$ be class of all total non-decreasing one-place functions $f : \mathbf{N} \to \mathbf{N}$ and $\mathbf{T}^+$ be subclass of $\mathbf{T}$ consisting of functions with infinite ranges. $f \lhd g$ denotes the statement $\exists y \forall x[y \leq x \Rightarrow f(x) \leq g(x)]$ where $f, g \in \mathbf{T}$. Then $X \lhd \mathbf{Y}$ for a set X and a class $\mathbf{Y}$ of sets means that there is a set $Y \in \mathbf{Y}$ such that $X \lhd Y$. $f \star g$ is superposition of one-place functions f, g. If $A \subseteq \mathbf{N}$ and A is infinite then $A[n]$ is the n-th element of A in the increasing order otherwise $A[n]$ is undefined. Thus we consider any infinite subset A of $\mathbf{N}$ as an element of $\mathbf{T}^+$.

Let $f \leq_T^z g$. Set $k(z, g, n) \rightleftharpoons \mu t[(\forall x \leq n)\forall y\{f(x) = y \Leftrightarrow (\exists u, v)[\langle x, y, u, v\rangle \in W_z \,\&\, D_u \subseteq g \,\&\, D_v \subseteq \overline{g} \,\&\, \pi_1^2(D_u \cup D_v) \subseteq \{0, \dots, t\}]\}]$. It is clear $\lambda n.[k(z, g, n)] \in \mathbf{T}$. We will write $f \leq_h g$ if $h \in \mathbf{T}$ and $\exists z[f \leq_T^z g \,\&\, \lambda n.[k(z, g, n)] \lhd h]$. Then $f \leq_{\mathbf{R}} g$ for $\mathbf{R} \subseteq \mathbf{T}$ denotes $\exists r[r \in \mathbf{R} \,\&\, f \leq_r g]$. Thus every subclass $\mathbf{R} \subseteq \mathbf{T}$ defines binary relation $\leq_{\mathbf{R}}$ on $\mathbf{T}$. (Earlier we have considered another way to introduce reduction complexity [3].)

THEOREM 1. (a) *Let* $\mathbf{R}$ *be a countable subclass* $\mathbf{T}$. *Then* ($\leq_{\mathbf{R}}$ *is reflexive and transitive*) $\Leftrightarrow$ ($\{I\} \lhd \mathbf{R}$ & $(\forall f, g \in \mathbf{R})[\{f \star g\} \lhd \mathbf{R}]$).

(b) *Let* $\leq_{\mathbf{R}}$ *and* $\leq_{\mathbf{H}}$ *be reflexive and transitive. Then* $(\leq_{\mathbf{R}} \subseteq \leq_{\mathbf{R}}) \Leftrightarrow (\mathbf{R} \lhd \mathbf{H})$.

Now we describe a recurrent construction of reducibilities of the above introduced kind. Let $\mathbf{R} \subseteq \mathbf{T}$. Define $\mathbf{A}_0(\mathbf{R}) = \{\mathbf{R}\}$. Then $\mathbf{A}_{n+1}(\mathbf{R})$ consists of all sets $q\alpha$ where $\alpha \in \mathbf{A}_n(\mathbf{R})$ & $q \in \alpha$ & $q\alpha = \{f \mid f \in \alpha \,\&\, f \leq_\alpha q\}$. Set $\mathbf{B}_n(\mathbf{R}) = \{\leq_\alpha \mid \alpha \in \mathbf{A}_n(\mathbf{R})\}$ and $\mathbf{B}(\mathbf{R}) = \bigcup_n \mathbf{B}_n(\mathbf{R})$.

We will say a class $\mathbf{R} \subseteq \mathbf{T}$ is regular if $f \star g, \max(f, g) \in \mathbf{R}$ for every $f, g \in \mathbf{R}$ and $(\forall f \in \mathbf{R})[I \lhd \{t \mid t \leq_f f\}]$. Note that $\mathbf{T}, \mathbf{T}^+$ are regular classes.

THEOREM 2. *If* $\mathbf{R}$ *is regular then every relation of* $\mathbf{B}(\mathbf{R})$ *is a reducibility.*

THEOREM 3. *For every* $\alpha\mathbf{T} \in \mathbf{A}(\mathbf{T}) \setminus \{\mathbf{T}\}$ *an infinite strictly increasing chain exists in the poset* $(\{\leq_\beta \mid \beta \in \mathbf{A}(\alpha\mathbf{T})\}, \subseteq)$.

Below we denote the relation $\leq_{X\mathbf{T}}$ by $\leq_X$. Let $K \rightleftharpoons \{i \mid i \in W_i\}$. Here we state Friedberg jump inversion theorem [1] specification only.

THEOREM 4. $(\forall A, C)\{A \lhd d_T(C \oplus K)$ & $K \leq_{C \oplus K} A \Leftrightarrow \exists B[B' \equiv_{C \oplus K} A]\}$.

We can choose $B \lhd C \oplus K$ and $B \leq_{C \oplus K} A$. (Let us note that if $K \leq_{\mathbf{T}} A$ then $K \leq_A A$).

The specification of Shoenfield jump inversion theorem will be formulated in the communication as enumeration complexity and related notions have to be introduced first.

[1] R. M. FRIEDBERG, *A criterion for completeness of degrees of unsolvability*, ***Journal of Symbolic Logic***, vol. 22 (1957), pp. 159–160.

[2] J. R. SHOENFIELD, *On degrees of unsolvability*, ***Annals of Mathematics. Second Series***, vol. 69 (1959), pp. 644–653.

[3] V. K. BULITKO, *Subturing reducibilities with bounded complexity*, ***Izvestia Vysshikh Uchebnykh Zavedeniĭ, Seriya Matematicheskaya***, vol. 1 (1992), pp. 7–17, Russian.

► RICHARD COLES, *A unique splitting theorem*.
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The structure of the recursively enumerable (r.e.) Turing degrees has been extensively studied over the past 50 years, but relatively little is known about nonsplitting results due to the complexity of the constructions required.

Sacks [1963] showed that every r.e. degree has a splitting into two r.e. degrees, and in contrast, Lachlan [1975] showed that there exists an r.e. degree **b** which has no splitting above some r.e. degree $\mathbf{a} < \mathbf{b}$. Harrington extended this result to show that bb could be $\mathbf{0}'$.

Recently Leonhardi [1995] has proved that given n there exists an r.e. degree $\mathbf{a} < \mathbf{0}'$ and n r.e. degrees above **a** which pairwise split bzp, but not $n + 1$ such r.e. degrees above **a**. We are able to add a (nontrivial) uniqueness to such splittings, and show that, given n, there is an r.e. degree **a** above which there are exactly n (nontrivially) unique r.e. degrees which pairwise split $\mathbf{0}'$.

▶ S. BARRY COOPER and ANDREA SORBI, *Noncapping in the Σ_2 enumeration degrees*.
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It is well known that the Σ_2 enumeration degrees coincide with the structure $\mathcal{D}_e(\leq \mathbf{0}')$, where $\mathbf{0}'$ is the enumeration degree of $\overline{K} = \{x : x \notin K\}$. An enumeration degree $\mathbf{a} \leq \mathbf{0}'$ is said to be *noncappable* if for no $\mathbf{b} \leq \mathbf{0}'$ and $\mathbf{b} \neq \mathbf{0}$ do we have $\mathbf{a} \cap \mathbf{b} = \mathbf{0}$. Noncappable enumeration degrees form a filter of $\mathcal{D}_e(\leq \mathbf{0}')$. The following theorem shows that this filter is not trivial.

Theorem 1. *There exist noncappable enumeration degrees strictly below* $\mathbf{0}'$.

Proof. Construct a Σ_2 approximation to a set A satisfying the following requirements:

$$\begin{array}{ll} N_\Phi : & \overline{K} \neq \Phi(A) \\ P_{\Phi,\Psi,B,W} : & \Phi(A) = \Psi(B_i) \Rightarrow \Gamma(A) \neq W \vee B \text{ r.e.}, \end{array}$$

where Γ is an enumeration operator to be constructed, Φ, Ψ are enumeration operators, and B is a Σ_2 set. ⊣

▶ LARS KRISTIANSEN, *A jump operator on honest subrecursive degrees*.
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Many articles have been written on the structure of different kinds of subrecursive degrees. After the middle of the seventies especially polynomial time reducibility between recursive sets has been investigated. Ladner shows density results and minimal pair results for such degrees in a classical paper. Moreover, he proves that neither the polynomial m- nor T-degrees of recursive sets are a lattice. (They are just upper semi-lattices.) After Ladner researchers have used refinements of his methods in further studies of the structure of polynomial time degrees of recursive sets.

The techniques from the study of reducibility between sets cannot be applied in the study of reducibility between honest functions. This area requires different methods. (Roughly speaking, a function is honest when the growth of the function mirrors the computational complexity of the function. Thus the characteristic function of a recursive set is certainly not honest.) Honest subrecursive degrees were studied in the sixties and early seventies by, among others, Meyer and Ritchie, Basu, and Machtey. In recent years this line of research has been rather dead. Machtey proved that structures of honest degrees are lattices, and a lot of rather strong density and embedding results are known. The main news in this paper is some theorems about a jump operator on the honest elementary degrees. (So the reducibility relation is "being (Kalmar) elementary in", but I believe that the results and proof methods can be generalized to other subrecursive reducibilities, e.g., "being primitive recursive in" and "being k-recursive in".)

Let $a \cup b$ and $a \cap b$ denote respectively the join and the meet of the degrees a and b in the lattice of elementary honest degrees. We introduce a jump operator $(\cdot')$ on the honest elementary degrees and define canonical degrees $0, 0', 0'', \ldots$ and low and high degrees analogous to the corresponding concepts for the Turing degrees. Among others, the following results are proved in the paper:

There exist low degrees, and there exist degrees which are neither low nor high. For any low degree b there exists a sequence of low degrees $\{a_i\}_{i\in\omega}$ such that $b \ll a_i$ and $a_i \ll a_{i+1}$.

(The relation $a \ll b$ holds iff the degree b contains a universal function for the degree a. So if $a \ll b$ then in some sense b lies far above a.) (It is an open problem whether high degrees exist.)

Every degree above $0'$ is the jump of some degree, moreover, for every degree c above $0'$ there exist degrees a, b such that $c = a \cup b = a' = b'$.

We have $a' \cup b' \leq (a \cup b')$ and $a' \cap b' \geq (a \cap b)'$. Moreover, $a' \cup b' \leq (a \cup b)'$ cannot be strengthened, i.e., there exist a, b such that $a' \cup b' = (a \cup b)'$ and there exist a, b such that $a' \cup b' < (a \cup b)'$. (It is an open problem whether there exist a, b such that $a' \cap b' > (a \cap b)'$.)

The jump operator is of course monotonic, i.e., $a \leq b \Rightarrow a' \leq b'$. We prove that every situation compatible with $a \leq b \Rightarrow a' \leq b'$ is realized in the structure, e.g., we have incomparable degrees a, b such that $a' < b'$ and incomparable degrees a, b such that $a' = b'$, etc.

So neither of the results (i) $a' \cup b' \leq (a \cup b)'$ and (ii) $a \leq b \Rightarrow a' \leq b'$ can be strengthened. Thus the situation is exactly the same as with the Turing degrees, i.e., (i) and (ii) are also the best possible results in the structure of the Turing degrees.

[1] L. Kristiansen, *Papers on subrecursion theory* (*Dr. Scient Thesis*), ***Research Report 217***, Department of Informatics, University of Oslo, 1996.

▶ A. S. MOROZOV, *Endomorphisms of the group of all recursive permutations*.
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All endomorphisms of the group of all recursive permutations are described.

Theorem 1. *Suppose* * *is an arbitrary homomorphism* $\mathrm{Aut}_r\,\omega \longrightarrow \mathrm{Aut}_r\,\omega$. *Then*

(1) * *is continuous, i.e., for all* $f \in \mathrm{Aut}_r\,\omega$, *for each finite set* $S \subset \omega$ *there exists a finite set* $U \subset \omega$ *such that* $U \subseteq \{x \in \omega \mid f(x) = x\}$ *implies* $S \subseteq \{x \in \omega \mid f^*(x) = x\}$.

(2) *There exists a partial recursive function* f *such that, for all* $n \in \omega$, $\kappa_n \in \mathrm{Aut}_r\,\omega$ *implies* $f(n)$ *is defined and* $(\kappa_n)^* = \kappa_{f(n)}$. *There exists a partial recursive function* g *such that, for all* $n \in \omega$, $\kappa_n \in \mathrm{Aut}_r\,\omega^*$ *implies* $g(n)$ *is defined and* $(\kappa_{g(n)})^* = \kappa_n$. *Therefore, there exists only countable number of endomorphisms of the group* $\mathrm{Aut}_r\,\omega$.

(3) *There exists an enumeration operator* Φ *such that for all* $f \in \mathrm{Aut}_r\,\omega$ $f^* = \Phi(f)$ *holds. There exists an enumeration operator* Ψ *such that for all* $f \in \mathrm{Aut}_r\,\omega^*$ $(\Psi(f))^* = f$ *holds.*

(4) *For all* $m \in \omega$, *the set*

$$\{\langle \bar{x}, \bar{y} \rangle \mid \bar{x} = x_1, \ldots, x_n, \bar{y} = y_1, \ldots, y_n, \exists f \in \mathrm{Aut}_r\,\omega\ f^*(\bar{x}) = \bar{y}\}$$

is recursive.

(5) *For all* $c, n \in \omega$, *the set* $(\mathrm{Orb}(c))^n$ *is a union of finitely many orbits of* n*–tuples.*

(6) *There exists a recursive model* $\mathcal{M}^*$ *such that* $\mathrm{Aut}_r\,\mathcal{M}^* = (\mathrm{Aut}_r\,\omega)^*$.

In addition, there exists an universal endomorphism which contains, in some sense, all endomorphisms of this group. This universal endomorphism is unique, with respect to some natural recursive similarity notion.

▶ KARL-HEINZ NIGGL, *A restricted computation model on Scott domains and its primitive recursive functionals*.
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The talk builds on both a simply typed term system $\mathcal{PR}^\omega$ and a computation model on arbitrary Scott domains D_σ via so-called *parallel typed while programs* (PTWP), a generalization of Cook's and Kapron's typed while programs to Scott domains. The former provides a notion of *partial primitive recursive functional* on Scott domains including a suitable concept of *parallelism* in the sense of Plotkin. Starting from the key observation that in the given partial context Kleene's schema of higher type simultaneous course-of-values recursion (SCVR) is not reducible to partial primitive recursion, extensions $\mathcal{PR}^{\omega e}$ and PTWP^e are studied that are closed under SCVR. The twist are certain type 1 *Gödel recursors* $\mathcal{R}_k$ for *simultaneous partial primitive recursion*. Formally, $\mathcal{R}_k \vec{g}\vec{H}xi$ denotes a function $f_i \in D_{\iota\to\iota}$, however, $\mathcal{R}_k$ is modeled such that f_i is a *finite* element or in other words, a *partial sequence*. In particular, no number theoretical function having no primitive recursive *bound* is definable in $\mathcal{PR}^{\omega e}$. Thus, $\mathcal{PR}^{\omega}$ stays within the realm of subrecursiveness. As for PTWP^e, the concept of higher type *writable variables* $U^{\iota\to\iota}$ is introduced, providing the possibility of creating and manipulating partial sequences.

It is shown that the PTWP^e-computable functionals coincide with those definable in $\mathcal{PR}^{\omega e}$ plus a constant $\mu^{(\iota\to\iota)\to\iota}$ for *sequential minimization*. In particular, the functionals definable in $\mathcal{PR}^{\omega e}$ denoted $\mathcal{R}^{\omega e}$ can be characterized by a subclass of PTWP^e-computable functionals denoted PPR$^{\omega e}$. Moreover, hierarchies $(\mathcal{R}_n^{\omega e})_{n\in\mathrm{N}}$ and $(\mathrm{PPR}_n^{\omega e})_{n\in\mathrm{N}}$ of strictly increasing complexity classes are introduced such that $\mathcal{R}_n^{\omega e} = \mathrm{PPR}_n^{\omega e}$ for all n. These results extend those for $\mathcal{PR}^{\omega}$ and PTWP, in particular, $\mathrm{PPR}_n^{\omega} = \mathcal{R}_n^{\omega} \subset \mathcal{R}_n^{\omega e}$ for all n.

Rounding off the talk, the closure of $\mathcal{R}^{\omega e}$ under SCVR is employed to define for each type σ the *enumeration functional* E^σ of all *finite elements* of D_σ, thus demonstrating that $\mathcal{PR}^{\omega e}$ satisfies a fundamental requirement for *finitistic* computation on Scott domains. The result should be contrasted to Plotkin's approach to defining all E^σ's, using his continuous approximation to the existential quantifier.

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[2] K. -H. Niggl, *Subrecursive hierarchies on the partial continuous functionals*, ***Ph.D. Thesis***, Munich, October 1994.

[3] K. -H. Niggl, *Non-definability of the Ackermann function with type* 1 *partial primitive recursion*, to appear in ***Archive for Mathematical Logic***, 1997.

[4] G. D. Plotkin, LCF *considered as a programming language*, ***Theoretical Computer Science***, vol. 5, 1977, pp. 223–255.

► VICTOR SELIVANOV, *On recursively enumerable structures*.
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Recursive algebra was established by several famous mathematicians including M. O. Rabin and A. I. Malcev. The key notions of a recursive and of a r.e. (or positive) structure were given in the early sixties. In the subsequent development of recursive algebra most attention was paid to recursive structures, though r.e. models of some theories (e.g., vector spaces) were considered in some details.

Recursive structures seem to be insufficient for a complete treatment of effectiveness in algebra for the following reasons: many interesting finitely presented structures are r.e. but not recursive (for the case of groups this was shown by P. S. Novikov and W. W. Boone); the so-called least model of a logic program is r.e. but not always recursive; natural classes of recursive structures may have no "acceptable" numerations.

In this talk we try to find a new flavour to the theory of r.e. structures and argue that r.e. structures are in some respects more "regular" and have a better theory than the recursive structures (a remote analogy to this is the situation with r.e. and recursive sets).

Our first main result states that the class of r.e. models of any given recursively axiomatisable quasivariety Q has an "acceptable" numeration (i.e., a uniformly r.e. sequence of Q-models to which any other such a sequence is reducible), and this numeration is unique up to recursive isomorphism of numerations.

We also show that the acceptable numeration for most natural theories Q has a nice characterization in terms of complete numerations. This enables us to consider natural decision problems on r.e. structures in a way similar to the study of index sets in recursion theory. We classify some of these problems and discuss the relation of this to the decidability of elementary theories and some applications to recursion theory.

Our second main result states the existence of a universal r.e. Q-model (i.e., an r.e. Q-model U such that for any r.e. Q-model A there exists an r.e. Q-model B with $A \times B$ recursively isomorphic to U). Again, the universal r.e. Q-model is unique up to recursive isomorphism of r.e. structures.

Note that our main results are similar to the existence of an acceptable numeration of r.e. sets and of a creative set, respectively.

Abstracts of contributed talks in the Special Session in Proof Theory

▶ MATTHIAS BAAZ AND RICHARD ZACH, *Generalizing theorems in algebraic closed fields*.

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Krajíček recently [2, Problem 23] asked the question: Is there a generalization result in the theory of real closed fields of the form: If $A(1 + \cdots + 1)(n$ occurrences of 1 is provable in length k for all $n \in \omega$, then $(\forall x)A(x)$ is provable? That question was answered affirmatively for the usual axiomatizations of the theory of real closed fields in [1]. We extend this problem to algebraic closed fields of characteristic 0 are axiomatized by the usual axioms of algebraic closure together with $1 + \cdots + 1 \neq 0(p$ occurrences of 1), where p is prime.

[1] M. Baaz and R. Zach, *Generalizing theorems in real closed fields*, ***Annals of Pure and Applied Logic***, 1995, to appear.

[2] P. Clote and J. Krajíček, *Open problems*, ***Arithmetic, proof theory, and computational complexity***, Oxford University Press, 1993, pp. 1–19.

▶ ANJOLINA GRISI DE OLIVEIRA and RUY J. G. B. DE QUEIROZ, *A new basic set of transformations between proofs*.

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Deductive systems based on the so-called Curry-Howard isomorphism [8] have an interesting feature: normalization and strong normalization (Church-Rosser property) theorems can be proved by reductions on the terms of the functional calculus. Exploring this important characteristic, we have proved these theorems for the *Labelled Natural Deduction—LND* [4] via a term rewriting system (*TRS*) constructed from the *LND*-terms of the functional calculus [2, 3]. The *LND* system is an instance of Gabbay's *Labelled Deductive Systems—LDS* [5], which is based on a generalization of the functional interpretation of logical connectives [6] (i.e., Curry-Howard isomorphism).

Proving the *termination* and *confluence* properties for the *TRS* associated to the *LND* system (*TRS-LND*) [2], we have in fact proved the normalization and strong normalization theorems for the *LND* system, respectively. The *termination* property guarantees the existence of a normal form of the *LND*-terms, while the *confluence* property its uniqueness. Thus, because of the Curry-Howard isomorphism, we have that every *LND* derivation converts to a normal form (**normalization theorem**) and it is unique (**strong normalization theorem**) [1, 3].

The significance of applying this technique in the proof of the normalization theorems lies in the presentation of a simple and computational method, which allowed the discovery of a new basic set of transformations between proofs, which we baptized as "ι (iota)-reductions" [1]. With this result, we obtained a confluent system which contains the η-reductions. Traditionally, the η-reductions have not been given an adequate status, as rightly pointed out by Girard in [7, p. 16], when he defines the *primary* equations, which correspond to the β-equations and the *secondary* equations, which are the η-equations. Girard [7] says that the system given by these equations is consistent and decidable, however he notes the following:

> "Although this result holds for the whole set of equations, one only ever considers the first three. It is a consequence of the *Church-Rosser property* and the *normalization theorem* (...)"

The first three equations, referred to by Girard, are the *primary* ones, i.e., β-equations.

Applying the so-called *completion procedure*, proposed by Knuth and Bendix in [9], to *TRS-LND*, the following term, which causes a non confluence in the system, is produced (i.e., a divergent critical pair is generated): This term rewrites in two different ways:

1. $\rightsquigarrow_\eta w(c)$ **2.** $\rightsquigarrow_\zeta \mathrm{CASE}(c, \upsilon a_1.w(\mathrm{inl}(a_1)), \upsilon a_2.w(\mathrm{inr}(a_2)))$

[The η-reduction for the $\vee$ connective is framed as follows [4]:

$$\dfrac{c : \mathbf{A}_1 \vee \mathbf{A}_2 \qquad \dfrac{[a_1 : \mathbf{A}_1]}{\mathrm{inl}(a_1) : \mathbf{A}_1 \vee \mathbf{A}_2}\vee\text{-intr} \qquad \dfrac{[a_2 : \mathbf{A}_2]}{\mathrm{inr}(a_2) : \mathbf{A}_1 \vee \mathbf{A}_2}\vee\text{-intr}}{\mathrm{CASE}(c, \upsilon a_1.\mathrm{inl}(a_1), \upsilon a_2.\mathrm{inr}(a_2)) : \mathbf{A}_1 \vee \mathbf{A}_2}\vee\text{-elim} \quad \rightsquigarrow_\eta \quad c : \mathbf{A}_1 \vee \mathbf{A}_2$$

('υ' is an abstractor, similarly to 'λ')

and the ζ-reduction is defined as follows:

$$\dfrac{\dfrac{p : \mathbf{A}_1 \vee \mathbf{A}_2 \quad \begin{matrix}[s_1 : \mathbf{A}_1] \\ d(s_1) : \mathbf{C}\end{matrix} \quad \begin{matrix}[s_2 : \mathbf{A}_2] \\ e(s_2) : \mathbf{C}\end{matrix}}{\mathrm{CASE}(p, \upsilon s_1.d(s_1), \upsilon s_2.e(s_2)) : \mathbf{C}}}{w(\mathrm{CASE}(p, \upsilon s_1.d(s_1), \upsilon s_2.e(s_2))) : \mathbf{W}}\,\mathrm{r} \quad \rightsquigarrow_\zeta \quad \dfrac{p : \mathbf{A}_1 \vee \mathbf{A}_2 \quad \dfrac{\begin{matrix}[s_1 : \mathbf{A}_1] \\ d(s_1) : \mathbf{C}\end{matrix}}{w(d(s_1)) : \mathbf{W}}\,\mathrm{r} \quad \dfrac{\begin{matrix}[s_2 : \mathbf{A}_2] \\ e(s_2) : \mathbf{C}\end{matrix}}{w(e(s_2)) : \mathbf{W}}\,\mathrm{r}}{\mathrm{CASE}(p, \upsilon s_1.w(d(s_1)), \upsilon s_2.w(e(s_2))) : \mathbf{W}}.$$

Whilst 'r' is usually restricted to an *elimination* rule, we have relaxed this condition: it is only required that 'r' does not discharge any assumptions from the other (independent) branch, i.e., that the auxiliary branches do not interfere with the main branch.]

The method of Knuth and Bendix says that when a terminating system is not confluent it is possible to add rules in such a way that the resulting system becomes confluent. Thus applying this procedure to *TRS-LND*, a new rule is added to the system:

$$\mathrm{CASE}(c, \upsilon a_1.w(\mathrm{inl}(a_1)), \upsilon a_2, w(\mathrm{inr}(a_2))) \rightsquigarrow_\iota w(c).$$

Since terms represent proof-constructions in the *LND* system, this rule defines a new transformation between proofs:

ι-reduction-$\vee$

$$\dfrac{c : \mathbf{A}_1 \vee \mathbf{A}_2 \qquad \dfrac{\dfrac{[a_1 : \mathbf{A}_1]}{\mathrm{inl}(a_1) : \mathbf{A}_1 \vee \mathbf{A}_2}\vee\text{-intr}}{w(\mathrm{inl}(a_1)) : \mathbf{W}}\,\mathrm{r} \qquad \dfrac{\dfrac{[a_2 : \mathbf{A}_2]}{\mathrm{inr}(a_2) : \mathbf{A}_1 \vee \mathbf{A}_2}\vee\text{-intr}}{w(\mathrm{inr}(a_2)) : \mathbf{W}}\,\mathrm{r}}{\mathrm{CASE}(c, \upsilon a_1.w(\mathrm{inl}(a_1)), \upsilon a_2.w(\mathrm{inr}(a_2))) : \mathbf{W}}\vee\text{-elim} \quad \rightsquigarrow_\iota \quad \dfrac{c : \mathbf{A}_1 \vee \mathbf{A}_2}{w(c) : \mathbf{W}}\,\mathrm{r}.$$

Similarly, the *iota* reduction for the existential quantifier is defined [1], since, similarly to $\vee$, the quantifier $\exists$ is a "Skolem type" connective (i.e. in the *elimination* inference for this type of connective is necessary to open local assumptions):

ι-reduction-$\exists$

$$\dfrac{c : \exists x^{\mathbf{D}}.\mathbf{P}(x) \qquad \dfrac{\dfrac{[t : \mathbf{D}] \quad [g(t) : \mathbf{P}(t)]}{\varepsilon y.(g(y), t) : \exists y^{\mathbf{D}}.\mathbf{P}(y)}\exists\text{-intr}}{w(\varepsilon y.(g(y), t)) : \mathbf{W}}\,\mathrm{r}}{\mathrm{INST}(c, \sigma g.\sigma t.w(\varepsilon y.(g(y), t))) : \mathbf{W}}\exists\text{-elim} \quad \rightsquigarrow_{\iota} \quad \dfrac{c : \exists x^{\mathbf{D}}.\mathbf{P}(x)}{w(c) : \mathbf{W}}\,\mathrm{r}$$

(where 'ε' is an abstractor).

With this result, we believe that we have answered the question as to why the η-reductions are not considered in the proofs of the normalization theorems (*confluence* requires ι-reductions). However, by applying a computational and well-defined method, the completion procedure, it seems that this problem of the non-confluence caused by η-reductions are solved.

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▶ MARIAGNESE GIUSTO and ALBERTO MARCONE, *Lebesgue numbers and reverse mathematics*.

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We study properties of Polish (complete separable metric) spaces within the framework of subsystems of second order arithmetic, carrying out work originally started by Simpson and Brown (see [3, 4, 5] for details on how Polish spaces are coded within these subsystems).

A Polish space $\hat{A}$ is said to be *Lebesgue* if every open covering $\langle U_n : n \in \mathbf{N}\rangle$ of $\hat{A}$ has a Lebesgue number, i.e., there exists $q \in \mathbf{R}^+$ such that

$$\forall x \in \hat{A}\ \exists n \in \mathbf{N}\ B(x, q) \subseteq U_n$$

where $B(x, q)$ denotes the open ball of center x and radius q.

$\hat{A}$ is said to be *Atsuji* if every continuous function from $\hat{A}$ into another Polish space is uniformly continuous (see [1, 2] for general information on Atsuji spaces). The equivalence between Atsuji and Lebesgue spaces is mentioned in [2].

As usual in the context of subsystems of second order arithmetic a Polish space is said to be *compact* if it is totally bounded.

We prove the following results.

THEOREM 1. RCA_0 *proves that the following are equivalent*:

(1) WKL_0.

(2) *Every compact Polish space is Lebesgue.*

(3) *The closed interval* [0, 1] *is Lebesgue.*

THEOREM 2. RCA_0 *proves that every Lebesgue Polish space is Atsuji.*

THEOREM 3. ACA_0 *proves that every Atsuji Polish space is Lebesgue.*

At this time we do not know whether the implication "Atsuji implies Lebesgue" is equivalent to ACA_0 or can be proved in a weaker system as WKL_0.

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► MARKUS MARZETTA and THOMAS STRAHM, *The μ quantification operator in explicit mathematics with universes*.

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Systems of explicit mathematics were introduced by Feferman in [1] in order to give a logical account to constructive mathematics, and they have soon become relevant for proof theory and the foundations of computer science. In [2] Feferman studies universes in explicit mathematics in order to prove Hancock's conjecture about the proof-theoretic strength of Martin Löf 's theories ML_n ; to this extent he considers fixed point theories $\widehat{ID}_n$ with proof-theoretic ordinal cofinal in Γ_0. Marzetta [7, 6] analyses a constructive theory UTN where universes are introduced via a so-called limit axiom. In particular, he shows that $|\,\mathrm{UTN}\,| = \Gamma_0$. On the other hand, Feferman and Jäger [3] establish the proof-theoretic strength of several theories of explicit mathematics in conjunction with the quantification operator μ.

The aim of the present work is to examine theories with both universes and μ operator. We show that the theory $\mathrm{UTN} + \mu$ is proof-theoretically equivalent to $\bigcup_n \widehat{\mathrm{ID}}_n$. In a first step, by an asymmetric interpretation, we eliminate the limit axiom in favour of finitely many universes, ending up with theories $\mathrm{UTN}_k + \mu$. The subsequent treatment of $\mathrm{UTN}_k + \mu$ makes use of new systems $\widehat{\mathrm{ID}\,\Omega}_{k+1}$ of Peano arithmetic with ordinals, which is introduced in Jäger [4] and extended in Jäger and Strahm [5]. Our ordinal analysis yields $|\widehat{\mathrm{ID}\,\Omega}_k| = \gamma_k$, where $\gamma_0 = \varepsilon_0$ and $\gamma_{k+1} = \varphi\gamma_k 0$ and, therefore, $\widehat{\mathrm{ID}\,\Omega}_k$ is proof-theoretically equivalent to $\widehat{\mathrm{ID}}_k$. As a consequence, we see that μ does not raise the proof-theoretic strength of UTN.

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► ANTON SETZER, *A type theory for Mahlo universes*.
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In our thesis [5] we determined the proof theoretical strength of Martin Löf's type theory (MLTT) with W-type and one universe (there was parallel work on this by E. Griffor and M. Rathjen, see [2]). One natural task is, to design stronger type theories and determine its precise strength. In this talk, we will make a first, very small step in this direction and give a type theory which has strength $|\psi_{\Omega_1}\Omega_{M+\omega}|$ and is slightly stronger than and was motivated by the theory KPM, introduced and analyzed in [4]. We conjecture that it is stronger than the super-universe construction introduced by Palmgren in [3] and P. Dybjers general formulation of simultaneous inductive-recursive definitions [1]. We have heard, that E. Griffor is currently working on more general concepts, by which he will probably go far beyond what is achieved in this abstract.

The theory is a an extension of (MLTT) with W-type and one universe (here called V) in the version à là Tarski, by having a rule, which assigns to every function f mapping pre-universes in V to pre-universes (pre-universe means, that we have just a collection of sets) a new element of V which is a new universe, closed under f. Therefore we have the rules

$$\frac{f : (\Sigma x \in V.T(x) \to V) \to (\Sigma x \in V.T(x) \to V)}{\mathrm{Fix}(f) : V} \qquad \frac{a : T(\mathrm{Fix}(f))}{S(f,a) : V}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f))}{\mathrm{Restr}(f,b,c) : \Sigma x \in T(\mathrm{Fix}(f)).T(S(f,x)) \to T(\mathrm{Fix}(f))}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f))}{S(f,\mathrm{Restr}(f,b,c)0) = f(< S(f,b), \lambda x.S(f,cx) >)0 : V}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad c : T(S(f,a)) \to T(\mathrm{Fix}(f)) \quad d : T(f(< S(f,b), \lambda x.S(f,cx) >)0)}{S(f,\mathrm{Restr}(f,b,c)1d) = f(< S(f,b), \lambda x.S(f,cx) >)1d : V}$$

and rules expressing, that $\mathrm{Fix}(f)$ is a sub-universe of V like:

$$\frac{b : T(\mathrm{Fix}(f)) \quad x : T(S(f,b)) \Rightarrow c : T(\mathrm{Fix}(f))}{\sigma_f x \in b.c : T(\mathrm{Fix}(f))}$$

$$\frac{b : T(\mathrm{Fix}(f)) \quad x : T(S(f,b)) \Rightarrow c : T(\mathrm{Fix}(f))}{S(f, \sigma_f x \in b.c) = \sigma x \in S(f,b).S(f,c) : V}.$$

We determine a lower bound by giving a direct well-ordering proof up to the proof theoretic strength. Here we will extend methods in [5], by using the fixed-point operator to introduce universes, for finding big ordinals $\psi_M(a)$. The upper bound for this system can be (work in progress) determined by interpreting in a similar way as in [5] the type theory in a Kripke-Platek style theory KPM$^+$, with axioms which guarantee the existence of one recursive Mahlo M and of ω admissibles above M.

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► STAN J. SURMA, *From the closure-theoretic deductive methodology to some of its non-standard alternatives*.

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In *Uber die Rolle den transfiniten Schlussweisen in einer allgemeinen Idealtheorie* published in ***Mathematische Nachtrichten***, vol. 7 (1952), pp. 165–182, Jurgen Schmidt showed that, loosely speaking, $\mathrm{Th}(\mathrm{Cn}(\mathrm{Th})) = \mathrm{Th}$ and $\mathrm{Cn}(\mathrm{Th}(\mathrm{Cn})) = \mathrm{Cn}$ where Th is a closure system and Cn is a closure operator over the powerset $P(S)$ of a set S. As a move in the same direction I describe four methodological frameworks, each based on a single primitive term, i.e., on Cons, on Ln, on Max and on Sep.

(i) Cons derives from the intuitive idea of consistency and, by the suggested axiomatisation, it is characterised as a non-trivial (i.e., $S \notin \mathrm{Cons}$), hereditary (i.e., $Y \in \mathrm{Cons} \cap 2_X$ *implies* $X \in \mathrm{Cons}$) and regular (i.e., *any* $X \in \mathrm{Cons}$ *extends to an inclusion-maximal* $Y \in \mathrm{Cons}$) property of subsets of S;

(ii) An intuitive motivation for Ln, a Lindenbaum operator, comes from the well-known Lindenbaum extension lemma. Ln is characterised as a non-trivial, (i.e., $\mathrm{Ln}(S) = \emptyset$), extensive (i.e., $\mathrm{Ln}(X) \subseteq 2_X$), inclusive (i.e., $\mathrm{Ln}(\emptyset) \cap 2_X \subseteq \mathrm{Ln}(X)$), and antimonotonic (i.e., $X \subseteq Y$ *implies* $\mathrm{Ln}(Y) \subseteq \mathrm{Ln}(X)$) and regular (i.e., $X = Y$ *for any* $X \in \mathrm{Ln}(\emptyset)$ *and* $Y \in \mathrm{Ln}(X)$) operator from $P(S)$ to $P(P(S))$;

(iii) Max, intuitively a property of maximality, e.g., that of deductive completeness is characterised as a non-trivial (i.e., $S \notin \mathrm{Max}$), and regular (i.e., $X = Y$ *for any* $X \in \mathrm{Max}$ *and* $Y \in \mathrm{Max} \cap 2_X$) property of subsets of S; and

(iv) Sep can be motivated on intuitive grounds as logical independence. It is axiomatised here as a non-trivial (i.e., $S \notin \mathrm{Sep}(A)$), hereditary (i.e., $Y \in \mathrm{Sep}(A) \cap 2_X$ implies $X \in \mathrm{Sep}(A)$), exclusive (i.e., $X \in \mathrm{Sep}(A)$ *implies* $A \notin X$), convex (i.e., $X \in \mathrm{Sep}(A)$ *implies* $\{B : X \notin \mathrm{Sep}(B)\} \in \mathrm{Sep}(A)$) and regular (i.e., $X \in \mathrm{Sep}(A)$ implies that there is $Y \in \mathrm{Sep}(A) \cap 2_X$ such that $B \in Y$ for any B and C such that $Y \cup \{B\} \in \mathrm{Sep}(C)$) operator from S to $P(P(S))$.

It is proved that each of these frameworks can be translated into the standard, closure-theoretic framework in a one-to-one and theorem-preserving fashion. In other words, for any $\alpha, \beta \in \{\text{Cn}, \text{Th}, \text{Cons}, \text{Ln}, \text{Max}, \text{Sep}\}$,

$$\alpha(\beta(\alpha)) = \alpha \text{ and } \beta(\alpha(\beta)) = \beta.$$

The Cons-based methodology and that based on Ln look as most interesting alternatives to the Cn-based methodology. For any α and β, and for any n-ary operation # on S, I define "α is (#)-saturated" and prove that

$$\alpha \text{ is (\#)-saturated iff } \beta \text{ is (\#)-saturated}$$

where Cons is (#)-saturated iff X is (#)-complete for any Cons-maximal X; Ln is (#)-saturated iff X is (#)-complete for any $X \in \text{Ln}(\emptyset)$; Max is (#)-saturated iff X is (#)-complete for any $X \in \text{Max}$; and Sep is (#)-saturated iff X is (#)-complete for any Sep-maximal X; and where X is (#)-complete iff the fact that $(\#A_1A_2\dots A_n) \in X$ is equivalent to the fact that $\#^*(A_1 \in X)(A_2 \in X)\dots(A_n \in X)$, for any $A_1, A_2, \dots, A_n$, where $\#^*$ is the meta-linguistic correlate of (the object-linguistic) #. Also for any α I define "α is (#)-normal" and prove that

$$\alpha \text{ is (\#)-normal iff } \alpha \text{ is (\#)-saturated.}$$

The (meta-linguistic) normalisation procedure for α in terms of # is somewhat similar to but simpler than that based on the well-known (object-linguistic) disjunctive-conjunctive normal form for # because it makes disjunction and conjunction dispensable. The explicit definitions of terms involved in the normalisation procedure are provided in the full text. An expression of the kind "$Y \in 2_X$" means "$X \subseteq Y$".

Abstracts of contributed talks in the Special Session in Model Theory

► L. MAYER and M. C. LASKOWSKI, *Models with few substructures*.
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In [1], D. MacPherson and J. H. Schmerl study countable structures having only countably many nonisomorphic substructures. They work in the context of a finite, binary relational language and classify all such structures. In the stable case, they show that every countable structure in a finite, binary language which has only countably many nonisomorphic substructures is cellular. We extend this result to the class of countable stable structures in any finite relational language. We also present an example showing that this result does not extend to infinite relational languages.

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► VLADIMIR A. MOLCHANOV, *Nonstandard free objects over pseudovarieties of finite algebraic*.
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After the discovery of Eilenberg's theory of varieties [3] the notion of "pseudovariety" plays an importent role in the theory of finite semigroups and its applications. By a pseudovariety is meant a class of finite algebraic systems which is closed under the formation of homomorphic images, subsystems and finite direct products. Eilenberg, Schützenberger, Ash, Rhodes, Reiterman, Almeida and others investigated connections between the theory of pseudovarieties and the classical theory of varieties (see, e.g., [1, 2]).

On the other hand, analogous problems in the theory of varieties of topological algebraic systems were investigated in [4] using an approach based on the principles of Robinson's nonstandard analysis. A reduction of the theory to the case of finite (discrete) algebraic systems permits us to develop in [5, 6] the theory of pseudovarieties with the help of methods of the nonstandard analysis. In this note we investigate the nonstandard construction of free algebraic systems over pseudovarieties.

Let X be a uniform space, A a finite algebraic Ω-system and $\mathbf{K}$ a class of finite algebraic Ω-systems. Consider a uniformly continuous mapping $\theta : X \to A$, and in particular, an extension $\theta : W(X) \to A$ and a nonstandard extension ${}^*\theta : {}^*W(X) \to A$. Denote $\varepsilon_{A,\theta} = \ker {}^*\theta$ and

$$E(\mathbf{K}, X) = \bigcap\{\varepsilon_{A,\theta} : A \in \mathbf{K} \text{ and } \theta : X \to A\}.$$

THEOREM 1. *Let X be a uniform space, Ω a type of algebraic systems with only a finite number of predicate symbols and $\mathbf{K}$ a pseudovariety of finite algebraic Ω-systems. Then the algebraic system $F(\mathbf{K}, X) = {}^*W(X)/E(\mathbf{K}, X)$, together with the canonical mapping $i_X : X \to F$ is the free $\mathbf{K}$-system over the space X, i.e., F is a X-generated algebraic system and, for any $A \in \mathbf{K}$ and a uniformly continuous mapping $\theta : X \to A$, there exists a uniformly continuous homomorphism $\varphi : F \to S$, such that $\varphi \circ i_X = \theta$.*

Using this result we obtain a nonstandard description of free objects over semidirect products of pseudovarieties of finite semigroups and investigate problems in the theory of finite semigroups connected with this operation.

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▶ MARCIN MOSTOWSKI, *Classifying finite order concepts*.
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The work presented is an attempt of describing the hierarchy of second and higher order concepts by means of the method of truth-definitions invented by Tarski.

Obtained new results were partially presented at **X** Latin American Symposium on Mathematical Logic, in Bogota, 24–29 July 1995. The method applied in their proofs works equally well for obtaining negative and positive results and for both restricted to finite models.

By a truth-definition in finite models for a class of formulae X we mean a formula $\varphi(x)$ such that for each $\psi \in X$ there is $k \in \omega$ and for each finite model M of cardinality $> k$, if M is a model for suitable theory of syntax for finite models, then

$$M \models \psi \equiv \varphi(\ulcorner \psi \urcorner).$$

For such truth-definition we obtain an analogon of Tarski theorem for finite models. However, truth-definitions in the finite case are much harder to obtain than in the infinite one. The easy example of such finite truth-definitions is $n+2$ order truth-definition for n-th order formulae, for any $n = 1, 2, \ldots$.

Abstracts of contributed talks in the Special Session in Non-monotonic Logic

▶ MATTHIAS BAAZ and CHRISTIAN G. FERMÜLLER, *Intuitionistic counterparts of finite-valued logics*.
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In recent years cut-free complete sequent calculi for the family of n-valued first-order logics have been investigated that arise by generalizing the classical binary sequent relation to an n-ary relation (see e.g., [3, 1, 2]). We extend these results by considering the family of logics that arise if we impose the same restriction to n-ary sequents that turns (binary) **LK**-sequents into **LJ**-sequents: Namely, allowing at most one formula to occur on some distinguished position of the sequent. Besides the interest in these new logics per se, we want to stress an important methodological motivation: In order to gain a better understanding of the relation between semantics and proof theory of classical and intuitionistic logics one needs to find a more general point of view with respect to which one can compare these particular logics. In particular, intuitionistic logic will appear as subcase of our "intuitionistic counterparts of finite-valued logics" in the same way as classical logic is just a particular finite-valued logic.

We define Kripke-style model structures and provide correctness and completeness proofs by generalizing Takeuti's proofs for the intuitionistic case. We also show that none (but pathological cases) of these logics admits a finite-valued truth table semantics. Moreover we investigate the admissibility of different candidates for the cut-rule.

This work is supported by FWF grant P10282–MAT.

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▶ SERGEI V. BABYONYSHEV, *Admissibility of inference rules in modal logics* $S4.1$ *and* $S4.2$.
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The modal and various kinds of intentional logics are used, in particular, in computer science for modelling knowledge representation systems. Such systems need a tool for derivation mechanism, which is usually based on some rule-like technique. On the other hand if we have a logic or more general deductive system then the problem of admissibility for rules is arising. The problem which is of interest is: what kind of inference rules do not expand our system after the join to the former system? Valid and truth types of the inference rules are investigated for instance in [1]. We restrict ourself only on structural syntactical rules of inference for modal propositional logics $S4.1$ and $S4.2$. This case we can consider only the rules with the unique premise.

DEFINITION. A rule of inference with parameters is a pair $A(x_i, p_j)/B(x_i, p_j)$, $(0 \leq i < n \leq j < n + k)$ where A,B are formulas and $\{p_j : n \leq j < n + k\}$, which is called set of parameters, is a subset of the set of variables.

DEFINITION. A rule $A(x_i, p_j)/B(x_i, p_j)$ of inference with parameters is admissible in a logic L iff for every set of formulas, $A_0, \dots, A_{n-1}$ if $A(A_i, p_j)$ is the theorem of L then $B(A_i, p_j)$ is the theorem of L.

The key concept of this paper is the notion of n-characterizing Kripke model for modal logic.

DEFINITION. A Kripke model T is n-characterizing for a modal logic L if for every formula A from n variables, A is a theorem of L iff A is valid on T.

The question about admissibility of inference rules is also interesting due to the fact that admissibility does not depend on the axiomatization or specific representation of system. The first criteria for admissibility in normal modal logic $S4$ and intuitionistic logic Int were obtained in [2]. In this work we use some results from [2] and [3].

As well known the logics $S4.1$ and $S4.2$ have a Kripke semantic and finite model property. The method used in the paper essentially depends on these facts. We also use the well-known

THEOREM (Semantic Criterion of Admissibility). *A rule with parameters $(A/B)(x_i, p_j)$ is admissible in a logic L provided that for every $T_n(L)$-n-characterizing model Kripke for L and for every formulistic valuation V on $T_n(L)$, if $A(x_i, p_j)$ is valid on $T_n(L)$ under valuation V, then $B(x_i, p_j)$ is valid on $T_n(L)$ under the same valuation.*

The main result is the following

THEOREM 1. *The problems of admissibility rules of inference with parameters in logics $S4.1$ and $S4.2$ are decidable.*

Using some tools from the proof of this theorem we can, in addition, prove

THEOREM 2. *There is an algorithm for constructing some solutions for logical equations in the modal propositional logics $S4.1$ and $S4.2$.*

COROLLARY 3. *The problems of admissibility rules of inference with parameters in logics $S4.1$ and $S4.2$ are NP-complete.*

[1] R. FAGIN, J. Y. HALPERN and M. Y. VARDI, *What is inference rule?*, ***Research report***, IBM Research Division Almaden Research Center, 1990.

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► J. HEIDEMA, W. A. LABUSCHAGNE, T. A. MEYER and H. ROSENBLATT, *A power-order semantics for non-monotonic logic*.
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Given a system of classical logic and a set Σ of axioms, consider the unary concept 'is entailed' and the binary notion 'is entailed by'. Each may be defined in terms of the frame consisting of the models of Σ: let $\wp\,\mathrm{Mod}(\Sigma)$ be the dual powerset Boolean algebra of sets of models of Σ ordered by $\supseteq$, then α is entailed (implicitly—by Σ) iff the set $\mathrm{Mod}(\alpha)$, comprising those models of Σ which are models also of α, is the minimum element, and α is entailed by

β (implicitly—in the context of the axioms Σ) iff $\mathrm{Mod}(\alpha) \supseteq \mathrm{Mod}(\beta)$.

In a system of non-monotonic logic, one wishes to replace the two classical concepts by the unary notion 'is plausible' and the binary notion 'is at least as plausible as' respectively. For example, non-monotonic systems based on minimal model semantics achieve this roughly as follows. Assume that in addition to the set Σ of axioms we have a preorder $\leq$ on the models of Σ. Then α is plausible iff $\mathrm{Mod}(\alpha) \supseteq X$, where X is the set of $\leq -$*minimal* models of Σ, and the relation 'is at least as plausible as' is the coarse relation that views any two plausible sentences as being each at least as plausible as the other.

We propose a power-order semantics for non-monotonic logic that lifts the preorder $\leq$ on models to a power-order $\leftharpoondown$ on sets of models. Given sentences α and β, we define α to be at least as plausible as β iff $\mathrm{Mod}(\alpha) \leftharpoondown \mathrm{Mod}(\beta)$, where in general for sets X and Y of models $X \leftharpoondown Y$ iff $(\forall y \in Y)(\exists x \in X)(x \leq y)$. And we define α to be plausible iff α is at least as plausible as Σ.

From these definitions it follows that

- If α is entailed (by Σ) then α is plausible (in the sense of power-order semantics).
- If α is entailed by β then α is at least as plausible as β.
- If α is plausible in the sense of minimal model semantics (and provided that models of Σ exist that are minimal with respect to $\leq$) then α is plausible in the sense of power-order semantics (relative to the power-order induced by $\leq$, of course).
- In terms of power-order semantics, α is plausible iff α is at least as plausible as $\neg\alpha$.

The value of the power-order semantics is that it permits us to represent reasoning that has no obvious representation using minimal model semantics. By way of illustration we treat two examples, one involving diagnostic reasoning (in the style of Reiter's model-based diagnoses) and the other involving a probabilistic intuition.

Abstracts of contributed talks in the Special Session in Theoretical Computer Science

► GERARD ALLWEIN and KATALIN BIMBÓ, *A system of ladder logic* (*PLC*).
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The Programmable Logic Controller is a commercial personnel and device protection tool used on control systems. The aim of this research is to give a formal description of the visual language, informally called "ladder logic", which comprises programs of the PLC, and to investigate—through formalism—the logical properties of ladder logic. Eventually, we hope to provide a basis for program verification of PLC programs.

We restrict our consideration to a proper subset of the actually existing PLC language which exhibits a logically interesting fragment in its own right. The language has two kinds of "conditions" ('⊣ ⊢', '⊣ / ⊢'), three kinds of "coils" ('–( )–', '–(L)–', '–(U)–'), an operator "once" ('[ons]'), two types of branching and a set of addresses. The branchings are denoted by a pair of '⌈' and '⌉', and their meaning depends on the symbols on the branches. The addresses are divided into two disjoint subsets, the set of internal addresses and the set of external addresses.

We show that it is possible to give a modal logical translation of the ladder logic, e.g., into a modal language with a unary modal operator 'θ', "at the previous step". However, the translation results in highly complex formulae already for simple diagrams. Therefore, we suggest a natural deduction style diagrammatic rule system to capture the notion of behaviorally equivalent ladders.

We give a version of a possible world semantics and we give also an algebraic semantics. We anticipate being able to prove that our rule system is complete w.r.t. defined semantics.

▶ DMITRY A. ARCHANGELSKY and MIKHAIL A. TAITSLIN, *Information systems*.
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Let G be an arbitrary set of binary relations. For each relation R from G, we define a relation R^s being symmetric to R that $R^{ss} = R$. We will consider all the relations obtained from above defined ones by means of set-theoretical operations: $\neg$, $\cap$, and $\cup$ —complement, intersection and union respectively. Let EREL be the set of all such relations. An abstract relational system of signature G is a pair $U = (\mathrm{OB}, G^U)$, in which OB is non-empty, and for each symbol R from G, a binary relation R^U on OB is assigned to the symbol. The set OB will be called the object set of U. We continue R^U on all the relational terms from EREL thereby terms $\neg R$, $R \cap S$, $R \cup S$ correspond to the complement of the relation R^U, the intersection of the relations R^U and S^U, and the union of the relations R^U and S^U respectively. More generally, we will consider quasi-systems $U = (\mathrm{OB}, \mathrm{EREL}\ U)$, in which OB is non-empty, and for each symbol R from EREL, a binary relation R^U on OB is assigned to the symbol. The set FOR of the formulas of the logic IL is defined as usually. FOR is the least set, contained set VAR and symbol d and closed respect to the propositional operations and respect to the modal operations (if R is a relational term and A is a formula, then $[R]A$ is a formula as well). $[R]A$ is true in a point $x \in \mathrm{OB}$ of a quasi-system U iff A is true in y for all $y \in \mathrm{OB}$ such that $xR^U y$. This conception of an information system has been introduced by Pavlak in [4]. The study has been continued in works of Pavlak and Orlowska [4]–[7] and in works of Vakarelov [1]–[3]. They had proposed some basic relations and had constructed a formal system of a modal logic describing the relations and some its Boolean combinations. The problem is to describe that, for example, $R \cap S$ and $\neg R$ are the intersection of R and S and the complement of R indeed. Our work is devoted to a generalization of the approach. A class of relation systems and a complete calculus construction method for these systems are proposed. As corollary of our main result, our paper contains a solution of a Vakarelov's problem [1]: to construct a formal system describing all the Boolean combinations of the basic relations. But a relation set unrestricted in some sense having been considered we also present a more formal and general version.

[This work was accomplished with the financial support of the Russian Foundation of Fundamental Researches (project code: 93-01-00490). The research described in this publication was made possible in part by Grant NYF000 from the International Science Foundation.]

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▶ ANATOLY P. BELTIUKOV, *Word stack register machines*.
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In this work the author gives a machine description of the word analogs of small function and predicate classes based on bounded recursion (by A. Grzegorczyk). It is proved that some above mentioned predicate classes coincide despite different growth rates of their generating functions. It is proved that the word analog of the unary functions class $\mathcal{E}_1^0$ has no finite basis with respect to superposition.

Consider a d-letter alphabet $\{a_1, \ldots, a_d\}$. Let $\mathcal{E}_d^0$ denote the word analog of the Grzegorczyk class $\mathcal{E}^0$ [1]. Let $\mathcal{E}_d(f)$ denote the similar class based on the function f as in [2]. Let A_* denote the class of all predicates (0-1-valued functions) of A.

Word stack register machine (WSRM) differs from the usual stack register machines [2] by having two kinds of stack registers: usual stack registers and command counters that count the number of value reassignment of corresponding stack register. WSRM have no general working register.

Word capacity of computation at WSRM is defined as the maximum of command counter values and stack register length. Let WSRM(f) denotes the set of all functions computable by WSRM within word capacity $f(x)$ where x is the maximum of the arguments values.

THEOREM 1. $\mathcal{E}_{d\ *}^0 = \text{WSRM(length)}$.

The construction of WSRM is used to prove the following

THEOREM 2. $E_{d\ *}^0 = E_d(x \| a_1^{\langle \text{length}(x)^{1-1/p} \rangle})_*$ *for all natural* p, *where* $\|$ *is concatenation,* $x^{\langle i \rangle} = x \ldots x$ (i *times*).

COROLLARY. *The class of unary functions* $E_{d\ 1}^0$ *has no finite basis with respect to superposition.*

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▶ JENS BLANCK, *Domain representability of metric spaces*.
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In this paper we consider effectivity for uncountable topological algebras. We do this in a uniform way using Scott-Ershov domains, i.e., consistently complete algebraic cpo's. Domain theory is a well-established general theory of approximations.

A topological algebra is domain representable by a structured domain if there is a continuous epimorphism from a substructure of the domain onto the topological algebra. Thus the algebra is represented as a quotient of a subspace of the representing domain. It has been shown that every locally compact topological algebra has a domain representation. We show that every metric algebra has a domain representation. Banach's fixed point theorem for complete metric spaces is then a consequence of the existence of least fixed points for continuous functions on the representing domains. These facts partially explain why both domains and metric spaces provide good semantics for programming languages. The fact that metric algebras, among others, are domain representable indicates the richness and versatility of the notion of domains in combination with a concept of totality.

We consider a natural notion of effective metric spaces and show that every such space can be represented by an effective domain. In this way the general effective theory of domains is exported to the represented algebra. As an example, Ceitin's theorem is proved using the generalisation of the Kreisel-Lacombe-Schoenfield theorem to domains. Applying the Kreisel-Lacombe-Schoenfield theorem in our situation forces us to simultaneously consider a weaker notion of effective domain, where the consistency relation on compact elements is not necessarily decidable.

▶ LI DAFA, *Improved Davis and Putnam's method.*
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The paper presents two sufficient conditions to check if a set of clauses is satisfiable. The conditions can be used with Davis and Putnam's method. And it is suggested to apply the splitting rule to the literal of which the number of occurrences in the set of clauses is greatest.

Some results about probability

RESULT 1.

$$\text{Let } p_1 = \sum_{i=1}^{m} p(E_i), \dots , p_k = \sum_{1 \leq < i_1 < i_2 < \cdots < i_k \leq m} p(E_{i_1} E_{i_2} \cdots E_{i_k}),$$

We obtain the following inequalities.

$$p\left(\bigcup_{i=1}^{m} E_i\right) \leq \sum_{i=1}^{2l+1} (-1)^{i+1} p_i, \text{ where } 2l+1 \leq m.$$

$$p\left(\bigcup_{i=1}^{m} E_i\right) \geq \sum_{i=1}^{2l} (-1)^{i+1} p_i, \text{ where } 2l \leq m.$$

Let S be a set of ground clauses and C a clause of the set S. F_C is a set of all interpretations falsifying the clause C. F_S is a set of all interpretations falsifying the set S. Clearly an interpretation I falsifies the set S iff it falsifies at least one clause of S, therefore $F_S = \bigcup_{c \in S} F_C$.

Let A be the atom set of the set S of ground clauses. If $|A| = n$, then from the discussion above there are 2^n interpretations of S. If $|F_S| = 2^n$ then clearly S is unsatisfiable. If $|F_S| < 2^n$ then S is satisfiable. **Clearly there are $2^n - |F_S|$ models of S.**

Let S be a set of ground clauses $C_1, \dots, C_m$. Let $s_1 = \sum_{i=1}^{m} |F_{C_i}|, \dots, s_k = \sum_{1 \leq < i_1 < \cdots < i_k \leq m} |F_{C_{i_1} \vee \cdots \vee C_{i_k}}|$, $C_i \vee C_j$ is a disjunction of clauses C_i and C_j.

We can derive the following results.

(1) If $\sum_{i=1}^{2l+1} (-1)^{i+1} s_i < 2^n$ for some l such that $2l + 1 \leq m$ then $|F_S| < 2^n$, that is, S is satisfiable.

Specially if $s_1 < 2^n$ then S is satisfiable.

(2) If $\sum_{i=1}^{2l} (-1)^{i+1} s_i \geq 2^n$ for some l such that $2l \leq m$ then $|F_S| = 2^n$ and S is unsatisfiable.

Specially if $s_1 - s_2 \geq 2^n$ then S is unsatisfiable.

Algorithm.

Step 1. Apply unit-clause rule
Step 2. Apply pure-literal rule
Step 3. If $s_1 < 2^n$ then S is satisfiable.
If $s_1 - s_2 \geq 2^n$ then S is unsatisfiable.
Step 4. Choose a literal l of which the number of occurrences among all literals in S of clauses, apply splitting rule to the literal.

Experiments. Pigeonhole Problem—hardest problem (by Cook and Reckhow 1979).

[This project supported by NSFC.]

▶ G. D'AGOSTINO, *Set-theoretic decidability results for modal theorem proving*.
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In [1,4], we proposed a *set-theoretic translation method* for modal logics that reduces derivability in a large class of propositional modal logics to derivability in a very weak first-order set theory, containing as axioms the standard axiomatizations of $\subseteq, \cup$, and $\setminus$ in terms of $\in$, a suitable axiomatization of Pow, and some simple consequences of Cantor's theorem on the number of subsets of a given set. Unlike most existing translation methods, the one we proposed applies to any normal complete finitely axiomatizable modal logic, regardless of whether it is first-order complete or an explicit semantics is available.

Based on the proposed translation method, we consider a suitable application of T-theorem proving (a specialized technique for automated theorem proving in first-order theories proposed in [7]) as a possibility for automatically testing modal derivability. A prerequisite to employing T-resolution in the context of a given theory T is the decidability of the class of T-derivable purely universally quantified formulae, which is the main topic of this note.

The decidability of classes very similar to the one we consider has already been proved by Cantone, Schwartz, and Ferro with respect to ZF [2,3]. However, such proofs do not apply to our (very restricted) context since they highly rely on axioms such as foundation, extensionality, and replacement. Our decidability proof is based on a suitable form of reflection theorem obtained by means of a technique first introduced in [6]. The main idea is the following: in order to establish whether there exists a model of our theory satisfying a formula $\varphi(x_1, \dots, x_n)$ (an unquantified formula written in the language of the theory), we assume that there exists a model M of it such that $M \models \varphi(x_1, \dots, x_n)$, and we concentrate our attention on n elements $a_1, \dots, a_n$ in the support of M satisfying φ. The goal is to show that, under this hypothesis, we can build another (*simpler*) n-tuple $a_1^*, \dots, a_n^*$ of elements in the support of a model M' of the theory still satisfying φ. The elements $a_1^*, \dots, a_n^*$ are completely described by a graph G whose size is bounded by a recursive function of n, in the sense that, in order to test the existence of (M' and) $a_1^*, \dots, a_n^*$, it is sufficient to test the existence of G, and this result guarantees the decidability.

The problem of determining G is combinatorially non-trivial. Notice that if in the formula $\varphi(x_1, \dots, x_n)$ we had no conjuncts of the form $\mathrm{Pow}(x_i) = x_j$, then we could define G simply as the graph having $a_1, \dots, a_n$ as nodes and $\in^M$ as binary relation. In such a case we could define a model M' with elements $a_1^*, \dots, a_n^*$ such that $a_i^* = \{a_j^* \mid a_j \in^M a_i\}$, and it would be easy to check that all our requirements are satisfied (recall that we do not have to deal with the extensionality axiom). In general, we can think of the map * as a way of *marking* some of the elements in each of the a_i and then take a_i^* as the set of (images with respect to $*$ of) marked elements in a_i. In order to deal with a literal of the form $\mathrm{Pow}(x_i) = x_j$, we need to mark more elements: at least all those elements which are subsets of the set of marked elements in a_i. Notice that if one simply does so and marks all such elements (subsets of a_i) without "care", new elements can turn out marked in a_i, and the marking process may not terminate. We solved the above problem processing the a_i's in a suitable order whose existence is guaranteed by the consequences of Cantor's theorem that we assume as axioms in our theory precisely for this purpose (the details of the proof are given in [5]).

We are currently trying to generalize the above decidability result in order to prove the decidability of classes of normal modal logics.

[Joint work with A. Montanari and A. Policriti.]

[1] J. VAN BENTHEM, G. D'AGOSTINO, A. MONTANARI and A. POLICRITI, *Modal deduction in second-order logic and set theory*, ***Research Report in the ILLC-series, ML-95-02***, University of Amsterdam, February 1995, to appear in the ***Journal of Logic and Computation***.

[2] D. CANTONE, *A decision procedure for a class of unquantified formulae of set theory involving the powerset and singleton operators*, ***Ph.D. Thesis***, New York University, 1986.

[3] D. CANTONE, A. FERRO and E. G OMODEO, ***Computable set theory***, vol. 1, Oxford University Press, Int. Series of Monographs on Computer Science, 1989.

[4] G. D'AGOSTINO, A. MONTANARI and A. POLICRITI, *A set-theoretic translation method for polymodal logics*, ***Proceedings of STACS'95***, München, Germany, LNCS 900, Springer-Verlag, March 1995, pp. 217–228, also published in ***Journal of Automated Reasoning***, vol. 15 (1995), pp. 317–337.

[5] ———, *Set-theoretic decidability results for modal logic*, ***Research Report***, Università di Udine, forthcoming.

[6] F. PARLAMENTO and A. POLICRITI, *Decision procedures for elementary sublanguages of set theory XIII. Model graphs, reflection and decidability*, ***Journal of Automated Reasoning***, vol. 7 (1991).

[7] A. POLICRITI and J. T. SCHWARTZ, *T Theorem proving I*, ***Research Report 08/92***, Università di Udine, July 1992, also published in ***Journal of Symbolic Computation***, vol. 20 (1995), pp. 315–342.

► EVGENY DANTSIN, *A randomized extension of arithmetic*.
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The notion of a randomized (interactive) proof is rather different from the standard notion of a proof in mathematial logic. The present paper aims at bridging the gap between these two notions. We define randomized proofs as derivations in formal arithmetic extended by one additional rule. This rule, called the random substitution rule, allows us to use the following inference (we omit details): suppose we have proved that an arithmetical formula $F(x)$ is true for "almost all values of x less than N", i.e., for all $x < N$ excepting not more than $\log N$ values; then we can derive $F(r)$ where r is picked randomly from $0, 1, \ldots, N-1$.

Of course, applications of this rule can lead to false conclusions and, therefore, a derivation of a formula S in arithmetic with random substitution does not guarantee that S is true. However, if all N's are sufficiently large in comparison with the number of applications of the rule then S is true with large probability. In particular, if we restrict ourselves to polynomially long derivations with exponentially large N's (in the size of S) then the error probability is exponentially small.

Such derivations in arithmetic with random substitution can be viewed as a special kind of interactive proof and, more exactly, as a special kind of Arthur-Merlin proof. Besides, as it is shown in the paper, our extension of arithmetic is enough to formalize the known technique for proving IP=PSPACE. We construct a PSPACE-complete set of arithmetical theorems and show that they have polynomially long proofs in arithmetic with random substitution (independently of the random numbers that appear) and the error probability of these proofs is exponentially small. Thereby, we give an example of arithmetical theorems having short proofs in the randomized extension of arithmetic but not having short proofs in arithmetic (assuming that PSPACE is not equal to NP).

▶ PAOLA GLAVAN and DEAN ROSENZWEIG, *Evolving algebras and (light) linear logic*.
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Extending our previous work on concurrent evolving algebras (and adapting it to Gurevich's new definition of a distributed EA), we have developed an EA-based computational model of cut-elimination for linear logic, related to 'linear CHAM' of Abramsky.

The model is adapted to 'light linear logic' introduced recently by Girard—in terms of linear CHAM, exponential boxes are taken out of terms and listed separately, so that a 'proof-expression' becomes a triple of sequences, of coequations, exponential boxes and (sequences of) terms. This provides a definition of 'light linear CHAM' which can be, by techniques we have previously used for an EA-based computational model of Milner's pi-calculus, steered so as to follow the lazy procedure of Girard. Since that procedure, for 'lazy sequents', runs in polytime, we obtain a class of highly distributed evolving algebra algorithms simulating exactly all polytime computations.

The model is then easily adaptable to 'elementary linear logic' as well, yielding the same kind of result for elementary computations.

The implications of these models remain to be analysed.

▶ MASAFUMI KUDOH and HISAO TANAKA, *On some oracle results on the bounded error probabilistic polynomial time complexity class* BPP.
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We show a measure one separation theorem and a Baire category separation theorem for subclasses of the relativized bounded-error probabilistic polynomial time complexity class, and also show existence of an oracle H for which the class $\{X : \mathrm{P}[X] \neq \mathrm{BPP}[H \oplus H]\}$ is comeager.

Strings mean finite binary strings, and oracles and languages mean sets of strings. Let k be a positive integer, and let X be an oracle. Then BPP$[k]$ (resp. BPP$[k, X]$) is the class of all languages accepted by a probabilistic $O(n^k)$ time bounded Turing machine (resp. oracle Turing machine with oracle X) with error probability uniformly bounded below $\frac{1}{2}$. Then clearly BPP$[k]$ is contained in BPP$[k+1]$. Then, by a standard method, we can show that for any k, $\mathrm{BPP}[k] = \mathrm{BPP}[k+1]$ implies $\mathrm{BPP}[k+1] = \mathrm{BPP}[k+2]$. However, whether BPP$[k]$ is properly contained in BPP$[k+1]$ is an open problem [3]. This seems to be a hard problem because they are 'semantic' classes but not 'syntactic' ones. So, we consider their relativized counterparts. Fortnow-Sipser [2] show a collapsing theorem: There is an oracle F such that $\mathrm{BPP}[F] = \mathrm{BPP}[1, F]$ holds, where BPP$[F]$ is the union of BPP$[k, F]$'s for all k. On the contrary we can show that there is an oracle A such that for every k, $\mathrm{BPP}[k, A] \neq \mathrm{BPP}[k+1, A]$. Now, how many such A's are there? For this question we have the following results. Let $\mathrm{SEP} = \{X : \forall k(\mathrm{BPP}[k, X] \neq \mathrm{BPP}[k+1, X])\}$. Then almost all oracles belong to SEP, namely:

THEOREM 1. *The measure of* SEP *is one.*

The class is also topologically large.

THEOREM 2. SEP *is comeager in the sense of Baire category.*

Next, we consider a relativized version of a problem in [1]. Bennett-Gill suggested the class $\mathrm{SECES} = \{X : \mathrm{P}[X] \neq \mathrm{BPP}[X]\}$ might be comeager. We give its relativized answer.

Namely

THEOREM 3. *There is an oracle H such that the relativized class* $\mathrm{SECES}^H = \{X : \mathrm{P}[X] \neq \mathrm{BPP}[H \oplus X]\}$ *is comeager, where $H \oplus X$ is the usual disjoint union of H and X.*

Note that for each H there is an A such that $\mathrm{P}[A] = \mathrm{BPP}[H \oplus A]$ holds. For the $\mathrm{P}[X] \neq ?\,\mathrm{NP}[X] \cap \mathrm{coNP}[X]$ question, we have a similar result. We do not know whether such X's constitute a comeager class.

[1] C. H. BENNETT and J. GILL, *Relative to random oracle A,* $\mathrm{P}^A \neq \mathrm{NP}^A \neq \mathrm{coNP}^A$ *with probability* 1, ***SIAM Journal on Computing***, vol. 10 (1981), pp. 96–113.

[2] L. FORTNOW and M. SIPSER, *Probabilistic computation and linear time*, ***STOC '89***, pp. 148–156.

[3] M. KARPINSKI and R. VERBEEK, *Randomness, provability, and the separation of Monte Carlo time and space*, ***Computation theory and logic*** (E. Borger, editor), Lecture Notes in Computer Science, vol. 270, Springer-Verlag, 1987, pp. 189– 207.

► OLIVER KULLMANN, *Algorithms and upper bounds for* SAT*-decision*.
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The aim of this talk is to survey algorithmic techniques for SAT-decision as well as corresponding estimation techniques for upper bounds on time complexity, and to argue that improved algorithms need improved estimation methods.

The algorithms we consider use polynomial reductions and branchings. For both an adequate understanding of "formula complexity" (in terms of a "measure") is essential, as we try to exemplify by the results and methods of [3] and [2] (containing the best known upper bounds for SAT-decision on NP-complete classes of CNF's).

Finally we compare our approach to a heuristical one as in [1] (obtaining some insights in the supposed nature of "branching rules"), and make suggestions for future work.

[1] A. VAN GELDER and Y. K. TSUJI, *Satisfiability testing with more reasoning and less guessing*, to appear in DIMACS Series in Discrete Mathematics and Theoretical Computer Science.

[2] O. KULLMANN, *A systematical approach to* 3-SAT*-decision, yielding* 3-SAT*-decision in less than* 1.5045^n *steps*, preprint, Frankfurt, 1994, accepted by ***Theoretical Computer Science***.

[3] O. KULLMANN and H. LUCKHARDT, *Deciding propositional tautologies: Algorithms and their complexity*, submitted to ***Information and Computation***.

► LEO MARCUS, *Formal dependence and independence relations in computer security*.
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Protection of information from unauthorized disclosure, or "confidentiality", is a major concern of computer security.

We discuss model-theoretic formulations of variants of confidentiality properties. These definitions all represent ways for information about one variable or set of variables to be "learned" by observing the behavior of other variables relative to a "specification" (theory) T. If information about a can be learned by observing B, then a is dependent on B.

In this model-theoretic formulation, the algorithmic nature of a computation is ignored; we focus solely on the possible sequences of states generated by (or allowed by, or satisfying) T. These specifications are our first-order theories and the sequences of states are the models. In the simplest case, information about program variable a is learned from the program variables B if there is a restriction of some T-computation to a and a restriction of a T-computation to B which cannot be jointly expanded to form a full T-computation.

The four main classes of confidentiality we discuss are simple dependence and independence, partial and probabilistic dependence, masked dependence (in which only certain properties of the restrictions of the T-computations can be used), and bounded (or finite) dependence (in which only initial segments of the restrictions of the T-computations can be used). These properties correspond to different possibilities for specifying the kind of confidentiality desired in a computer system, for example an operating system or a database system.

The definitions of simple dependence and independence satisfy the axioms for dependence or independence as given by Baldwin [1]; a similar axiom system was considered by Pearl [2]. The other definitions violate symmetry in general. The definitions, given in a general model-theoretic setting, may be of interest in their own right. This level of abstraction appears to be the appropriate one at which the essential distinctions between the varieties of confidentiality can be isolated and studied.

In real world computer security, a typical situation has users and objects divided into classes, where users at a higher class have access to objects (data) at lower levels, but not the reverse. This is obviously an asymmetric relation. There are two aspects to this situation: a prohibition and a permission. Our definition of simple independence relates only to the prohibition aspect: lower level user b cannot deduce anything about upper level user a's computation by looking only at his own (b's) computation (and using information available through T) and vice versa. The asymmetry enters through the other half of confidentiality, the permission aspect, which is formalized in masked independence: a user a is not restricted to look only at his own computation, but can examine that of users through a mask, which essentially specifies what information about the other users a is allowed to observe directly. Each user level will have a different mask. The prohibition side of masked independence is that user a cannot deduce more about another user than what he is permitted by the mask.

[We thank the Aerospace Sponsored Research Program for support and Telis Menas for helpful discussions.]

[1] J. T. Baldwin, *First-order theories of abstract dependence relations*, ***Annals of Pure and Applied Logic***, vol. 26 (1984), pp. 215–243.

[2] Judea Pearl, ***Probabilistic reasoning in intelligent systems: Networks of plausible inference***, Morgan Kaufmann Publishers, San Mateo, California, 1988.

► DAN E. WILLARD, *Introspective semantics for Turing machine languages*.
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We describe a generalization of the IS(A) systems from [2, 3, 4], called ISTM(A), which recognizes the usefulness of Turing Machines as an aid for proving theorems. The deduction method employed by ISTM(A) may be any of Gentzen's cut-free version of the sequent calculus, semantic tableaux or resolution. Define $F(a_1, a_2, \dots, a_j)$ to be a **non-growth function** iff $F(a_1, a_2, \dots, a_j) \le \text{Maximum}(2, a_1, a_2, \dots, a_j)$. With the exception of Successor, all the functions recognized by ISTM(A) will be non-growth functions. The non-growth functions include Subtraction, Division, Logarithm, Square-Root, Right-Shift, Maximum and Mininimum to simulate the basic arithmetic functions. Given an arbitrary universal Turing Machine $\mathcal{U}$, with k distinct right-sided tapes, ISTM(A) will differ from the prior IS(A) systems by containing the following three classes of Turing Simulation functions:

(1) $\text{Tape}_i(Z, x, t)$ designating a number 0, 1, or 2, according to which letter is stored on

$\mathcal{U}$'s i-th tape in position x at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes.

(2) $\text{Head}_i(Z, t)$ indicating the address of the i-th tape's head at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes.

(3) $\text{State}_i(Z, t) = 1$ if the machine $\mathcal{U}$ is in state i at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes. $\text{State}_i(Z, t) = 0$ otherwise.

Given a fixed constant λ and an arbitrary initial axiom system A, $\text{ISTM}(A)$ is defined to include the following three groups of axioms:

Group-1. An initial finite set of Π_1 axioms which formally define the properties of $\text{ISTM}(A)$'s base functions and of the two relations $=$ and $<$.

Group-2. An infinite axiom schema which for each Π_1 sentence Φ, asserts if a proof of Φ from A exists then Φ.

Group-3. For arbitrary x and arbitrary tuple Z, the assertion that the machine $\mathcal{U}$ is unable to produce a proof from $\text{ISTM}(A)$ of $0 = 1$ on any of its tapes when it runs for x^λ units of time.

Some of our main theorems about $\text{ISTM}(A)$ are listed below:

(1) If the union of A with Group-1 (above) is consistent then $\text{ISTM}(A)$ is consitent.

(2) The above theorem is not true for all A when $\text{ISTM}(A)$'s Group-1 axioms are merely expanded to incorporate Addition as a total function.

(3) $\text{ISTM}(A)$ may use a version of deduction that permits Gentzen-style cuts provided the Successor function is removed from Group-1. (Added Comment: Robert Solovay [1] has proven a theorem which implies that $\text{ISTM}(A)$, with Gentzen-style cuts, can be inconsistent if Successor is not removed.)

(4) The several techniques, itemized in our preceding papers [2, 3, 4], to strengthen $\text{IS}(A)$'s Group-3 axiom schema are also applicable for strenghtenning $\text{ISTM}(A)$. For example, Group-3 could state that if for some Z, x and Σ_1 formula Ψ, $\mathcal{U}$ can construct in time $O(x^\lambda)$ a proof from $\text{ISTM}(A)$ of $\forall x\, \Psi(x)$ then $\Psi(\bar{c})$ is valid for each fixed $\bar{c}$

[1] R. SOLOVAY, private communications, April 1994.

[2] D. WILLARD, *Self-verifying axiom systems*, ***Proceedings of the Third Kurt Gödel Symposium***, 1993, pp. 325–336, published in Springer-Verlag Lecture Notes in Computer Science, vol. 713.

[3] ———, *Self-verifying axiom systems and the incompleteness theorem*, talk presented at the 1994 Annual Meeting of the ASL, summary of talk in this JOURNAL (1995), pp. 136–137, the journal version of this article available as University of Albany TR 93-10 and to appear in ***Journal of Symbolic Logic***.

[4] ———, *Stronger variants of self-verifying axiom systems*, talk presented at the Association for Symbolic Logic's 1995 Winter Meeting, summary of talk available in this JOURNAL (1995), pp. 371–372.

Abstracts presented by title

► STANLEY BURRIS and ANDRÁS SÁRKÖZY, *Fine spectra and first-order limit laws*.
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Using Feferman-Vaught techniques we show a certain property of the fine spectrum of an admissible class of structures leads to a first-order law. The condition presented is best possible in the sense that if it is violated then one can find an admissible class with the same fine spectrum which does not have a first-order law.

Our first result for verifying that the above condition actually holds is to show that the count function of an admissible class has regular variation with a certain uniformity of convergence. This condition applies to a wide range of admissible classes, including those satisfying Knopfmacher's Axiom A, and those satisfying Bateman and Diamond's condition.

The second result applies to the case that the sizes of the structures in an admissible class K are all powers of a single integer, and there is a uniform bound on the number of K-indecomposables of any given size.

► D. A.BREDIKHIN, *Graphs calculus and quasieqational theories of relation algebras*.
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Let **K** be the class of algebras which is isomorphic to ones whose elements are binary relations and whose operations are the following: relation product $\circ$, relation inverse $^{-1}$, intersection $\cap$ and the diagonal relation Δ. This class was introduced by B. Jónsson [1]. The equational theory of **K** was characterized in [2,3]. Here the quasiequational theory of **K** shall be described.

Graphs Calculus. A (2-pointed labeled) graph is a structure $G = (V, E, i, o)$ where V is a set, $i, o \in V$ and $E \subset V \times N \times V$ $(N = 1, 2, \dots)$. We write that $G_1 \prec G_2$ if there exists an homomorphism from G_2 to G_1. Denote by $G_1[u, v, G_2]$ a graph which is obtained from the disjoined union of G_1 and G_2 by identifying i_2 with u and o_2 with v.

Let us consider a formal system $\mathcal{L}$ determined as follows. Formulae of $\mathcal{L}$ are expressions of the form $\varphi = G_1 \Rightarrow G_2$ where G_1 and G_2 are graphs. Axioms: $G \Rightarrow G$. Rules of inference:

$$\frac{G_1 \Rightarrow G_2, G_2 \Rightarrow G_3}{G_1 \Rightarrow G_3}, \quad \frac{G_1 \Rightarrow G_2, G_2 \prec G_3}{G_1 \Rightarrow G_3}, \quad \frac{G_1 \Rightarrow G_2}{G[u, v, G_1] \Rightarrow G[u, v, G_2]}.$$

We write that $\varphi_1, \dots, \varphi_n \vdash \varphi_0$ if φ_0 deduce from $\varphi_1, \dots, \varphi_n$.

Let $(A, \cdot, ^{-1}, \wedge, 1)$ be an algebra of the type $(2, 1, 2, 0)$. With each its term p can be associated a graph $G(p)$ (see in [2,3]).

THEOREM 1. *The quasi-identity* $\bigwedge_{k=1}^{n} p_k = q_k \to p_0 = q_0$ *belongs to the quasiequational theory of* **K** *if and only if* $\varphi_1, \dots, \varphi_n \vdash \varphi_0$ *and* $\tilde{\varphi}_1, \dots, \tilde{\varphi}_n \vdash \tilde{\varphi}_0$ *where* $\varphi_k = G(p_k) \Rightarrow G(q_k)$ *and* $\tilde{\varphi}_k = G(q_k) \Rightarrow G(p_k)$.

This result can be generalized to classes of algebras of relations with primitive-positive operations.

[1] B. JÓNSSON, *Representation of modular lattices and relation algebras*, ***Transactions of the American Mathematical Society***, vol. 92 (1959), pp. 449–464.

[2] H. ANDRÉKA and D. A. BREDIKHIN, *The equational theory of union-free algebras of relations*, ***Algebra Universales***, to appear.

[3] D. A. BREDIKHIN, *The equational theory of algebras of relations with positive operations*, ***Izvestia Vysshikh Uchebnykh Zavedenii, Matematicheskaya***, vol. 3 (1993), pp. 23-30, Russian.

► ZINOVY DISKIN, *Towards graph-based algebraic model theory for computer science*.
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For theoretical support of modern information technologies Computer Science needs a model theory framework for studying structures over graphs whose nodes are sets and edges are functions. A suitable concept was developed in category theory under the name of *sketches*. However, while the familiar notion presupposes a few fixed diagram properties (commutativity, (co)limitness), the applications require the possibility to set arbitrary vocabularies of diagram properties.

DEFINITION 1. (i) *A (finitary) predicate signature* is a set π of symbols called *markers* s.t. each $m \in \pi$ is ssigned with a (finite) graph D_m called the *arity shape* of m.

A π-sketch is a couple $S = (G, \mathcal{D})$ where G is a graph and $\mathcal{D} = (\mathcal{D}_m, m \in \pi)$ is a family with $\mathcal{D}_m \subset \mathbf{Hom}(D_m, G)$ a collection (maybe, empty) of diagrams *marked* by m. *A morphism* of π-sketches $S \to S'$ is a graph morphism $h\colon\ G \to G'$ s.t. $\delta; h \in \mathcal{D}'_m$ as soon as $\delta \in \mathcal{D}_m$.

(ii) *An operation signature over π* is a set φ of *operation markers* s.t. each $q \in \varphi$ is assigned with a π-sketch inclusion $D_q^{\text{in}} \subset D_q^{\text{out}}$ with D_q^{out} called the *output shape* of the operation and D_q^{in} *the input* one. An operation is called *finitary* if its output shape is finite.

A π-sketch A is said to be *φ-algebra* if for any $q \in \varphi$ there is defined a mapping

$$q^A\colon\ \mathbf{Hom}_\pi(D_q^{\text{in}}, A) \to \mathbf{Hom}_\pi(D_q^{\text{out}}, A)$$

s.t. $q^A(\mu)\lceil_{D_q^{\text{in}}} = \mu$ for any morphism $\mu\colon\ D_q^{\text{in}} \to A$.

Since an operation marker is simultaneously a predicate marker of arity D_q^{out}, the construction can be iterated. Actually, a *signature σ* of markers should be a partly ordered set s.t. the arity sketch of a marker q can involve only markers preceding q.

Starting from these basic definitions, a conceptual framework for graph-based model theory is proposed in the paper. The main technical contribution is the accurate development of the machinery of diagram operations over generalized sketches defined above. Specifically, constructions of composing such operations and of the free algebra generated by a sketch were elaborated.

Given a signature σ of predicate and operation markers, a notion of *σ-tree* is defined in a suitable way. Very roughly, this is a tree whose nodes are labeled by pairs (q, δ) with δ a diagram $\delta\colon\ D_q^{\text{out}} \to V$ and V a fixed pool of variables. With any σ-tree T there are correlated (i) a corresponding σ-sketch $\mathrm{Ske}(T)$ obtained by merging all diagrams and (ii) its subsketch $\mathrm{In}(T)$ obtained by merging all diagrams in the leaves.

DEFINITION 2. A *σ-specification* is a pair $\mathbf{S} = (S, X)$ with S a finitary σ-sketch and X a distinguished subobject of its carrier called *the input object* of $\mathbf{S}$. $\mathbf{S}$ is called *well-formed* (wf) if there is a σ-tree T s.t. $S = \mathrm{Ske}(T)$ and $X = \mathrm{In}(T)$.

PROPOSITION 3 (Sketch parsing). *There is an algorithm either extracting from any given specification $\mathbf{S} = (S, X)$ a tree $T = \mathrm{Tree}(\mathbf{S})$ s.t. $S = \mathrm{Ske}(T)$ and $X = \mathrm{In}(T)$, or terminated with failure proving that $\boldsymbol{S}$ is not wf. So, the question of whether a given σ-specification is well-formed is decidable.*

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In [4] it is proved that a reduced ideal-product $\prod_\Psi^\Lambda \mathcal{X}_i$ (Λ and Ψ being, respectively, an ideal and a filter on the index set I) preserves the separation axioms T_k, $k = 0, 1, 2, 3, 3\frac{1}{2}$, iff the following, named $(\Lambda\Psi)$-condition, holds: $\forall A \in \Psi\, \forall B \notin \Psi\, \exists L \in \Lambda\, (L \subseteq A \cap B^c \wedge L^c \notin \Psi)$, which is, on the other side, equivalent to the condition that the set $(\Lambda/\equiv) \setminus \{\mathbf{0}\}$ is dense in $(P(I)/\equiv) \setminus \{\mathbf{0}\}$, where, for $A, B \in P(I)$, $A \equiv B$ iff $\exists F \in \Psi\ A \cap F = B \cap F$.

For $k = 0, 1, 2$ the inverse implication ($\prod^{\Lambda}_{\Psi} \mathcal{X}_i$ is T_k-space $\Longrightarrow \{i \in I \mid \mathcal{X}_i$ is T_k-space$\} \in \Psi$) holds independently of $(\Lambda\Psi)$-condition, while it is needed when regularity or T_3-separation axiom is considered. In particular, for $k = 1, 2$, under $(\Lambda\Psi)$-condition both implications follow directly from the fact that the so-called $\varphi_{n\text{-sep}}$ formula is 1-formula, which means: $\prod^{\Lambda}_{\Psi} \mathcal{X}_i \models \varphi_{n\text{-sep}}$ iff $|\{i \in I \mid \mathcal{X}_i \models \varphi_{n\text{-sep}}\}| = 1$ in the Boolean algebra $P(I)/\equiv$ ([5], [6]).

The generalization of the Feferman-Vaught theorem to reduced ideal-products implies that a reduced ideal-product of topological spaces preserves L_t-equivalence and provides the short proof of the Łos theorem for ultraproducts of topological spaces ([7]).

Similarly, from the generalizations of the known results from classical model theory concerning Horn sentences, it follows ([8]):

(CH) An L_t-sentence φ is preserved under reduced products of basic L_2-structures iff there exists a Horn L_2- sentence ψ such that for any basic structure $\mathcal{A}$, $\mathcal{A} \models \varphi \Longleftrightarrow \psi$ (in notation, $\varphi \overset{b}{\Longleftrightarrow} \psi$);

A disjunction of Horn L_t-sentences is preserved under powers of basic structures.

The last propositions and there consequences could be of use in proving some preservation theorems.

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[9] M. Ziegler, *A language for topological structures which satisfies a Lindström theorem*, ***Bulletin of the American Mathematical Society***, vol. 82 (1976), pp. 586–570.

► ZARKO MIJAJLOVIĆ, *Expansions of countable models*.
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Works of Kueker, Reyes, Barwise, Makkai and others, show that certain sets of model-theoretic objects related to a countable model $\mathbf{A}$, as $\operatorname{Aut}\mathbf{A}$ for example, behave as analytic subsets of the Cantor discontinuum. Based on the properties of Lindenbaum algebras we show that the set of all countable expansions of a countable model $\mathbf{A}$ to a Σ^1_1 theory T of $L_{\omega_1\omega}$ has the same property. For example, the following is proved for a class of second order sentences:

Theorem. *Let* $\mathbf{A}$ *be a countable model of a countable first-order language L, and let L' be a countable first-order expansion of L. If φ is an infinitary Σ^1_1 sentence in L' of the form $\exists P_1 \exists P_2 \ldots \psi(P_1, P_2, \ldots)$, where $\psi(P_1, P_2, \ldots)$ is an $L''_{\omega_1\omega}$ sentence, where $L'' = L' \cup \{P_i | i \in \omega\}$, P_i, $i \in \omega$ are new predicate symbols, then the number of expansions of the model* $\mathbf{A}$ *to L' that are models of φ satisfies* CH, *i.e., this number is either $\leq \aleph_0$, or it is equal to $2^{\aleph_0}$.*

The proof of this theorem is based on the properties of analytic subsets of the Cantor space, and a variant of Suslin operation.

As an example we obtain Kueker's Theorem: If $\mathbf{A}$ is a countable model of a countable language L, then $\operatorname{Aut}\mathbf{A}$ satisfies CH. To see this let $L(F) = L \cup \{F\}$, where F is a unary function symbol, and T be the theory of $L(F)$ which states that F is an automorphism in respect to symbols of L. Then obviously there is a one-to-one correspondence between expansions of $\mathbf{A}$ to $L(F)$ that are models of T, and automorphisms of $\mathbf{A}$. Therefore $|\operatorname{Aut}\mathbf{A}|$, according to the stated theroem, satisfies CH.

In a similar way as in the preceding examples, one can show that the set of all prime ideals of a countable commutative ring also satisfies CH. In other words, the Zariski space of a countable commutative ring satisfies CH.

As the last example, it is easily deduced from the above theorem by designing an appropriate sentence φ, Kueker-Reyes Theorem:

If $\mathbf{A}$ *is a countable model in a countable language* L *and* $P \subset A$, *then the set* $\{Q \subset A | (\mathbf{A}, Q) \cong (\mathbf{A}, P)\}$ *satisfies* CH.

The above technique can be used in other situations, too. For example, I. Farah gave a new short proof of the Morley theorem on the number of countable models, based on Silver's result on the number of classes of an analytic equivalence relation.

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In this paper we answer a question of D. Monk, asked in [1], about the characterization of Galois extensions of Boolean algebras.

Definition. Let B be a Boolean algebra. Automorphisms $f, g \in \operatorname{Aut} B$ are strongly distinct if for every $b \in B^+$ there exists $s \in B$, so that $f(s) \cdot b \neq g(s) \cdot b$.

Definition. Let $C < B$. B is Galois extension of C if B is a finite extension of C, and there exists a group G of strongly distinct automorphisms from $\operatorname{Aut}_C B$ so that $\operatorname{Fix} G = C$.

Galois extensions are relatively complete.

One direction of the following theorem was proved in [1], and the question about the other direction was asked.

Theorem. *Let* $C < B$. B *is Galois extension of* C *iff* B *is a finite relatively complete extension of* C *having a reduced set of generators* $\langle u_1, \dots, u_k\rangle$ *over* C *such that* $\operatorname{ind} p(u_i) = 1$, $i \leq k$.

Corollary. *Let* $C < B$. B *is a Galois extension of* C *iff* $B \cong C \oplus 2^k$, $k \in N$.

We also define a weaker notion of pseudo-galois extensions, and in the attempt to give their characterization we end up with some open questions on transitive embeddings of permutation groups, or equivalently on the existence of irreducible polynomials with the same Galois group.

[1] J. D. Monk, *Automorphism groups*, ***Handbook of Boolean algebras*** (J. D. Monk), vol. 2, North-Holland,1989, pp. 517–546.